Evaluation of Three Impedance Eduction Methods for Acoustic Liners Under Grazing Flow

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Abstract

The growth of the aircraft transportation sector coupled with the urbanization of airport surrounding areas brought with them stricter regulations regarding aircraft noise, mainly during take-off and landing, where engine noise dominates. This paper focuses on the comparison of three different indirect methods to measure the impedance of acoustic liners used in the treatment of aircraft engines. Data obtained from Finite Element simulations of a test setup considering different flow velocities were fed as input to the three methods, which attempted to find the correct impedance imposed in the numerical model for each flow speed. An analysis was made in order to evaluate the capacity of the methods to handle higher order modes in the test duct and the effect of the position of the flush-mounted microphones on the duct side-walls. It was found that, even though the methods assume plane-wave propagation in the hard-wall portions of the duct, they are able to converge to correct results if the microphones are positioned in nodal positions of the higher order modes cut-on on the frequency range under analysis.

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1. Introduction

One of the main sources of noise in an aircraft is its engine, especially during take-off and landing. Since engine noise is generally dominated by tonal frequency components, associated with blade passage frequencies, an efficient way of noise treatment in this application are acoustic liners[1].

An important property of an acoustic liner is the acoustic impedance, especially under grazing flows. Its determination is not a trivial task, and while it has been tried for decades by multiple research groups to develop a reliable impedance determination method, it remains a challenge and a topic of interest. The most common methods published in the last years are the so-called “impedance eduction methods”, that usually consist in measuring the acoustic field in a duct where a liner sample is subject to grazing flow. A numerical or analytical model is used to calculate the acoustic field for a given impedance value, which is varied in an optimization until the calculated field converges to the measured one.

Santana *et al.*[2] devised a method, hereafter called the Two-Port Method (TPM), that uses two-port matrices to describe the lined section of a duct. These analytical matrices are used in an optimization procedure from which the longitudinal wavenumbers and thus the unknown impedance are found. Elnady *et al.* [3] used the mode-matching technique to couple the acoustic fields in hard-wall and lined sections of a duct. The acoustic field is calculated and compared to the measured one until a matching impedance is found. This method will be referred to as the Mode-Matching Method (MMM).

In a previous paper [4], these two techniques were implemented, and validated by means of Finite Element Method (FEM) simulations. This validation was carried-out in different flow speeds, including no-flow and up to Mach 0.3. Both methods were able to find the imposed impedance within good tolerances. However, in a second paper [5] the methods were fed test data from a new test rig built in the Vibration and Acoustics Laboratory at Federal University of Santa Catarina, and showed difficulty converging at a few frequencies. It was later found that, among other factors, the initial guess for the optimization played an important role in avoiding convergence to incorrect, local minima in the objective function.

Several other inverse methods can be found in the literature [6, 7]. They all share the difficulties of relying in an optimization: multiple evaluations of - sometimes complex - objective functions, unreliable convergence, difficulty of setting an appropriate initial guess for the impedance - or for the wavenumbers, which can be even more complicated. Most of them also require some form of the two-microphone method [8], which brings an additional source of error to the techniques.

To overcome some of the above cited problems, Jing *et al.* [9] proposed a direct technique. In Jing's method, hereafter called Straight-Forward Method (SFM), several microphones are positioned opposite to the lined wall of a duct, and their measurements used to find the propagation constants (axial wavenumbers) and amplitudes of all propagating modes by means of Prony's numerical method. The most reliable wavenumber is then used to calculate the unknown impedance. This technique has the advantages of directly using the microphone measurements, without the need for the two-microphone decomposition, and not requiring an optimization procedure to be carried-out at each frequency, greatly reducing computational cost. As it will be seen later, it also has a few limitations, mostly regarding the number and positioning of microphones, which affect the valid frequency range for its use.

In this paper, the TPM, MMM and SFM are briefly described. Then, FEM models of a rectangular duct with a lined section are built to validate the three methods. An uniform mean flow of Mach 0.3 is added to the model. Additionally, two excitations will be tested. First, a plane wave will be imposed on the inlet. Then, all modes cut-on in the frequency range of the simulation will be imposed on the inlet. This is to test if the methods, although considering plane-waves incident on the lined section, are able to handle the higher order modes. Going forward, the position of the microphones on the duct wall will be changed from the mid-height of the duct to an arbitrary position. Again, this is to test if measuring the acoustic field in a nodal point of the first transverse mode in the duct plays any role on the impedance results.

