# Design and Experimental Validation of a Plate with Internally Resonating Lattices for Low-frequency Vibro-acoustic Control

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**Summary**

A flexible plate and an internal resonating lattice are combined in an assembly which is characterized by high damping performance and tuned vibration attenuation at low frequencies.
The resonating lattice consists of an elastomeric frame equipped with a metallic inclusion which is designed to resonate at selected frequencies. The system achieves high damping performance by combining the frequency-selective properties of internally resonating structures, with the energy dissipation characteristics of their constituent material.
Furthermore, tuning and modifying the layout of the resonant lattice allows for tailoring of the resonating properties so that vibration attenuation is obtained over desired frequency ranges. Experimental results show the performances of the resonant assembly and suggest its potential application in vibrations and noise reduction.

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## 1. Introduction

Over the last decades, research on effective methods for suppressing noise and vibration levels has been performed both in the automotive and the aircraft industry [3]. The control of the radiated noise due to the coupling between the vibration of the flexible boundary of the considered structure and the motion of the surrounding fluid is one of the major concerns.
Increasing customer demand for improved comfort environments have prompted researchers into investigating innovative techniques to reduce noise levels. Lightweight solutions are needed which can provide global broadband noise control without compromising vehicle performance and efficiency. It is acknowledged that classical passive solutions such as acoustic blankets add considerable weight and are effective only at high frequencies.
For this reason a different passive strategy for vibration mitigation and noise suppression has been proposed with the aim to improve the control capabilities of the overall structure in the low frequency regime. This technique is based on the use of locally resonating structures embedded in an elastic plate.



In the literature it is possible to find numerous examples that clearly show the advantages associated to the use of non-conventional materials for the design of structures with outstanding vibro-acoustic properties. This is the case of composites featuring a negative stiffness phase, which can provide extreme levels of stiffness and damping [6]. In addition, the exploitation of structural effects such as topology, geometry and local resonances has led to the development of material systems with extraordinary electromagnetic and acoustic properties as in the case of metamaterials [4]. In acoustic metamaterials specifically, locally resonant phases lead to strong attenuation which is directly associated to exceptional reduction in sound transmission [2] and vibration mitigation [7].
This concept has been explored for structural bars and beams whose internally resonant behavior is the result of their coupling with periodically spaced spring-mass resonator arrays [9]. An attenuation band is generated at the resonance frequency of spring-mass absorbers connected to a primary structure. The results in [10] also show that the attenuation bandwidth can be expanded by tuning the various resonators at different frequencies within a range of interest, which suggests the potential of resonance grading as an effective strategy for broadband vibration control.
Confinement of vibrations has also been demonstrated for 2D periodic configurations. For example, inertial

amplification is exploited in [11] for low frequency effectiveness with limited mass penalties. An interesting analysis of periodic structures is presented in [5], where various kinds of waves that can propagate in the structure are discussed and the effect of local resonances of the lattice is illustrated.
This paper applies the concept of internally resonant structures to provide a plate assembly with high damping performance and noise suppression at selected frequencies bands. The system achieves high damping performance by combining the frequency-selective properties of internally resonating structures, with the energy dissipation characteristics of their constituent material. With this paradigm it is therefore possible to improve the acoustic properties of the plate by only modifying the properties of the embedded lattice thus avoiding to degrade the vibroacoustic properties of the primary structure.
The embedded lattice is a multi-degree of freedom structure consisting on a set of flexible structures periodically arranged over the surface of the plate. The lattice is characterized by frequency band-gaps in different frequency ranges, which can be conveniently tuned through the selection of the characteristic parameters of the lattice. There is a number of such parameters that define the lattice lay-out so that tunability of the dynamic behavior can lead to attenuation over desired ranges of frequency.
Following this introduction, the paper describes the considered concepts for vibration attenuation in Section 2, while their implementation and design in a plate-like assembly are presented in Section 3. Section 4 presents preliminary experimental results associated to the fully coupled system consisting in a finite-extent plate equipped with a resonant units. Conclusions and recommendations for future investigations are provided in Section 5.

