# Simulation of Aerodynamically Generated Sound Using Hybrid Aeroacoustic Method

Johan Hammar
KTH Royal Institute of Technology, Centre for ECO2 Vehicle Design, KTH Aeronautical and Vehicle Engineering, Teknikringen 8, SE 10044 Stockholm, Sweden, jhammar@kth.se

Creo Dynamics AB, Westmansgatan 37, SE 58216 Linköping, Sweden.

Ciarán J. O'Reilly
KTH Royal Institute of Technology, Centre for ECO2 Vehicle Design, KTH Aeronautical and Vehicle Engineering, Teknikringen 8, SE 10044 Stockholm, Sweden

Gunilla Efraimsson
KTH Royal Institute of Technology, Centre for ECO2 Vehicle Design, KTH Aeronautical and Vehicle Engineering, Teknikringen 8, SE 10044 Stockholm, Sweden

**Summary**
A numerical method to simulate aerodynamically generated sound and its propagation is presented in this paper. The transient flow field solution is established using a compressible 2D Navier Stokes solver. The source terms are then defined by using Howe's vortex sound aeroacoustic analogy and are evaluated from the flow solutions. The propagation of acoustic waves from these sources is then performed using the wave expansion method (WEM). This is a discretization method suitable for solving wave propagation through inhomogeneous potential flows. The method is tested on a flow of a rectangular open cavity. The flow conditions are a free stream Mach number of M=0.5 and Reynolds number of Re=1500.. The numerical results are compared to experimental and numerical results from other studies of the same configuration.

## 1. Introduction

In the recent years, the possibility to preform large-scale computations has enabled the use of aeroacoustic analogies for a broader range of applications. This is since a larger portion of the turbulent flow field can be resolved this gives the opportunity to compute the time-dependent aeroacoustic sources for higher Reynolds number flows. The aeroacoustic analogy of Lighthill [1] has since its introduction been the starting point for a major part of flow induced noise computation. Since its derivation, other analogies based on the one of Lighthill have also been proposed. For instance Curle's equation [2], where the effect of solid surfaces is included in the solution and is accounted for by sources introduced on the surfaces. Curle's solution has then been extended to account for the effect of source motion in the analogy by Ffowcs-Williams and Hawkings [3]. Vortex based analogies have also been formulated from Lighthill's analogy by for instance Powell and Howe [4, 5]. These might give a less extended source region than the sources in Lighthill's analogy and could therefore present an advantage.

The most common use for these aeroacoustic sources is to assume that the sources are compact and then calculate the propagated sound waves with an integral method using a free-field Green's function. Another method proposed by Oberai [6] is the variational form of Lighthill's analogy which has enabled the use of Finite Element Formulations for solving an inhomogenus wave equation in the form of a boundary value problem. The advantage of this is that the propagation can be evaluated for complex geometries where the scattering and background flow effects can be accounted for in the numerical solution.

One efficient discretization for solving acoustic propagation through an inhomogeneous flow is the Wave Expansion Method (WEM). WEM is based on the Green's function discretization that was firstly developed by Caruthers et al [7]. The WEM discretization has recently been further developed by O'Reilly [8], Liu [9] and others who have shown it to be highly efficient in solving acoustic wave propagation. The method has been shown to give accurate solutions down to a spatial resolution of two points per wave length. The aim of this work is to take the first steps towards evaluating the possibility of using the Wave Expansion Method for the propagation of sources defined by a Vortex sound based aeroacoustic analogy.

## 2. Theoretical background

### 2.1. Aeroacoustic analogy

Powell formulated an aeroacoustic analogy which highlights the significance of vorticity as an acoustic source. In this formulation which is based on the total enthalpy, the Lamb vector, L=(ω×v), acts as a source where v is the velocity vector and ω is the vorticity vector. The vorticity based formulation can have the advantage in that the source region can be less geometrically extended than the source region from Lighthill's analogy. This would mean that a smaller region of the flow would have to be used to evaluate the aeroacoustic sources.
Howe further showed that for low Mach numbers this can reduce to a wave equation as

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\boldsymbol{B} = \mathrm{div}(\boldsymbol{\omega} \times \mathbf{v}) \quad (6)$$

where the total enthalpy, B, relates to pressure as

$$p(\mathbf{x}, t) = \rho_0 B(\mathbf{x}, t) \quad (7)$$

$c_0$ and $\rho_0$ is the farfield speed of sound and density. In this case, the flow simulations will be used to compute the Lamb vector in the source region. This will then be used as a volume distributed source in the propagation. When the acoustic analogy is used in combination with flow simulations this is usually the aeroacoustic source term that would be computed from the flow solver.
The distributed sources can then be propagated by integrating the convolution of the source with a Green's function over the source region. This is very convenient when the free field Green's function can be used and the propagation is not affected by the scattering of surfaces. When this is not the case, a tailored Green's function has to be used. This is usually much more complicated or impossible to solve analytically and therefore it is often more useful to solve the propagation with a volume or surface based numerical method such as BEM, FEM or WEM.