2. Impedance Eduction Techniques

The TPM and MMM are impedance eduction methods based on measuring the acoustic field in hard-wall sections before and after a lined section of a duct with uniform mean flow. Analytical models for the acoustic propagation in the duct are compared to the measured data, and the liner impedance can be found using an optimization approach. The SFM, on the other hand, uses the well-known Prony's numerical method to adjust a series of exponential functions to the measured acoustic field along the lined section, from which the modal amplitudes and wavenumbers, and thus the impedance, are found.

All three methods assume a straight, rectangular duct, with a liner sample on a wall covering a section of length l of the duct, as shown in Figure 1.

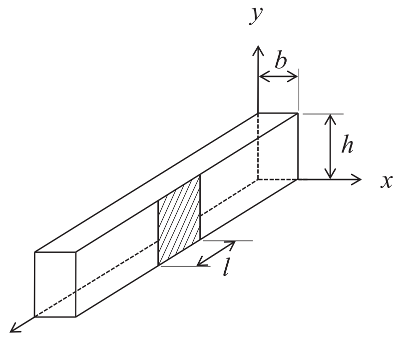

Figure 1. Rectangular duct of height h and width b, with a section of length l where an impedance Z_w was imposed.

This duct can be seen as consisting of three different ducts: a hard-wall inlet section (1), followed by a lined section (2), followed by a hard-wall outlet section (3). The acoustic field in the n-duct can be written as a summation of all Q modes that propagate inside it as given by

$$p_n = \sum_{q=1}^{Q} a_{ni}^{(q)} \Phi_{ni}^{(q)} e^{-jk_{zni}^{(q)} z} + \sum_{q=1}^{Q} a_{nr}^{(q)} \Phi_{nr}^{(q)} e^{jk_{znr}^{(q)} z} \quad (1)$$

where the indexes i and r represent the incident and reflected waves that propagate, respectively, in the z^+ and z^- direction, q is the index of the mode, $a_n^{(q)}$ its amplitude, $\Phi_n^{(q)}$ its 2D mode-shape in the xy plane, and $k_{zn}^{(q)}$ its wavenumber in the z direction, that satisfies the dispersion relation

$$k_x^2 + k_y^2 + k_z^2 = (k_0 \pm M k_z)^2, \quad (2)$$

where k_0 is the wavenumbers ω/c_0, c_0 being the speed of sound in air and M the mean flow Mach number in the z direction.

Using the hard-wall boundary conditions, the wavenumbers and mode-shapes can be easily calculated in ducts 1 and 3. In duct 2, calculation of the wavenumbers in directions x and z depend on the unknown impedance Z_w. Below plane-wave cut-off frequency in direction y, however, $k_{z2i} = k_{z2r} = 0$, and from the dispersion relation, Eq. 2, the wavenumbers in directions x and z are related by

$$\begin{aligned} k_{x2i}^{(q)} &= \sqrt{(k_0 - M k_{z2i}^{(q)})^2 - (k_{z2i}^{(q)})^2}, \quad \text{and} \\ k_{x2r}^{(q)} &= \sqrt{(k_0 + M k_{z2r}^{(q)})^2 - (k_{z2r}^{(q)})^2}. \end{aligned} \quad (3)$$

Application of the Myers [10] boundary condition for the acoustic impedance in the presence of flow to the wall at $x = b$ where the unknown impedance Z_w is applied results in

$$Z_w = jZ_0 \frac{k_0}{k_{x2i}^{(q)}} \left(1 - M \frac{k_{z2i}^{(q)}}{k_0}\right)^2 \cot(k_{x2i}^{(q)} b), \quad (4)$$

where Z_0 is the characteristic impedance of the fluid. This equation is used by the three methods to find the impedance from the wavenumbers, or to determine the wavenumbers for a given impedance value.