## 2. Plate with resonant inclusions

The structure considered in this work is a plate equipped with a finite number of resonant inclusions periodically arranged over a portion of the plate. The primary structure is made of aluminum, the resonant unit composed by a two ligaments frame attached to an hollow cylinder containing one cylinder made respectively of liquid silicone rubber and steel. The mechanical properties of each component are summarized for convenience in Table I.
The cylinder has a diameter of 10 mm with a height of 10 mm.
By properly selecting the dynamic properties of the resonant unit it is possible to suppress the response of the primary structure at selected frequencies.

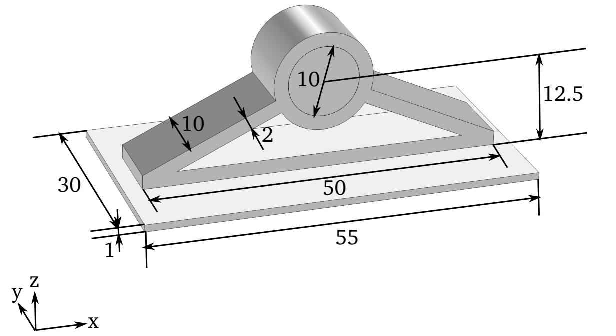


Figure 1. Unit cell of the assembly: plate + resonant unit. All dimensions in mm.

## 3. Wave propagation in two-dimensional periodic structures

By assuming that the resonant behavior of the periodic lattice is not heavily influenced by the boundary condition of the finite structure is thus possible to restrain the study of the composite structure to the sole periodic lattice attached to the plate and arranged along the x and y direction as depicted in Figure 1.
Wave propagation in the resulting 2D periodic structure is investigated through the analysis of a unit cell and the application of the Bloch theorem [1]. A schematic of the considered plate configuration and associated unit cell is shown in Figure 1.
The motion of the periodic domain, according to Bloch's theorem, may be expressed as follows:

$$\mathbf{u}\left(\mathbf{r},\mathbf{n}\right) = e^{\mu\cdot\mathbf{r}}\mathbf{u}_0 \tag{1}$$

where $\mathbf{u}$ denotes the generalized displacement of point $\mathbf{r}$ belonging to the cell at location $n$ within the assembly.
The displacement $\mathbf{u}_0$ describes the generalized displacements of a single cell while $\mu = [\mu_x, \mu_y]$ is the vector of the propagation constants. The propagation constants are complex numbers $\mu_i = \delta_i + i\varepsilon_i$ whose real and imaginary parts denote attenuation and phase constants, respectively. The propagation constants are equal to the wave-number component $k_i$ in the direction of wave propagation, multiplied by the spatial period of the domain in the corresponding direction and therefore they are non-dimensional quantities. They describe the nature of elastic waves propagating in the 2D periodic structure: purely imaginary propagation constants correspond to waves which are free to propagate, while the existence of a real part indicates that amplitude attenuation occurs as elastic waves propagate from one cell to the next.
The behavior of the unit cell can be conveniently described through a discretized equation of motion.
A general formulation for the cell's equation of motion can be expressed as:

$$\mathbf{K}_D\left(\omega\right)\mathbf{d} = \mathbf{f}. \tag{2}$$

Table I. Physical properties of the assembly.

| | Steel | Liquid Silicone Rubber | Aluminum |
|---|---|---|---|
| Young's modulus | 210 $GPa$ | 2 $MPa$ | 70 $GPa$ |
| Poisson's ratio | 0.33 | 0.4 | 0.33 |
| Mass density | 7850 $kg/m^3$ | 1100 $kg/m^3$ | 2700 $kg/m^3$ |

where $\mathbf{K}_D$ is the dynamic stiffness matrix of the cell. In Equation (2) $\mathbf{d}$ and $\mathbf{f}$ are, respectively, vectors of generalized nodal displacements of the cell and associated forces:

