Inverse estimation of the elastic and anelastic properties of anisotropic foams - study of the static/dynamic separation

Jacques Cuenca[1,2], Christophe Van der Kelen[1], Peter Göransson[1]
[1] Marcus Wallenberg Laboratory for Sound and Vibration Research, KTH Royal Institute of Technology, Teknikringen 8, SE-10044 Stockholm, Sweden.
[2] Siemens Industry Software, Interleuvenlaan 68, B-3001 Leuven, Belgium.

Summary
This paper investigates the modelling and characterisation of the porous frame of anisotropic open-cell foams. The main objective is to find a suitable model for describing the elastic and anelastic properties of the material by making as few assumptions as possible. The proposed model is based on a fractional differential equation, taking into account the deformation memory of the material in a versatile and compact manner. In the frequency domain, this results in an augmented Hooke's law, where the stiffness matrix of the porous frame consists of a superposition of a fully-relaxed, frequency-independent elastic part, and a dynamic, frequency-dependent anelastic part. In order to estimate the properties of the material and to determine if the elastic and anelastic parts share the same material symmetry, two separate experiments are performed. A static photometry setup is designed, where a cubic sample of material is compressed along each of the three directions of space while the deformation is recorded on the four exposed faces. Furthermore, a dynamic measurement of a set of transfer functions between each pair of opposed faces of the sample is performed. The characterisation methodology consists of an inverse estimation of the parameters of the model. This is acheved by replicating each experiment as a finite element simulation and fitting the model by using an optimisation algorithm. The static and dynamic observations serve as a basis for discussing the independence of the elastic and anelastic properties of the material.

PACS no. xx.xx.Nn, xx.xx.Nn

1. Introduction

Porous foams exhibit varying degrees of anisotropy as a result of their manufacturing processes. This can be directly observed in the geometry of the microstructure and influences the macroscopic properties of the solid phase of the foams. Isotropic models may be sufficient for certain applications, especially for describing sound absorption. However, recent studies reveal that relatively low degrees of anisotropy can have a significant impact on the behaviour of structures involving porous materials such as multilayer systems [1, 2].

The morphology of the porous microstructure is partly determined by its chemical composition, which also governs the macroscopic relaxation properties of the material. In order to account for such relaxation properties in a realistic manner, a causal representation of the deformation memory mechanisms of the material must be used. A wide range of stress-strain models [3] satisfy the causality requirements and enable the description of the behaviour of materials ranging from soils to polymers [4, 5]. A common aspect between these models is that in the frequency domain the stress-strain relations take the form of an augmented Hooke's law. In the latter, the stiffness moduli of the material consist of a constant term describing the elastic, fully-relaxed deformation, and a frequency-dependent term describing the anelastic deformation.

Although anisotropy and relaxation properties may appear to be uncorrelated as they respectively relate to spatial symmetry and to time, they exhibit a rather intriguing interconnectedness. In fact, no intrinsic requirement exists for the elastic and anelastic properties to share the same type or orientation of anisotropy.

The present paper proposes an inverse methodology for the estimation of the elastic and anelastic properties of anisotropic open-cell foams. This is illustrated in the case of a melamine foam by separately using static and dynamic observations. The differences between the material properties obtained from these two observations are discussed as well as the implications of the assumption of collinearity between elasticity and anelasticity.

2. Anisotropic anelastic frame model

The constitutive equations for anisotropic porous materials using Biot's formalism [6, 7] show that the properties of the solid frame are determined by the solid material that composes it and are independent from the rest of the properties of the porous material. Therefore, a partial characterisation of the frame alone is possible by performing static or quasi-static measurements. However, this will only allow to characterise the zero-frequency limit of the deformation. In the case of open-cell porous materials, it is possible to perform frequency-dependent measurements by placing the material inside a vacuum chamber in order to remove the fluid. Indeed, in the absence of fluid or at zero frequency, Biot's eqs reduce to a Hooke's law for the frame, in the form

$$\sigma_i(\omega) = H_{ij}(\omega)\varepsilon_j(\omega), \qquad (1)$$

where σ_i and ε_j denote the components of the stress and strain tensors and H_{ij} are the terms of the stiffness matrix.