### 2.2. Wave expansion method

The Wave Expansion Method (WEM) is a physically based discretization method that can be used for solving wave propagation in the frequency domain.

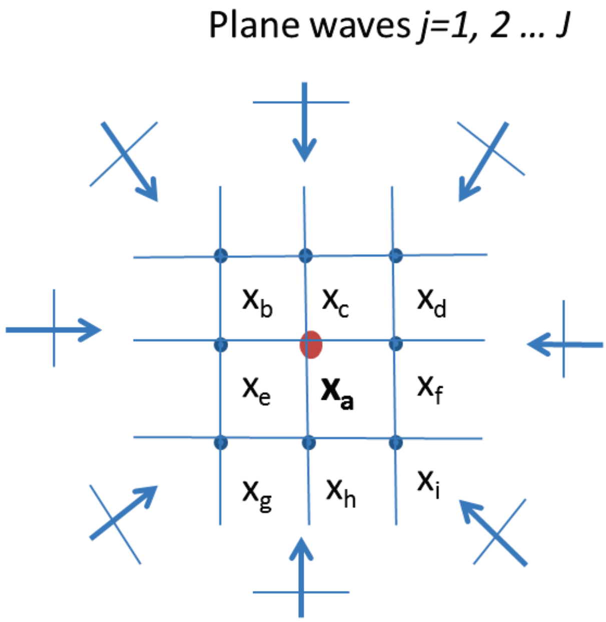


Figure 1. Stencil used in the Wave Expansion Method discretisation.

#### 2.2.1. Discretization

A schematic of the discretization in WEM is shown in Figure 1. When solving for pressure, the pressure at node a is approximated by a superposition of fields defined by $J$ plane waves with amplitude $\gamma_j$ and direction vector $\boldsymbol{\alpha}_j$. The pressure at node a is there by described

$$p_a = \sum_{j=1}^{J} \gamma_j e^{i\kappa \boldsymbol{\alpha}_j \cdot \mathbf{x}_a} = \mathbf{h}_a \boldsymbol{\gamma} \quad (9)$$

where

$$\mathbf{h}_a = \left[e^{i\kappa\boldsymbol{\alpha}_1\cdot\mathbf{x}_a} \quad e^{i\kappa\boldsymbol{\alpha}_2\cdot\mathbf{x}_a} \quad \ldots e^{i\kappa\boldsymbol{\alpha}_j\cdot\mathbf{x}_a}\right]$$

and

$$\boldsymbol{\gamma} = \left[\gamma_1 \ \gamma_2 \ \ldots \ \gamma_J\right]^T$$

The pressure at the neighboring nodes are then approximated by the same waves given by

$$\mathbf{p}_{nb} = \mathbf{H}\boldsymbol{\gamma} \tag{10}$$

where

$$\mathbf{p}_{nb} = \left[p_b \ p_c \ p_d \ p_e \ p_f \ p_g \ p_h \ p_i\right]^T$$

and

$$\mathbf{H} = \left[\boldsymbol{h}_b^T \ \boldsymbol{h}_c^T \ \boldsymbol{h}_d^T \ \boldsymbol{h}_e^T \ \boldsymbol{h}_f^T \ \boldsymbol{h}_g^T \ \boldsymbol{h}_h^T \ \boldsymbol{h}_i^T\right]^T$$

The number of plane waves does not have to correspond to the number of neighboring nodes. Thus when the number of plane waves are more than the number of neighboring nodes the system is underdetermined. The wave's amplitudes are then calculated by premultiplying with the Moore-Penrose pseudo-inverse of H.

$$\boldsymbol{\gamma} = \mathbf{H}^{+}\mathbf{p}_{nb} \tag{11}$$

The pressure at node a can then be related to the pressure at the neighbouring nodes as.

$$p_a = \mathbf{h}_a\mathbf{H}^{+}\mathbf{p}_{nb} \tag{12}$$

As this procedure is performed for each node in the computational domain a system of equations in the form of equation (13) can be assembled.

$$\mathbf{Kp} = \mathbf{Q} \tag{13}$$

Where K is an unsymmetrical and sparse stiffness matrix and Q will act as a source term.