$$\begin{aligned}\mathbf{d} &= [\mathbf{d}_L\, \mathbf{d}_R\, \mathbf{d}_T\, \mathbf{d}_B\, \mathbf{d}_{LB}\, \mathbf{d}_{LT}\, \mathbf{d}_{RB}\, \mathbf{d}_{RT}\, \mathbf{d}_I] \\ \mathbf{f} &= [\mathbf{f}_L\, \mathbf{f}_R\, \mathbf{f}_T\, \mathbf{f}_B\, \mathbf{f}_{LB}\, \mathbf{f}_{LT}\, \mathbf{f}_{RB}\, \mathbf{f}_{RT}\, \mathbf{f}_I]\end{aligned} \tag{3}$$

where $\mathbf{d}_I$ and $\mathbf{f}_I$ denote the generalized displacements and forces internal to the unit cell.
Imposing periodicity conditions on the generalized displacements and equilibrium conditions on the generalized forces yields:

$$\begin{array}{ll} \mathbf{d}_R = e^{\mu_x}\mathbf{d}_L & \mathbf{f}_R = -e^{\mu_x}\mathbf{f}_L \\ \mathbf{d}_T = e^{\mu_y}\mathbf{d}_B & \mathbf{f}_T = -e^{\mu_y}\mathbf{f}_B \\ \mathbf{d}_{LT} = e^{\mu_y}\mathbf{d}_{LB} & \mathbf{f}_{LT} = -e^{\mu_y}\mathbf{f}_{LB} \\ \mathbf{d}_{RB} = e^{\mu_x}\mathbf{d}_{LB} & \mathbf{f}_{RB} = -e^{\mu_x}\mathbf{f}_{LB} \\ \mathbf{d}_{RT} = e^{\mu_x+\mu_y}\mathbf{d}_{LB} & \mathbf{f}_{RT} = -e^{\mu_x+\mu_y}\mathbf{f}_{LB} \end{array} \tag{4}$$

Equations (4) can be rewritten in the following matrix form:

$$\mathbf{d} = \mathbf{A}\mathbf{d}^r \;\; \mathbf{f} = \mathbf{B}\mathbf{f}^r \tag{5}$$

where $\mathbf{d}^r$ and $\mathbf{f}^r$ are the reduced vector of nodal displacements and nodal forces.
Substituting Equation (4) into Equation (2) , premultiplying the resulting Equations for its complex conjugate transpose $\mathbf{A}^H$ and assuming $\mathbf{f}_I = 0$ gives:

$$\mathbf{K}_D^r(k, f)\, \mathbf{d}^r = \mathbf{0}, \tag{6}$$

where $\mathbf{K}_D^r$ is the reduced dynamic stiffness matrix.
Equation (6) is an eigenvalue problem whose solution depends on the wavenumber $\mathbf{k}$.
The approach used to evaluate the dispersion relation of the two-dimensional periodic domain consists in setting the attenuation part of the propagation constants to zero and varying the wavenumber $k_i$ in the first Brillouin zone.
By solving the eigenvalue problem (6) with respect to frequency for a specific combinations of $k_x$ and $k_y$ in the considered range yields a set of functions

$$f = f(k_x, k_y), \tag{7}$$

which are known as phase constant surfaces and represent the dispersion characteristics of the domain. The phase constant surfaces are 2D representations of the dispersion relations for the considered periodic domain and provide a wealth of information on the dynamics of propagating waves. Frequency gaps between subsequent surfaces correspond to attenuation in all directions and therefore identify the stop bands or band gaps, typical of all periodic structures. This approach is very convenient since it allows to calculate the dispersion relation of the wave-guide along a given direction by just specifying the module of the wave-numbers $k$ and the angle of propagation $\theta$ and solving for the frequency $f$. The components of the wavenumber $k_x$ and $k_y$ are related to $k$ and $\theta$ through the following simple relation

$$\begin{aligned} k_x &= k\cos(\theta), \\ k_y &= k\sin(\theta). \end{aligned} \tag{8}$$

In case of fully reactive systems the solution of the associated eigenvalue problem is particularly simple since each terms in the matrix $\mathbf{K}_D^r$ are linear.
The presence of band-gaps can be highlighted by calculating the group velocity of the propagating waves [8].
The group velocity vector is given by

$$\mathbf{C}_g = \nabla_k \omega = \frac{\mathbf{I}}{E_{tot}}, \tag{9}$$

where $\mathbf{I}$ is the intensity vector

$$\mathbf{I} = -\frac{1}{2} Re\left(\int_\Omega \sigma \cdot u_t^*\right) dV, \tag{10}$$

and $E_{tot}$ the total energy

$$E_{tot} = \frac{1}{4}\int_\Omega \left(\rho u_t^* \cdot u_t + \varepsilon^* \cdot \sigma\right) dV. \tag{11}$$

