On representativeness of the representative cells for the microstructure-based predictions of sound absorption in fibrous and porous media

Tomasz G. Zieliński
Department of Intelligent Technologies, Institute of Fundamental Technological Research of the Polish Academy of Sciences, ul. Pawinskiego 5B, 02-106 Warsaw, Poland. E-mail: tzielins@ippt.pan.pl

Summary
Realistic microstructure-based calculations have recently become an important tool for a performance prediction of sound absorbing porous media, seemingly suitable also for a design and optimization of novel acoustic materials. However, the accuracy of such calculations strongly depends on a correct choice of the representative microstructural geometry of porous media, and that choice is constrained by some requirements, like, the periodicity, a relative simplicity, and the size small enough to allow for the so-called separation of scales. This paper discusses some issues concerning this important matter of the representativeness of representative geometries (two-dimensional cells or three-dimensional volume elements) for sound absorbing porous and fibrous media with rigid frame. To this end, the accuracy of two- and three-dimensional cells for fibrous materials is compared, and the microstructure-based predictions of sound absorption are validated experimentally in case of a fibrous material made up of a copper wire. Similarly, the numerical predictions of sound absorption obtained from some regular Representative Volume Elements proposed for porous media made up of loosely-packed identical rigid spheres are confronted with the corresponding analytical estimations and experimental results. Finally, a method for controlled random generation of representative microstructural geometries for sound absorbing open foams with spherical pores is briefly presented.

PACS no. xx.xx.Nn, xx.xx.Nn

1. Introduction

Sound absorption in air-saturated rigid porous media can be very well predicted by the Johnson-Champoux-Allard model [1] or its enhanced versions, provided that all model parameters are known. Those parameters can be divided into two groups: the inherent parameters of air and the so-called transport parameters which depend purely on the micro-geometry of the solid frame (skeleton) of porous medium.

A microstructure-based approach has been developed for the problem of the macroscopic propagation of acoustic waves in a porous medium, where the estimations of the transport parameters are derived from the micro-geometry of the rigid solid frame (skeleton). This approach has been recently applied to various porous and fibrous materials [2, 3, 4, 5, 6, 7, 8]. In this approach, of the utmost importance are the periodic Represenative Volume Elements (RVEs) and a question of their representativeness for particular porous media. The present work discusses the microstructural approach and the question of RVE for a fibrous material made up of a metal wire, a granular layer of plastic spherical beads, and a foam with spherical pores.

2. Sound absorption of porous media

2.1. Experimental testing

Sound absorption of porous media can be measured in the impedance tube (see Figure 1) using the so-called two-microphone transfer function method [9]. A sample of porous material of known thickness is set at the rigid termination at one end of tube. A loudspeaker, mounted at the other end, generates plane harmonic acoustic waves which propagate in the tube, penetrate the sample, and are reflected: partially from the surface of sample, and completely from the rigid termination at the back of the sample. A standing-wave interference pattern results due to the superposition of forward- and backward-travelling waves. By measuring acoustic pressure at two distinct positions using two calibrated microphones, the transfer function is determined, which allows to calculate the surface

Figure 1. Impedance tube and a fibrous or granular sample inside the tube.

acoustic impedance at the sample's surface, and then the reflection and absorption coefficients.

In this paper, the impedance tube measurements will be confronted with the results of modelling based on representative microstructures for two porous media (see Figure 1): a fibrous material woven from a copper wire and a granular layer composed of plastic spherical beads.

2.2. Microstructural modelling

The one-dimensional configuration of porous layer of known thickness (i.e., the sample in the tube) subjected to harmonic excitation by an acoustic wave can be solved analytically using the Helmholtz equation, provided that the solid frame of porous medium is stiff and motionless (rigid), the porosity is open, and finally, the (effective) speed of sound c_e in such porous material is known. It is in fact, an effective complex-valued function of frequency (to reflect the dispersive and absorptive properties of porous medium), which can be related to the effective density ϱ_e and effective bulk modulus K_e of porous medium in the standard way, namely: $c_\mathrm{e} = \sqrt{K_\mathrm{e}/\varrho_\mathrm{e}}$. Those are also complex and frequency-dependent functions, and they are related to the constant density ϱ_f and bulk modulus K_f of the actual fluid filling the porous medium (typically, it is the air) in the following way:

$$\varrho_\mathrm{e}(\omega) = \frac{\varrho_\mathrm{f}\,\alpha(\omega)}{\phi}\,, \qquad K_\mathrm{e}(\omega) = \frac{K_\mathrm{f}}{\phi\,\beta(\omega)}\,,$$

where ω is the angular frequency, ϕ is the open porosity, and

$$\beta(\omega) = \gamma_\mathrm{f} - \frac{\gamma_\mathrm{f}-1}{\alpha'(\omega)}\,.$$

Here, γ_f is the ratio of specific heats for the fluid in pores (air), whereas $\alpha(\omega)$ and $\alpha'(\omega)$ are complex-valued frequency dependent functions describing the visco-thermal interaction between the fluid in pores (air) and the solid skeleton (rigid frame) of porous medium. The Johnson-Champoux-Allard-Pride-Lafarge (JCAPL) model provides analytical formulas to estimate these functions.