In order to derive a realistic model of the frame, a causal representation of the deformation must be used. In the present work, Eq. (1) is obtained in the form of an augmented Hooke's law from a fractional differential constitutive model[3, 8, 9]. This leads to the stiffness matrix in the form

$$H_{ij}(\omega) = C_{ij} + \frac{B_{ij}\,(i\omega/\beta)^\alpha}{1 + (i\omega/\beta)^\alpha}, \qquad (2)$$

which thus appears as a superposition of two terms. C_{ij} are the components of the fully-relaxed stiffness matrix, which represents the elastic effects of the deformation. The second term represents the anelastic effects, where B_{ij} is a matrix giving their magnitude, β is the relaxation frequency and α is the order of the fractional derivative in the constitutive model.

The main advantage of using a fractional differential constitutive model is that the formulation consists of a reduced number of parameters compared to other (otherwise equivalent) models such as a superposition of discrete relaxation processes [3].

3. Material symmetry

Previous observations on melamine foams [10] motivate the assumption that C_{ij} presents orthotropic symmetry, i.e. it is determined by 9 independent moduli. Due to the variability and inherent randomness of the foam manufacturing processes, the planes of material symmetry are not necessarily parallel to the faces of the produced materials. A major contribution of the present work is to consider the orientation of the natural coordinates of the foam as unknown, which is described by 3 independent angles [8, 9, 11, 12].

In principle, the elastic and anelastic properties of the frame do not necessarily share the same material symmetries. In fact, no constitutive requirement exists for this to be the case. This implies that both C_{ij} and B_{ij} may be considered orthotropic with unknown orientation angles, for a total of 24 independent unknown parameters.

The main question that arises at this point is whether the elastic and anelastic properties of a particular foam share the same material symmetry or not.

Intuitively, since the manufacturing process results in a geometrical microstructure that is a priori common to all types of deformation, it is natural to assume that the anelastic part of the stiffness matrix is collinear to the elastic part, as

$$B_{ij} = b\,C_{ij}, \qquad (3)$$

where b is a scalar. The resulting stiffness matrix is then

$$H_{ij}(\omega) = C_{ij}\left(1 + \frac{b\,(i\omega/\beta)^\alpha}{1 + (i\omega/\beta)^\alpha}\right). \qquad (4)$$

Whether a particular material satisfies Eq. (2) or Eq. (4) requires additional considerations and/or experimental work. The present characterisation methodology is a first attempt at discussing such a distinction.

4. Characterisation method

An estimation of the properties of the frame is here proposed by means of independent static and dynamic measurements. Both estimations are based on the same general principle, presented hereafter.

4.1. General methodology

An inverse estimation methodology is used, which consists in fitting the model onto measured data. The set of model parameters yielding the best fit are then the estimated properties of the material. The parameter search is formulated as an optimisation problem where the objective function, i.e. the function to be minimised, is defined as a measure of the distance between the model and the experiment. This problem is solved using an optimisation algorithm, namely the globally convergent method of moving asymptotes [13].

In the present case, the setup consists of a cubic sample of foam subjected to a static or dynamic load. A set of displacements or acceleration transfer functions are respectively retrieved. The experimental setup is replicated in a finite element model, using the stiffness matrix defined above. This numerical model is updated within the optimiser until the objective function stabilises below a prescribed tolerance.

4.2. Elastic part of the stiffness matrix

Figure 1 represents the setup used for the static characterisation. The cubic sample of porous material is compressed under the action of a constant force and the resulting displacement is measured at N points on the four visible faces of the sample by means of a 3D optical technique [14]. This is repeated with the sample oriented in the 3 directions of space.

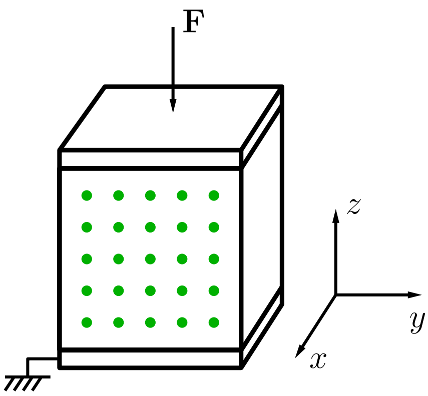

Figure 1. Setup used for the static measurements. A cubic sample of foam compressed between two plates and the displacement on the surface is measured by using two cameras.