Results for the first configuration are presented in Figure 2, which show the variation of the wavenumber $k_x$ over the 0-700 Hz range. The results obtained with the plate equipped with lattice of resonant units are compared with the case of bare plate which is considered the reference configuration. The wavenumber along the x direction show the presence of several frequency bands where waves can not propagate. The controlled configuration, represented by a red line, features three band-gaps centered at 66, 110 and 300 Hz. The third band gap presents a very interesting feature, the presence a two close modes of the frame that interact with the primary structure originating two band gaps that coalesce in a wider single band gap. To help the reader, each band-gap has been highlighted with a colored box. The presence of this attenuation band can be attributed to the impedance mismatch generated

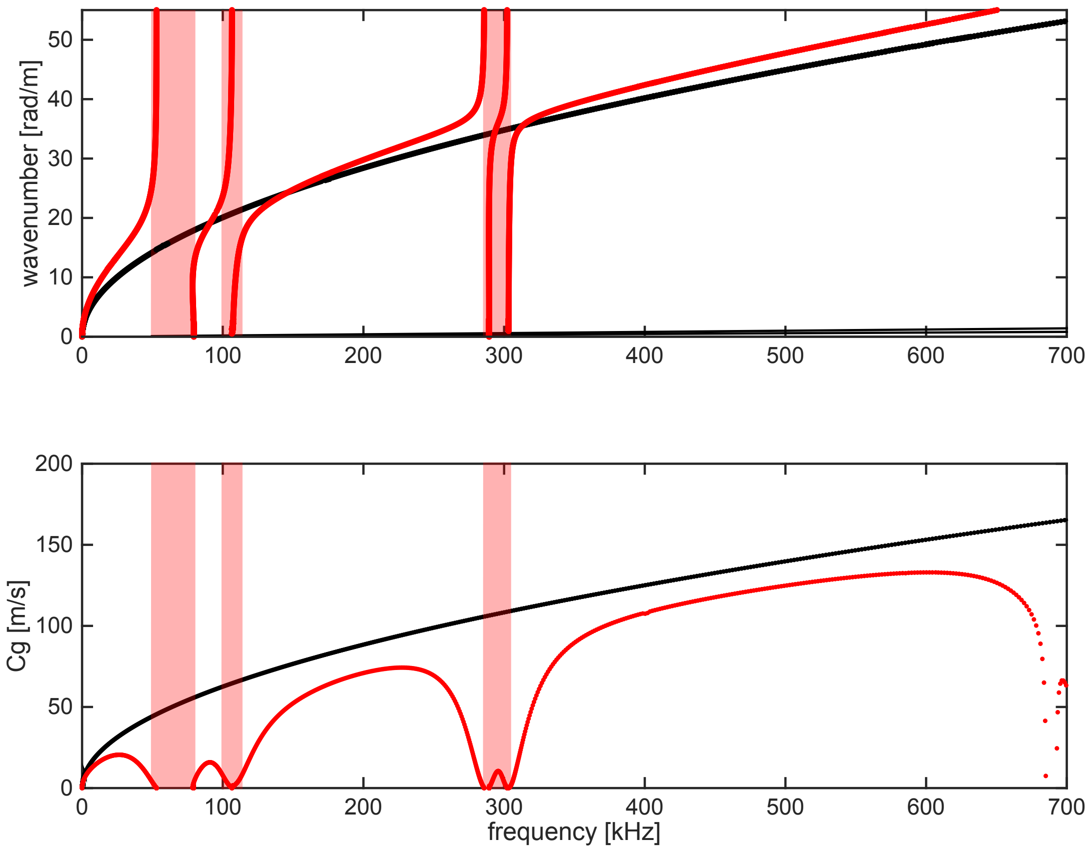


Figure 2. Dispersion relation and group velocity of the resonant unit (2 ligament LSR frame): bare plate (black dots), plate+ RU (red dot). In this simulation the thickness of the plate is 1 mm.