The function $\alpha(\omega)$ is called the dynamic (viso-inertial) tortuosity function and the formula for it depends on the kinematic viscosity of air (fluid in pores) and some purely geometric parameters of solid frame, namely: the total open porosity, the (classic, inertial) tortuosity, the static viscous tortuosity, the static (viscous) permeability, and the characteristic length for viscous forces. The function $\alpha'(\omega)$ is the thermal analogue of dynamic tortuosity and its JCAPL-model formula contains the kinematic viscosity and Prandtl number of air (fluid in pores) and the following geometric parameters of solid frame: the total porosity, the static thermal tortuosity, the static thermal analogue of permeability, and the characteristic length for viscous effects. Obviously, both formulas depend also on the angular frequency ω.

Thus, there are eight purely geometric parameters in the JCAPL model, the so-called transport parameters of porous medium, namely: the porosity, the tortuosity, the static viscous and thermal tortuosities, the viscous and thermal permeabilities, and the visocus and thermal characteristic lengths. Provided that there exists a periodic cell representative for the micro-geometry of porous medium, the transport parameters can be calculated directly from microstructure by solving three static analyses defined on the fluid domain of periodic cell, namely:

- the Stokes flow (i.e., the steady viscous flow) caused by a unit pressure gradient constant throughout the fluid domain, with no-slip boundary conditions on the fluid-solid interface;
- the steady heat transfer caused by a unit heat source constant in the fluid domain, with isothermal boundary conditions on the fluid-solid interface;
- the Laplace problem (the potential flow) with a unit source constant in the fluid domain.

In fact, two of the transport parameters (the porosity and thermal length) are determined directly from the micro-geometry. The remaining six parameters are found by averaging (over the cell volume) and rescaling the fields found by solving the problems listed above. This approach will be applied below for three various rigid porous media: a fibrous material of copper wire, a granular layer of spherical beads, and a randomly generated microstructure of open foam.

3. Examples of absorption estimation using various representative cells

3.1. Fibrous material

A silvered copper wire with diameter 0.5 mm was used to manufacture two fibrous samples by weaving and

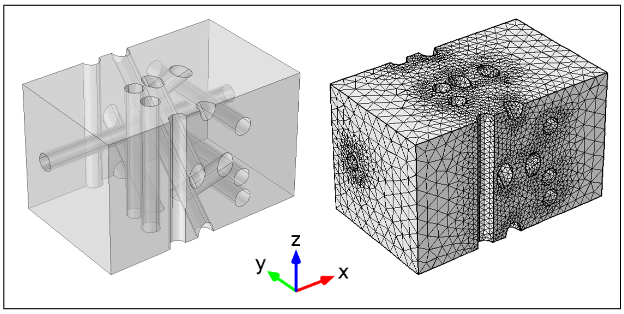

Figure 2. Representative Volume Element (RVE) for a fibrous material with porosity 90%.

fitting pieces of the wire into the impedance tube of diameter 29 mm (see Figure 1). Two cylindrical samples were manufactured in that way: one is 30 mm in height and the other 60 mm. The smaller one was made up from 10 m and the taller one from 20 m of copper wire, so that both samples have the same porosity of 90%.

The sound absorption was measured in the impedance tube, in the frequency range from 100 Hz to 6 kHz, for both samples separately, and also for two-layered configurations with total height 90 mm, composed from the fibrous samples set closely one after another, that is, one sample in front and the other behind (i.e., 30 mm+60 mm), and conversely (i.e., 60 mm+30 mm). The acoustic absorption coefficient was also computed form the microstructural modelling where the Representative Volume Element presented in Figure 2 was used. The RVE is periodic and contains only straight fibres which are mainly grouped in a layer normal to the direction of propagation X. In a simplified way, such a configuration to some point resembles the distributions of locally straight fibres in the real samples.

The results of measurements and calculations are presented together in Figure 3. One should notice that two experimental curves obtained for the two-layered configurations of the same total height of 90 mm are nearly identical, which proves that the same fibrous material is always manufactured, although the procedure involves manual weaving of copper wire. One should also observe that the computed absorption curves are very similar to their experimental counterparts which means that the proposed RVE – based on the assumption that the fibres are locally straight – is in fact representative.