### 2.2.2. Introduction of sources

If multipole point sources are introduced, solving for instance the inhomogeneous Helmholtz equation, with WEM discretization, the correct amplitude of the resulting pressure field might be difficult to achieve with this kind of wave based method. However a robust way to introduce a point monopole source was shown by Liu [9], where the point source was distributed to the neighboring nodes using free field Green's functions. Given the stencil in Figure 2 the pressure at node 1 from a source at node s can be

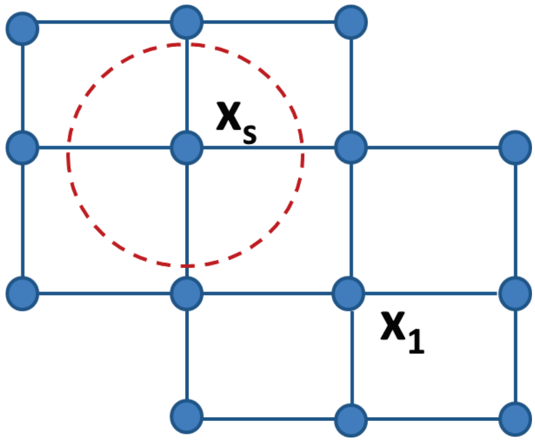


Figure 2. Stencil used for evaluation of a point source at node s.

evaluated using a Green's function. The pressure in node 1 is then formulated as

$$p_1 = \boldsymbol{h}_1\boldsymbol{\gamma} + q \tag{14}$$

where q is a monopole source

$$q = i\omega\rho_0 Q_s G(x_1/x_s) \tag{15}$$

as described in Sec. 2.2.1. the pressure at the neighboring nodes are included through

$$\boldsymbol{p}_n = \boldsymbol{h}_n\boldsymbol{\gamma} + \boldsymbol{Q_1} \tag{16}$$

where $Q_1$ is the source distribution at the corresponding nodes evaluated with the green's function

$$\boldsymbol{Q_1} = i\omega\rho_0 Q_s G(x_1/x_s) \tag{17}$$

Since the solution is not given at the source node this node is left out from the evaluation at node 1. This gives the following expression for the pressure in node 1,

$$p_1 - \boldsymbol{h}_n\boldsymbol{H}_n^{+}\boldsymbol{p_n} = q - \boldsymbol{h}_n\boldsymbol{H}_n^{+}\boldsymbol{Q_1} \tag{18}$$

In a similar fashion the Green's function of a multipole source can be introduced. For a monopole the Green's function in 2D is

$$G^{2D}(\boldsymbol{x}, \boldsymbol{x}_0) = \frac{-i}{4}H_0^{(2)}(k|\boldsymbol{x} - \boldsymbol{x}_0|) \tag{19}$$

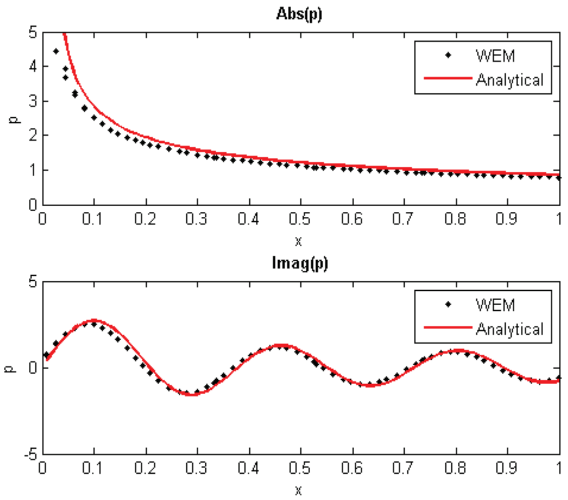


Figure 3. Acoustic pressure from a dipole point source in a 2D circular domain. The x-axis corresponds to the strongest radiationg direction of the dipole.

where $H_0^{(2)}$ is the zeroth order Hankel function of the second kind, i=$\sqrt{-1}$, $k$ is the wave number and $\mathbf{x}$ the location of the receiver. For a dipole source the resulting Green's function is

$$\partial_m G^{2D}(\boldsymbol{x},\boldsymbol{x}_0) = \frac{-ik}{4} cos\theta \, H_1^{(2)}(k|\boldsymbol{x}-\boldsymbol{x}_0|) \tag{20}$$

Here $H_1^{(2)}$ is the first order Hankel function of second kind. $\theta$ is the angle with regard to the dipoles strongest radiation direction.