by the added mass and stiffness of the resonant unit. By calculating the group velocity it is possible to localize the position and better estimate the wideness of the band-gaps as clearly shown in Figure 2, band-gaps occur when the group velocity goes to zero. This fact is easily understood if one consider the classical representation of the group velocity

$$\mathbf{C}_g = \frac{d\omega}{dk}, \tag{12}$$

only valid for conservative systems. In correspondence of zero group velocity the dispersion relation becomes a straight line orthogonal to the frequency axis which is consistent with the results depicted in Figure 2.

## 4. Experimental Results

Experimental investigations are performed on a plate equipped with a single resonant unit. The plate is excited by a random signal in the 0-5000 Hz frequency band by mean of a shaker placed in the lower corner of the plate. The out-of-plane motion is measured by an accelerometer located in the opposite corner of the plate. The input and the output signal are acquired

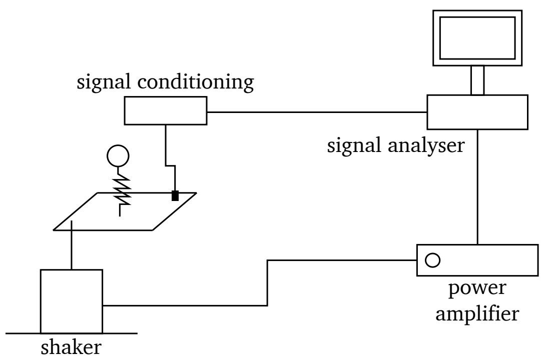


Figure 3. Sketch of the experimental apparatus.

and processed by a SigLab measurement system as depicted in Figure 3.

In Figure 4 the frequency response function of the assembly is compared to the response of the bare plate in the 0-500 Hz frequency band. By comparison it is evident that the addition of the resonant inclusion determine a visible modification in the dynamic of the system especially in the 50-100 Hz range. This results is consistent with the calculations performed in Section 3. In this frequency band the part of the

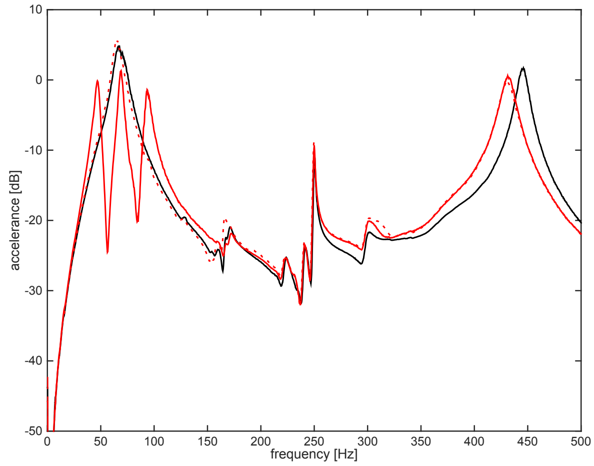


Figure 4. FRF of the assembly in the 0-500 Hz frequency band: bare plate (black solid line), plate + rubber frame + inclusion (red solid line) and plate + rubber frame (red dotted line)

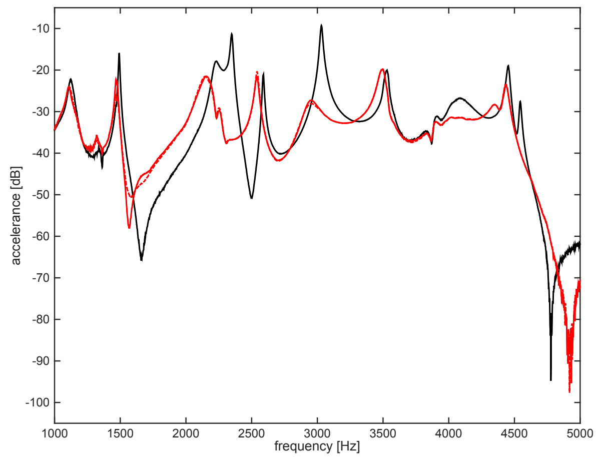


Figure 5. FRF of the assembly in the 0-5000 Hz frequency band: bare plate (black solid line), plate + rubber frame + inclusion (red solid line) and plate + rubber frame (red dotted line)