The experimental data thus consists of N displacement points on 4 sample faces for 3 directions of space. The objective function is formulated as [11, 12]

$$f_0(\mathbf{x}) = 1 + 10^3 \sum_{m=1}^{12} \sum_{k=1}^{N} \left| u_{mk}(\mathbf{x}) - u_{mk}^{(\exp)} \right|^2, \quad (5)$$

where $\mathbf{x}$ is the set of unknown parameters, and $u^{(\exp)}$ and u are respectively the experimental and modelled displacements. The constants 1 and 10^3 are added for numerical stability.

4.3. Anelastic part of the stiffness matrix

As a first approximation and in order to keep the number of unknowns to a minimum, the dynamic characterisation is performed under the assumption of collinearity of the elastic and anelastic parts.

Figure 2 represents the setup used for the dynamic characterisation. The cubic sample of porous material is placed between a vibrating plate on the bottom and a seismic mass on top. The purpose of the latter is to enforce a resonant behaviour in the frequency range of interest, so as to facilitate the differentiation between the different types of motion. The experimental data extracted from this setup consists of four transfer functions between the acceleration of the bottom plate and the acceleration at the corners of the seismic mass. These are respectively measured with an accelerometer and a laser vibrometer. Furthermore, the measurement is repeated with the sample oriented in the 3 directions of space.

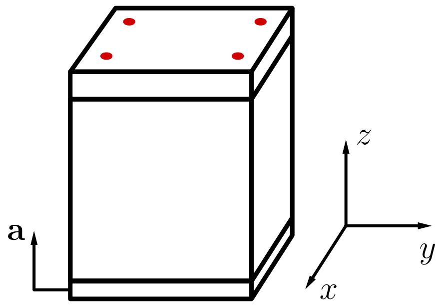

Figure 2. Setup used for the dynamic measurements. A cubic sample of foam is placed between a vibrating plate at the bottom and a seismic mass on top, where the acceleration is measured using a laser vibrometer.

The setup thus provides 12 transfer functions which contain N frequency points each. The objective function is formulated as

$$f_a(\mathbf{x}) = 1 + \sum_{m=1}^{M} \sum_{n=1}^{N} \left| \frac{h_m(\omega_n, \mathbf{x}) - h_m^{(\exp)}(\omega_n)}{h_m^{(\exp)}(\omega_n)} \right|^2, (6)$$

where $\mathbf{x}$ is the set of unknown parameters, and $h^{(\exp)}$ and h are respectively the experimental and simulated transfer functions.

5. Results

The present section summarises the results obtained from each of the two procedures for a sample of melamine foam [9, 12]. The parameters of the stiffness matrix modelled in Eq. (4) are reported in Tab. I as estimated using a static or dynamic measurement.

The table shows that the estimated properties appear to be different depending on whether the observation is static or dynamic. This suggests that a single model for the anisotropy of the material is not sufficient and therefore that in the present case the elastic and anelastic properties of melamine foam are not collinear.

However, as reported in recent investigations [11], the static setup yields uncertain results for the shear moduli and thus the parameterisation of the solution in terms of engineering constants is not unique. This arises from the fact that the deformation imposed upon the sample is purely compressional, which does not involve the shear moduli. Therefore, a direct comparison between the two columns of Tab. I is not possible. Instead, a partial comparison by using an appropriate choice of parameterisation may be done [12].

Parameter	Static estimation	Dynamic estimation	unit
E_1	124.2	448	kPa
E_2	161.8	211	kPa
E_3	219.2	170	kPa
G_{23}	(3234.8)	104	kPa
G_{31}	(4203.0)	124	kPa
G_{12}	(28.3)	101	kPa
ν_{21}	-0.35	0.445	-
ν_{13}	0.35	-0.514	-
ν_{32}	0.63	0.433	-
β	813	-	krad/s
α	0.333	-	-
b	0.296	-	-
ϕ_1	0.038	-0.118	rad
ϕ_2	-0.01	0.030	rad
ϕ_3	-0.76	0.131	rad

Table I. Estimated properties for a melamine foam. The static setup used here is insensitive to shear, therefore the corresponding moduli are presented in gray.