### 2.2.3. Propagation of sound with mean flow

The propagation of acoustic waves in an irrotational, steady, homentropic flow can be described by the acoustic potential $\phi$ and is governed by.

$$\frac{1}{\bar{\rho}}\nabla\cdot(\bar{\rho}\nabla\phi) - (i\omega + \bar{\mathbf{u}}\cdot\nabla)\left(\frac{1}{\bar{c}^2}(i\omega + \bar{\mathbf{u}}\cdot\nabla)\phi\right) = 0 \tag{21}$$

where $\bar{\mathbf{u}}, \bar{\rho}, \bar{c}$ are the local mean velocity, density and speed of sound. The pressure and velocity are then related to the acoustic potential as $p = -i\omega\bar{\rho}\phi - \bar{\rho}\bar{\mathbf{u}}\nabla\phi$ and $\mathbf{u} = \nabla\phi$. If the mean flow gradients are neglected, Equation 21 can be formulated as

$$\nabla^2\phi - (\mathbf{M}\cdot\nabla)^2\phi - 2ik\mathbf{M}\phi + k^2\phi = 0 \tag{22}$$

where $\mathbf{M}$ is the local Mach vector, k is the local wave number. Considering these to be constant over the stencil used at a point $\mathbf{x}_a$, the fundamental plane wave solution may be determined by solving for the roots of the characteristic equation.

$$k = \pm\frac{\omega}{c(1\pm\mathbf{M}\cdot\boldsymbol{\alpha})} \tag{23}$$

where ω is the angular frequency and α is an arbitrary unit vector in the direction of the plane wave propagation. These waves can then be used in the wave expansion method [7, 8].

## 3. Flow results

The acoustic field caused by the flow over a 2D open cavitiy has been investigated numerous times over the past 50 years. More recent work that

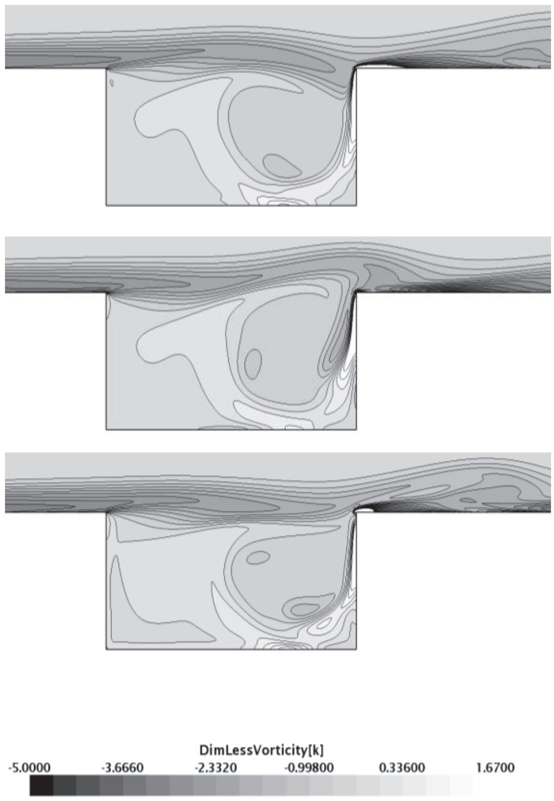


Figure 4. Contours of dimentionless vorticity in the range ωD/U=-5 to 1.67 for three consecutive time steps. Where U is the freestream velocity.

resembles the geometrical setup used here was performed by Rowley et.al [10], where numerical simulations for a range of cavity configurations and flow conditions where studied. Depending on the depth to length ratio, the speed of the incoming flow and the boundary layer thickness upstream of the cavity, the flow in a 2D open cavity is usually divided into two different flow modes. The first one is referred to as shear layer mode. In this mode vorticies are rolling up at the leading edge of the cavity. The vorticies are then convected downstream by the main flow. As this vorticial flow interacts with the trailing edge of the cavity an acoustic wave forms and propagates towards the leading edge. This acoustic wave then interacts with the leading edge shear layer creating a feedback mechanism when vorticity once again rolls up at the leading edge. Due to this feedback mechanism it is crucial that the flow simulations include compressible effects to capture this interaction of the trailing and leading edge. If the length to depth ratio of the cavity is increased the flow will enter a wake mode behavior. In this mode a vortex is formed and will start to grow inside the cavity which does not have the same acoustic feedback mechanism.