kinetic energy of the assembly is confined in the resonant unit. Moreover, it is important to point out that the rubber frame itself does not affect the dynamic of the assembly, a small shift of the resonant frequencies is observed (red dotted line) if compared to the reference measurement (black solid line). On the other hand, by adding the metallic inclusion the first resonant peak is split in two secondary resonances (solid red line).

At higher frequencies the viscoelastic properties of the liquid silicone rubber determine a suppression of the plate's response in the 2000-4000 Hz frequency range. This phenomenon is particularly evident for the peaks located at 2000 and 3300 Hz as depicted in Figure 5. At higher frequency the addition of the metallic inclusion does not affect the dynamic behavior of the assembly, above 500 Hz the response associated to plate equipped with the resonant units is exactly superposed to the response of the plate equipped with the sole rubber frame.

In practical application the modal content of the structure that one may wants to control is not precisely known. Therefore it of the paramount importance to be able to adjust the values of each resonant unit in order to achieve the best performance. With this in mind a further analysis has been performed with the aim to determine the range of frequency that can be 'covered' by the control device once the mass of metallic inclusion is varied.

In Figure 6 the reference measurement (solid black line) is compared with the response of the plate equipped with the resonant unit with a solid (red line) and an hollow (blue line) inclusion. As expected a diminution in the inclusion's mass determine a shift of the resonance toward higher values. Concretely the control resonant frequency is moved from approximately 70 to 100 Hz.

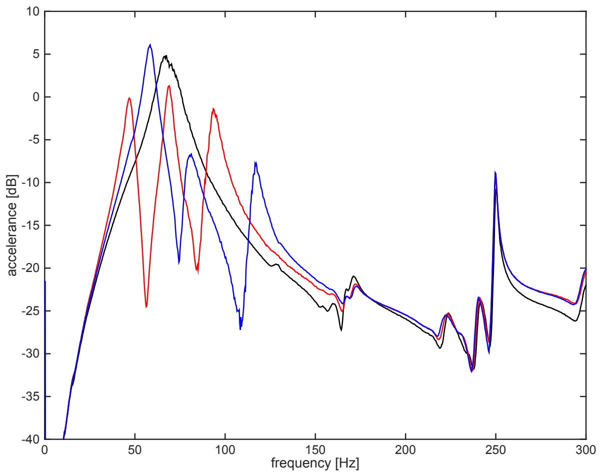


Figure 6. FRF of the assembly in the 0-500 Hz frequency band: bare plate (black solid line), plate + rubber frame + inclusion (red solid line) and plate + rubber frame + lightened inclusion (blue solid line)

## 5. Concluding remarks

Strategies have been proposed for the design of structural assemblies able to control the dynamic response of a two-dimensional flexible structures. In the present study a periodic resonant lattice is integrated into a plate to enhance the vibro-acoustic properties of the assembly.

The design of internal resonators is based on the concept of distributed vibration absorbers, which has received significant attention in the recent literature on acoustic metamaterials. The lattices are tuned to provide out-of-phase motion with respect to the primary structure, which leads to the confinement of energy and band-gap behavior that prevent the onset of vibrations. Resonant masses are added as stiff inclusions attached on the top of an elastomeric cylinder. The

lattice structures are realized with a soft rubber material that dissipates the vibrational energy captured by the resonators through structural damping.
Numerical investigations demonstrate that periodic arrangements exhibit wide band-gaps at low frequencies. The results associated to the study of the fully coupled system also show that the addition of the resonant inclusions can be beneficial to the attenuation of radiated sound over a specific frequency band.
Preliminary experimental results have shown how that addition of this device could modify the dynamic behavior of the assembly.
Further experimental studies will be performed by considering multiple resonant units attached to a finite extent plate with the aim to show the applicability of the concept to real-life structure.

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