2.1. Two Port Matrix Method (TPM)

If only one dominating mode is considered to propagate in the entire duct, then Eq. 1 for the lined section, i.e., duct 2, is reduced to

$$p_2 = a_{2i}\Phi_{2i}e^{-jk_{z2i}z} + a_{2r}\Phi_{2r}e^{jk_{z2r}z}. \tag{5}$$

The acoustic velocity distribution can also be derived [2], and by using it with Eq. 5, expressions for the relations between pressure and velocity before (index $2in$) and after (index $2out$) the test section can be written, resulting in a transfer matrix, $[T]$, of the form [11, 2]

$$\begin{Bmatrix} p_{2in} \\ u_{2in} \end{Bmatrix} = [T] \begin{Bmatrix} p_{2out} \\ u_{2out} \end{Bmatrix}, \text{where } [T] = \begin{bmatrix} \frac{Z^+e^{-jk_{z2i}l}+Z^-e^{jk_{z2r}l}}{Z^++Z^-} & \frac{Z^+Z^-\left(e^{-jk_{z2i}l}-e^{ik_{z2r}l}\right)}{Z^++Z^-} \\ \frac{e^{-jk_{z2i}l}-e^{jk_{z2r}l}}{Z^++Z^-} & \frac{Z^-e^{-jk_{z2i}l}+Z^+e^{jk_{z2r}l}}{Z^++Z^-} \end{bmatrix} \tag{6}$$

Eq. 6 can be considered the main equation of the TPM. It is a system of two equations and two unknowns (k_{z2i} and k_{z2r}) that when solved provides the wavenumbers in the axial direction in the lined duct. Using Eq. 3 and Eq. 4, the unknown impedance is found. Of course, the method implies that pressure and velocity before and after the test section are known, which can be achieved using the two-microphone method or a more general technique [4].

There is still an important consideration: the effect of the hard-soft wall transition. Santana *et al.* suggests it to be incorporated to the present method through the addition of a new 2x2 transfer matrix, $[T_{tr}]$, which represents an infinitesimal transition element before and after the lined section. It adds four new unknowns to the analysis, resulting in a total of 6 unknowns. Since Eq. 6 is a system of two equations, two additional independent measurements have to be made, which can be achieved by means of the two-source or two-load techniques [12]. The final system consists of 6 equations and 6 variables, which can be solved, for instance, via an optimization procedure.

2.2. Mode-Matching Method (MMM)

The MMM, different from the TPM, can take into account as many modes as necessary in each duct. It still assumes that the only mode propagating towards both sides of the lined section is a plane wave mode, such as that the acoustic fields in ducts 1 to 3 can be written as

$$p_1 = a_{1i}^{(1)}\Phi_1^{(1)}e^{-jk_{z1i}^{(1)}z} + \sum_{q=1}^{Q} a_{1r}^{(q)}\Phi_1^{(q)}e^{jk_{z1r}^{(q)}z}, \tag{7}$$

$$p_2 = \sum_{q=1}^{Q} a_{2i}^{(q)}\Phi_{2i}^{(q)}e^{-jk_{z2i}^{(q)}z} + \sum_{q=1}^{Q} a_{2r}^{(q)}\Phi_{2r}^{(q)}e^{jk_{z2r}^{(q)}(z-l)}, \tag{8}$$

$$p_3 = \sum_{q=1}^{Q} a_{3i}^{(q)}\Phi_3^{(q)}e^{-jk_{z3i}^{(q)}(z-l)} + a_{3r}^{(1)}\Phi_3^{(1)}e^{jk_{z3r}^{(1)}(z-l)}. \tag{9}$$

First, continuity of pressure and velocity is assumed at the interfaces of duct 1 to duct 2, and from duct 2 to duct 3. Elnady *et al.* then apply the cited boundary conditions to end up with a system of $4Q$ equations and $4Q$ unknowns, the modal amplitudes $a_{1r}^{(q)}$, $a_{2i}^{(q)}$, $a_{2r}^{(q)}$ and $a_{3i}^{(q)}$, for q from the first mode to the Q-th mode.

The required inputs to the system of equations are the incident plane-wave amplitude in duct 1, $a_{1i}^{(1)}$, and the exit reflection coefficient $R_e^{(1)} = a_{3r}^{(1)}/a_{3i}^{(1)}$. Both inputs can be easily computed using procedures already mentioned in this paper. It is also required to know the wave numbers for each mode, in each direction, in each duct. In the hard sections, they are easily computed from Eq. 2 by applying the mentioned assumptions. In the lined section, however, they have to be computed from an expected impedance value by solving together equations 3 and 4.

From solving this system of equations for an expected impedance value, the acoustic field can be computed at any position in the ducts with equations 7 to 9. It makes sense then to use the microphones positions already used to compute $a_{1i}^{(1)}$ and $R_e^{(q)}$ to compare the calculated acoustic field to the measured one. From that, a cost function is built, and by minimizing it the unknown impedance can be found.