6. Conclusion

The present study highlights the implications of the loss of generality in the representation of the material symmetry in the frame of open-cell foams. The main question that arises is whether the elastic (static, fully-relaxed) properties share the same natural coordinates as the anelastic (dynamic) properties.

The estimated material properties differ when observed statically or dynamically, which suggests that the assumption of elastic-dynamic collinearity may oversimplify the real behaviour of the material. However, the setup used here for the measurement of static displacements is insensitive to shear, which yields a non-unique solution when expressed in terms of engineering constants.

In order to render the comparison possible, it is necessary to use a different parameterisation or to perform a static shear measurement. Moreover, in order to guarantee the feasibility of the model, a simultaneous estimation of the elastic and anelastic properties is required.

The collinearity of elastic and anelastic properties is a crucial aspect in the understanding of porous materials. It is necessary for the determination of whether a particular characterisation method yields a complete or partial representation of the material properties.

The ultimate goal in the characterisation of porous materials is to establish models determined by a minimum number of parameters. The measured material properties can then be used in methods for the prediction and analysis of the acoustic behaviour of anisotropic structures [6, 7].

References

[1] E. Lind Nordgren, P. Goransson, J.-F. Deu, and O. Dazel. Vibroacoustic response sensitivity due to relative alignment of two anisotropic poro-elastic layers. *J. Acoust. Soc. Am.*, 133(5):EL426–EL430, 2013.

[2] J.P. Parra Martínez, O. Dazel, P. Göransson, and J. Cuenca. Acoustic response modelling of multilayered structures with anisotropic porous materials. In *SAPEM 2014*, Stockholm, 2014.

[3] K. Dovstam. Augmented Hooke's law based on alternative stress relaxation models. *Computational Mechanics*, 26(1):90–103, 2000.

[4] M. Caputo and F. Mainardi. A new dissipation model based on memory mechanism. *Pure Appl. Geophys.*, 91(1):134–147, 1971.

[5] T. Pritz. Analysis of four-parameter fractional derivative model of real solid materials. *J. Sound. Vib.*, 195:103–115, 1996.

[6] N.E. Hörlin and P. Göransson. Weak, anisotropic symmetric formulations of Biot's equations for vibroacoustic modelling of porous elastic materials. *Int. J. Num. Meth. Eng.*, 84(12):1519–1540, 2010.

[7] J.P. Parra Martínez, O. Dazel, P. Göransson, and J. Cuenca. Acoustic behaviour and internal energies of multilayer systems including anisotropic poroelastic materials. *Submitted*, 2015.

[8] J. Cuenca and P. Göransson. Inverse estimation of the elastic and anelastic properties of the porous frame of anisotropic open-cell foams. *J. Acoust. Soc. Am.*, 132(2):621–629, 2012.

[9] J. Cuenca, C. Van der Kelen, and P. Göransson. A general methodology for inverse estimation of the elastic and anelastic properties of anisotropic open-cell porous materials—with application to a melamine foam. *J. Appl. Phys.*, 115(8):–, 2014.

[10] L. Jaouen, A. Renault, and M. Deverge. Elastic and damping characterizations of acoustical porous materials: Available experimental methods and applications to a melamine foam. *Applied Acoustics*, 69(12):1129–1140, 2008.

[11] C. Van der Kelen, J. Cuenca, and P. Göransson. A method for characterisation of the static elastic properties of the porous frame of orthotropic open-cell foams. *Int. J. Eng. Sci.*, 86:44–59, 2015.

[12] C. Van der Kelen, J. Cuenca, and P. Göransson. A method for the inverse estimation of the static elastic compressional moduli of anisotropic poroelastic foams - with application to a melamine foam. *Submitted*, 2015.

[13] K. Svanberg. A class of globally convergent optimization methods based on conservative convex separable approximations. *SIAM Journal on Optimization*, 12(2):555, 2002.

[14] R. Guastavino and P. Göransson. A 3D displacement measurement methodology for anisotropic porous cellular foam materials. *Polymer Testing*, 26:711–719, 2007.