3.2. Granular medium

An artificial granular medium was composed from identical plastic beads with diameter of 5.9 mm: the beads were loosely poured into the impedance tube (with diameter of 29 mm, see Figure 1) set up vertically, so that a layer of beads, 106 mm thick and with a roughly flat surface, was formed. The acoustic absorption coefficient was measured for the layer in the frequency range from 500 Hz to 6 kHz (see Figure 6).

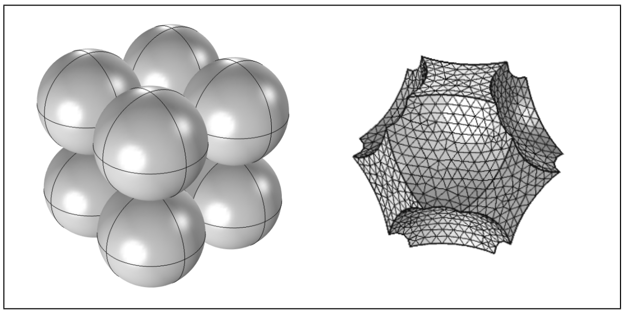

Figure 4. Simple Cubic (SC) arrangement of slightly overlapping spheres and the corresponding RVE of fluid domain.

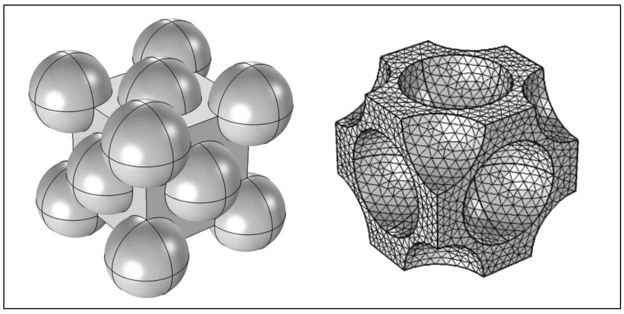

Figure 5. Face Centered Cubic (FCC) arrangement of spheres slightly shifted apart and the corresponding RVE of fluid domain.

Since all the beads in the granular layer are identical with the same size and the same weight of 0.3 g, the layer's porosity could be easily determined by simply measuring the total weight of layer, so that the number of solid beads in layer could be estimated as 380, which gives the total porosity of approximately 42%.

The microstructre-based approach was applied for granular media of that kind by Gasser et al. [10], Lee et al. [11], and Zielinski [8, 12]. Some relevant analytical estimations were given by Boutin and Geindreau [13, 14]. Following the works by Zielinski [8, 12], two micro-geometries were constructed to represent the granular medium of plastic spherical beads. Both are based on the well-known simple regular periodic arrangements of spheres, namely: the Simple Cubic (SC) arrangement which contains only one complete sphere (composed from equal parts of eight spheres, see Figure 4), and the Face Centred Cubic (FCC) arrangement containing four complete spheres in the periodic cube (see Figure 5). However, the SC arrangement has porosity of 47.6%, which is superior than the actual measured porosity, whereas the FCC arrangement has porosity of 26%, which is inferior. Therefore, in order to make these arrangements in some way representative, the porosity is set to the actual value of 42% by letting the spheres to slightly overlap in the SC case, and shifting them slightly apart in the FCC case. This can be clearly observed in the meshed RVEs of fluid domain presented in Figures 4 and 5.