A detailed derivation and the full system of equations can be seen on the original paper [3].

2.3. Straight-Forward Method (SFM)

As seen in the previous sections, the TPM and the MMM rely on pressure measurements on the hard-wall sections of the duct. The MMM uses them to calculate incident pressure and reflection coefficient, and the TPM, to calculate pressure and velocity at the liner's leading and trailing edges. The SFM instead uses pressure measurements along the lined section.

If n equally-spaced microphones are positioned in the wall opposite to the liner sample, the pressure at the j-th microphone could be rewritten, from Eq. 1, as a sum of exponentials:

$$p(z_j) = \sum_{q=1}^{k} A^{(q)}e^{\mu^{(q)}z_j} \tag{10}$$

where $\mu^{(q)} = -jk_z^{(q)}$ for downstream (q odd) and $\mu^{(q)} = jk_z^{(q)}$ for upstream (q even) traveling waves, $k = 2Q$, Q still being the number of modes considered in the solution, and $A^{(q)}$ is the product of the wave amplitude by its mode-shape at the measured duct height.

Prony's method [13] is then used to find the A_0 and $\mu^{(q)}$ for the k waves taken into account. The $\mu^{(q)}$

give the wavenumbers $k_z^{(q)}$, and from here and on, the same procedure outlined in the TPM explanation is followed: calculate k_x and them the impedance from Eq. 3 and Eq. 4.

Since there are $2k$ unknowns to find, it is necessary to make measurements on at least $2k$ points, i.e., $n \geq 2k$ for a determined (equal case) or overdetermined (greater-then case) system.

3. Numerical Validation

The generation of input data for validating the three methods was carried-out using numerical models based on FEM. The model followed the geometry of the grazing flow impedance eduction test rig built at the Federal University of Santa Catarina [1, 5], i.e., duct cross-section of 0.04 by 0.10, liner sample of 0.20 m in length and covering the entire duct height (0.10 m), 4 microphones before and after the lined section for the TPM and MMM and 10 microphones along the lined section for the SFM.

The mesh, seen in Figure 2, was built with 13 elements per wavelength of the highest frequency of interest (3500 Hz). The models were solved in FFT ACTRAN 13 [14], which allows exciting the models with a mode of a specific order or all modes cut-on within a given frequency range using the Modal Duct boundary condition. As mentioned in the introduction of this paper, both cases will be used to test the robustness of the methods in the presence of higher order modes. The modes cut-on in simulated frequency range are modes of order (1,0), which cuts-on at 1700 Hz and has a nodal line on the original microphones height ($x = h/2$), and (2,0), which cuts-on at 3400 Hz and has nodal lines on thirds of the duct height.

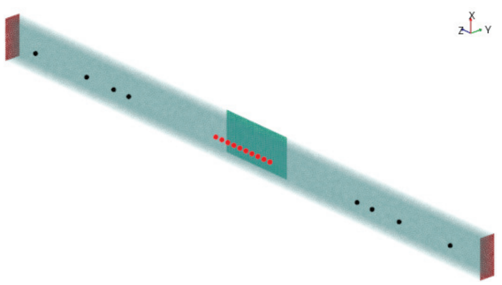

Figure 2. FEM model for the validation. The green elements on the wall represent the liner region. In red, inlet and outlet faces. The dark points are microphones for the TPM and MMM and the red points, for the SFM.

In sections 3.1 and 3.2, all microphones are positioned on the mid-length of the ducts height ($x = h/2 = 0.05$ m), as shown in Figure 2, which is a nodal line of the first transverse mode in the duct. In sections 3.3 ands 3.4, the microphones will be put at an arbitrary height $x = 0.015$, in an effort to remove them from nodal lines of the first two transverse modes.

3.1. Plane-wave excitation and Original Microphone Positions

In this section, the microphones are kept on their original positions, i.e., $x = h/2$. The Modal Duct boundary condition is set to impose a plane-wave on the inlet. The results obtained by the three methods can be seen in Figure 3.