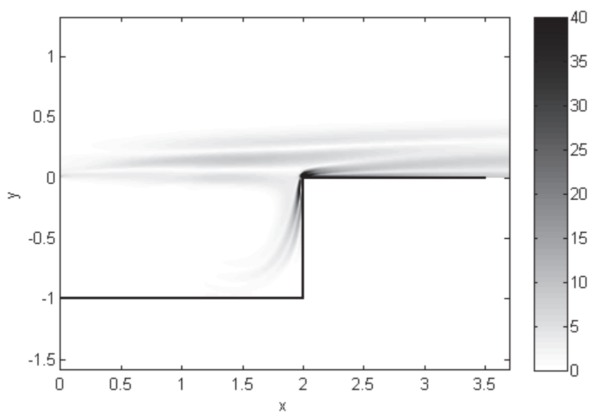


Figure 5. The absolut value for the divergence of the Lamb vector for the first tonal component of the cavity flow nondimensionalised by div(L)*D^2/U^2.

For this work a 2D cavity at a Reynolds number of Re=1500, Mach number of M=0.5 and length to depth ratio of L/D=2 is chosen as a test case. Under these conditions the cavity flow can be described by a shear layer mode. The flow simulations are performed in the commercial flow solver package CCM+. To capture the delicate acoustic coupling shear layer interaction with the trailing edge a grid of 440,000 nodes has was used for the 2D CFD simulations. Figure 4 show the dimensionless vorticity in the cavity at three consecutive times. The shear layer interaction with the trailing edge is the key noise source as strong vorticies will form on both sides of this edge. The flow in the cavity has a very strong periodic behavior. In the flow simulations the first tonal component of the cavity flow was at a Strouhal number, $St=fL/V = 0.215$, where $f$ is the frequency, $L$ is the cavity length and $V$ is the free stream velocity. Due to this strong periodic behavior the only frequency considered for the propagation in this paper corresponds to this first cavity tone. After the flow field was statistically converged to a satisfying periodic behavior, the aeroacoustic sources were saved for a total of 2000 timesteps. FFT was then used to transform the sources into the frequency domain. Since the grid requirements for CFD are much higher than the grid needed to resolve the acoustic waves near the source region, a coarser acoustic mesh of the cavity with ~60000 nodes was generated for the acoustic propagation. The aeroacoustic sources were then interpolated onto the acoustic nodes using linear interpolation. Figure 5 shows the absolute divergence of the lamb vector given from the CFD. The obvious sources appear to be the shear layer passing over the cavity and interacting with the trailing edge of the cavity.

## 4. Aeroacoustic results

The equivalent sources for the main cavity tone frequency are propagated in a domain with a circular outer boundary located 100 cavity depths from the leading edge of the cavity. At this boundary a non-reflective boundary condition is imposed. To implement non-reflective outer boundary conditions is not always easy for volume discretizing methods. In the WEM this can be enforced by only considering the plane waves in the outgoing directions. A uniform background flow of Mach=0.5 is used for the simulations. Since the WEM will incorporate the volume distributed sources as point sources, each nodes equivalent source will be evaluated using the area of the corresponding faces. The acoustic pressure in figure (6) shows a forward directivity of the first cavity tone which correspond previous literature and the CFD solutions.

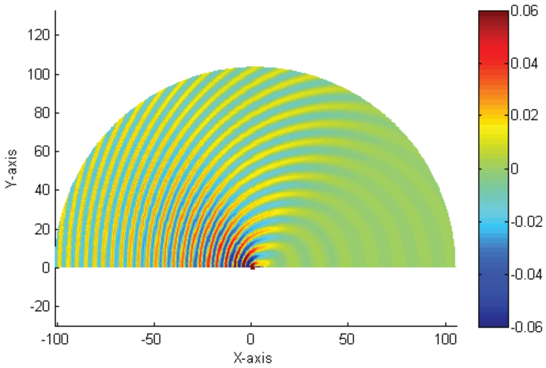


Figure 6. Real part of the acoustic pressure, normalized by the dynamic pressure, for the first cavity tone.

## 5. Conclusions

In this paper a method using vortex based aeroacoustic sources evaluated in CFD together with a plane wave based discretization method for the wave propagation is proposed. The discretization is proven to give accurate solutions at very few points per wave length and may therefore provide an efficient method for aeroacoustic computations in frequency domain. The CFD results of the vorticity in the source region compare well with previously presented results given from literature. In this first step methods for introducing sources of monopole and dipole type in the WEM have been proposed and compared to analytical solutions with good agreement. There is however still a need to investigate the coupling of the aeroacoustic sources from the CFD solution to the propagation solver.

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