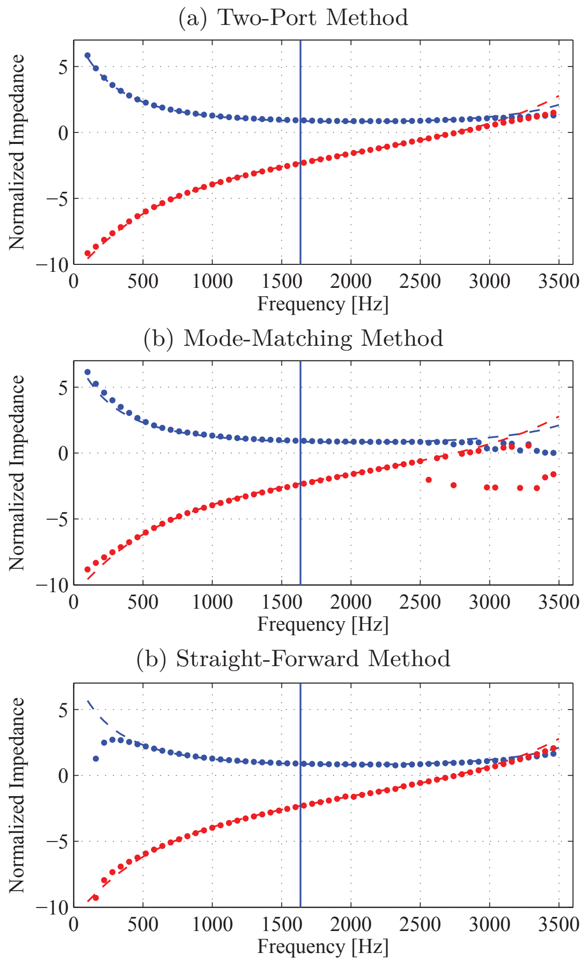

Figure 3. Plane-wave, and microphones on $x = h/2$. – – Reference (Real Part) – – Reference (Imaginary Part) • Educed (Real Part) • Educed (Imaginary Part).

It can be seen that the three methods were able to find the imposed impedance. The TPM and MMM showed unstable results for the upper frequency range, although the SFM showed good results up to 3500 Hz.

On the lower frequency range, however, the SFM diverges from the imposed impedance. This is explained by the effects associated with the hard-soft wall transitions cited in section 2.1, that are captured by the microphones closer to the liner's leading and trailing edges. This is supported by three evidences: it does not happens in no-flow results (not shown in this article for the sake of brevity); it is alleviated by using only the microphones further away from the transitions and aggravated by using microphones closer to the them; it affects only the lower-frequency range, where wavelengths are bigger.

3.2. All Modes and Original Microphone Positions

With the microphones still on $x = h/2$, now the Modal Duct boundary condition is set to impose all modes cut-on on the frequency range. The results for all three methods are presented in Figure 4.

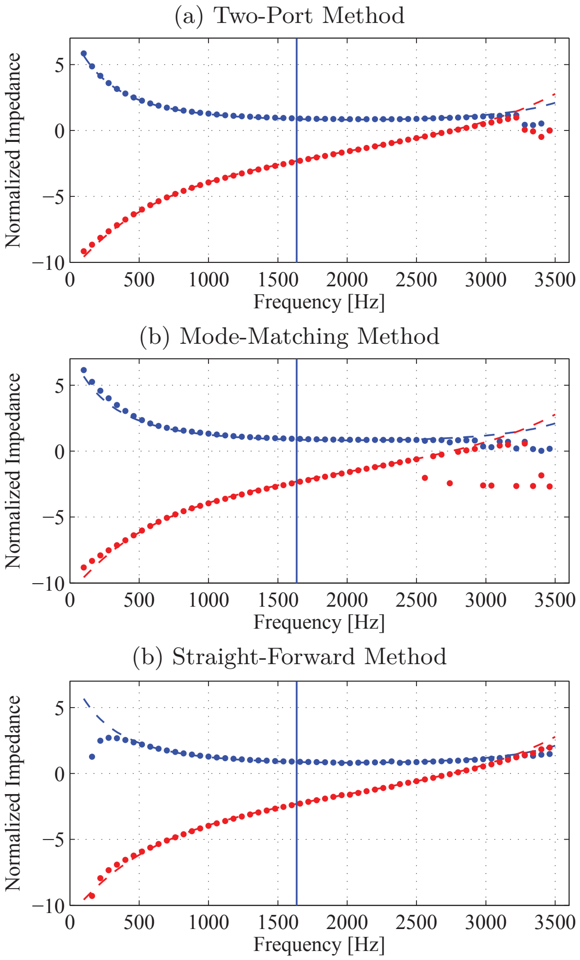

Figure 4. All modes, and microphones on $x = h/2$. – – Reference (Real Part) – – Reference (Imaginary Part) • Educed (Real Part) • Educed (Imaginary Part).

The results presented in Figure 4 show mostly the same behavior as those in Figure 3, i.e., good results at lower frequencies, but instabilities at higher frequencies for the TPM and MMM; good results at higher frequencies and divergence in lower frequencies for the SFM for reasons already mentioned.

3.3. Plane-wave excitation and Modified Microphone Positions

For this section and the next one, the microphones are removed from $x = h/2$, where there is a nodal line of the first transverse mode, and positioned at $x = 0.015m$. This section shows results for a plane-wave excitation, seen in Figure 5.

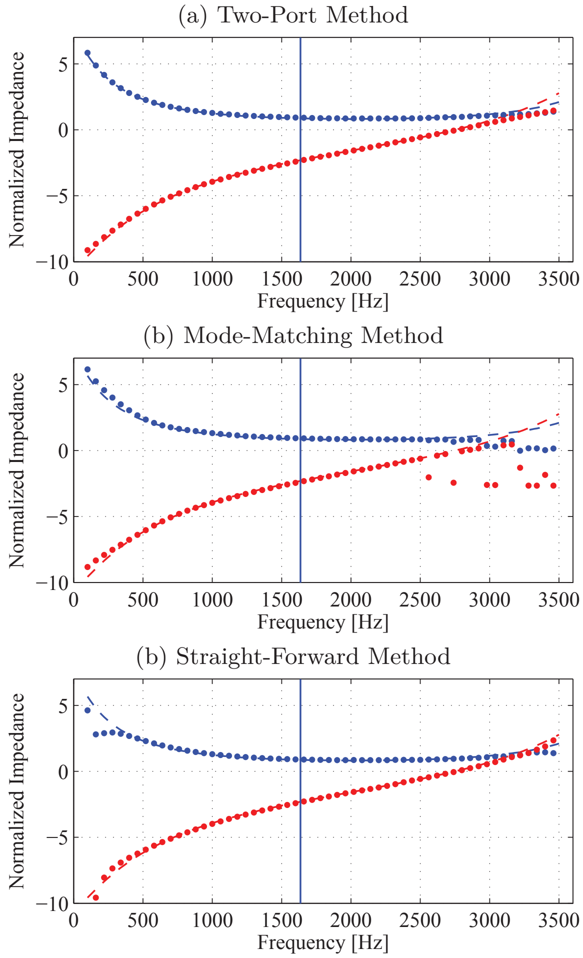

Figure 5. Plane-wave, and microphones on $x = 0.015$ m. – – Reference (Real Part) – – Reference (Imaginary Part) • Educed (Real Part) • Educed (Imaginary Part).

It can be seen that if plane-wave excitation is used, removing the microphones from the nodal line of the first transverse mode does not affects the results. In fact, the results found in this case are very similar to the two previous results.

3.4. All Modes and Modified Microphone Positions

In this section, the microphones are again positioned at $x = 0.015$ m on the duct wall. The results are shown in Figure 6. As soon as the first transverse mode cuts-on at around 1700 Hz (actually 1636 Hz because of the convection effect), all methods start to diverge from the imposed impedance, which indicates that the methods' plane-wave hypothesis loose validity.

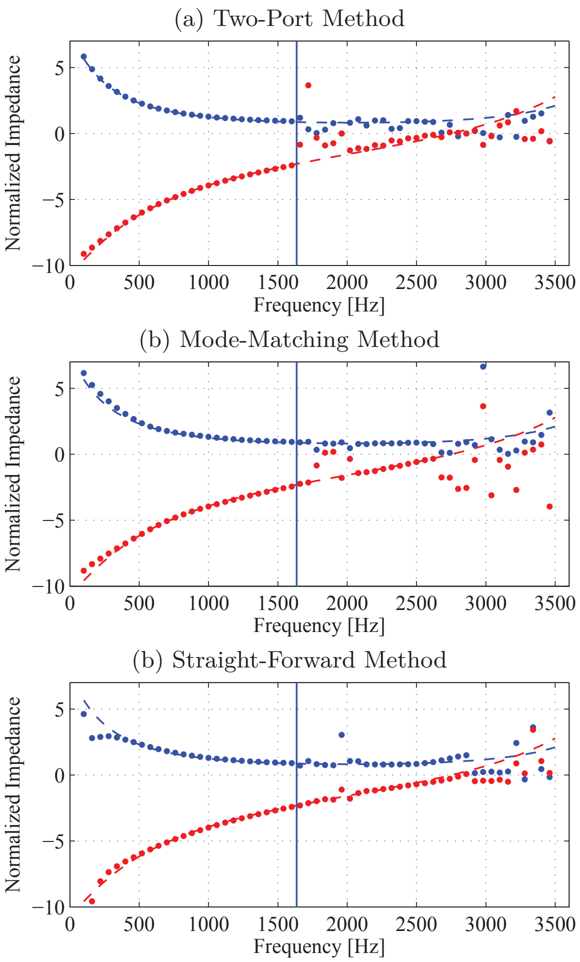

Figure 6. All modes, and microphones on $x = 0.015$ m. Reference (Real Part) Reference (Imaginary Part) Educed (Real Part) Educed (Imaginary Part).

4. Conclusions

In this paper, three impedance measurement techniques were implemented and validated by means of numerical simulation results. Since these methods assume plane-wave propagation in the hard-wall sections, it was desired to check whether they were able to find the correct results when higher order modes are present in the acoustic field. For that, the analysis frequency range was chosen in order to have two higher-order modes cut-on.

It was found that if one is able to select the microphone positions to be at the nodal line of the undesired cut-on mode, then that mode is not taken into account (because it is not measured) and the methods are able to find the impedance. If that is not the case, convergence will depend on whether the excitation is a plane-wave or if the undesired mode is also excited.

If the model is excited with a plane-wave, despite the scattering on the hard-soft wall transitions that excites higher order modes, the position of the microphones will not play any role. If, however, all modes are excited at the inlet, then they will propagate and destabilize the solution.

As a side note, care must be exercised when placing microphones too close to the liner's edges, because transition effects might contaminate the measurements and thus the solution.

References

[1] P. G. Serrano: Desenvolvimento de uma bancada de determinação da impedância acústica na presença de escoamento tangencial. Diploma Thesis. Federal University of Santa Catarina, 2014.

[2] L. D. Santana, W. De Roeck, W. Desmet, P. Ferrante: Two-port indirect acoustic impedance eduction in presence of grazing flows. 17th AIAA/CEAS Aeroacoustics Conference 2011, 2011.

[3] T. Elnady, H. Bodén, B. Elhadidi: Validation of an inverse semi-analytical technique to educe liner impedance. AIAA journal **47** (2009) 2836–2844.

[4] A. A. Medeiros, Z. Masson, P. G. Serrano, D. C. dos Reis, J. A. Cordioli: Comparison of mode-matching and two-port formulations for acoustic impedance eduction of liners under grazing flow. Forum Acusticum 2014, Krakow, 2014.

[5] A. Amador Medeiros, Z. Neves Masson, P. Giordani, J. A. Cordioli: Impedance eduction of acoustic liners under grazing flow using the mode-matching method and the two-port method. SAE Brasil International Noise and Vibration Colloquium 2014, 2014.

[6] M. G. Jones, W. Watson, D. Nark: Impedance eduction in ducts with higher-order modes and flow. Proceedings of the 15th AIAA/CEAS Aeroacoustics Conference, 2009.

[7] T. L. Parrott, W. R. Watson, M. G. Jones: Experimental validation of a two-dimensional shear-flow model for determining acoustic impedance. Tech. Rept. NASA Technical Paper 2679, NASA, 1987.

[8] I. Standard: 10534-2, acoustics: Determination of sound absorption coefficient and impedance in impedance tubes.

[9] X. Jing, S. Peng, X. Sun: A straightforward method for wall impedance eduction in a flow duct. The Journal of the Acoustical Society of America **124** (2008) 227.

[10] M. Myers: On the acoustic boundary condition in the presence of flow. Journal of Sound and Vibration **71** (1980) 429–434.

[11] M. L. Munjal: Acoustics of ducts and mufflers with application to exhaust and ventilation system design. Wiley New York (NY) et al., 1987.

[12] M. Åbom: Measurement of the scattering-matrix of acoustical two-ports. Mechanical Systems and Signal Processing **5** (1991) 89–104.

[13] R. Hamming: Numerical methods for scientists and engineers. Courier Corporation, 2012.

[14] ACTRAN 13. Free Fields Technology, Belgium, 2013.