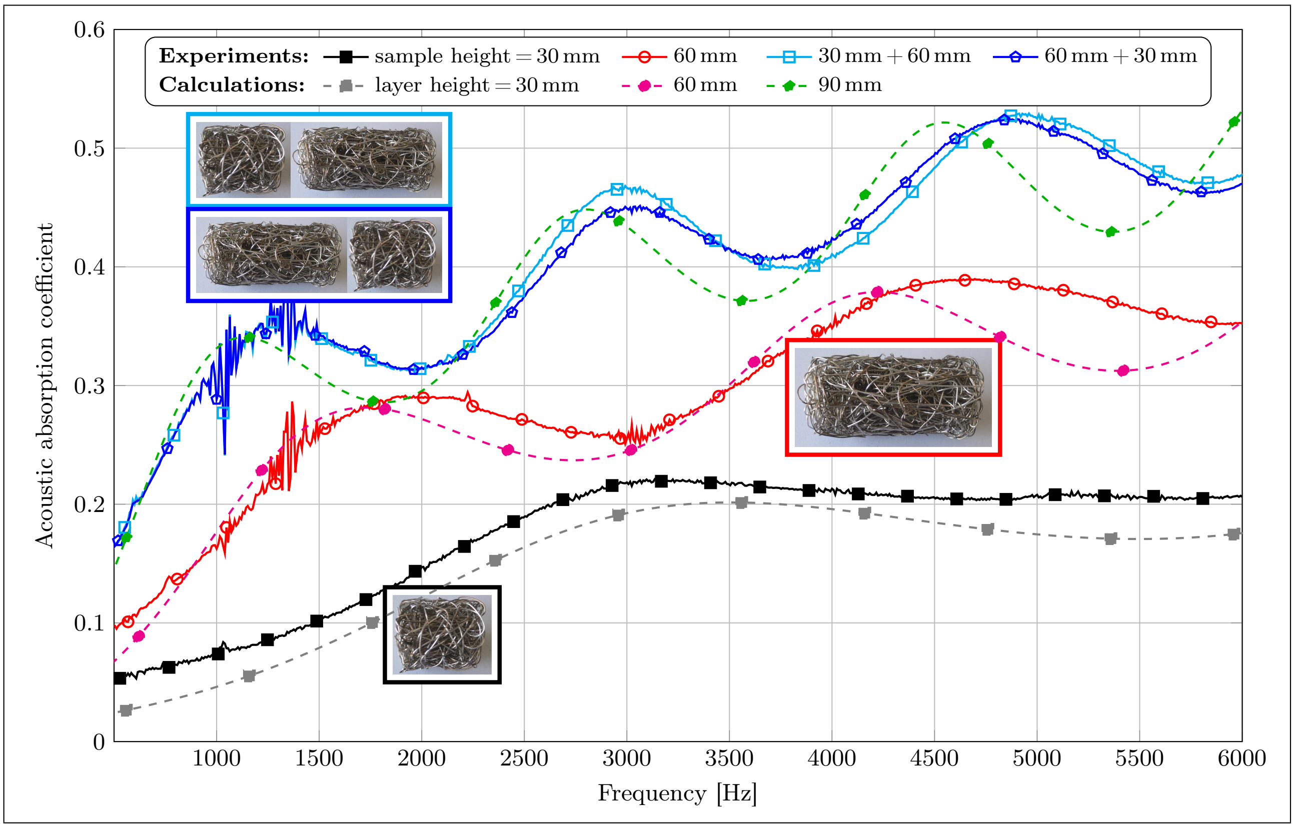

Figure 3. Acoustic absorption of fibrous samples of tangled copper wire.

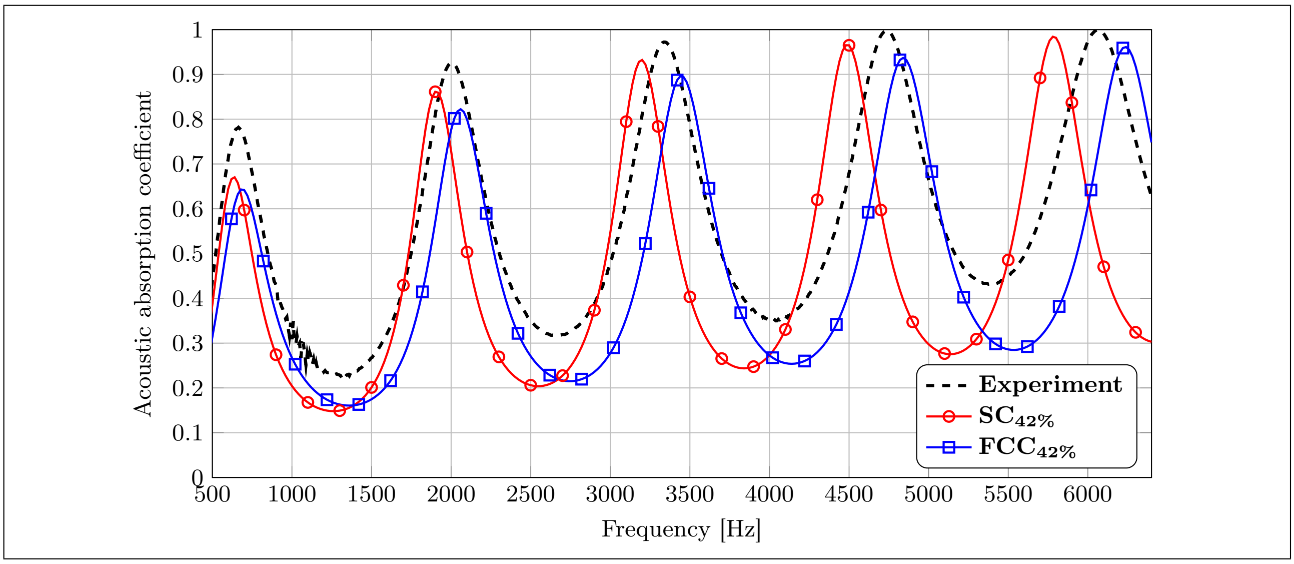

Figure 6. Acoustic absorption of a 106 mm thick porous layer of identical rigid beads (spheres).

The representative cells $SC_{42\%}$ and $FCC_{42\%}$ were used by the finite element analyses in order to calculate the transport parameters from such representative microstructures. The calculated parameters served in the JCAPL model and eventually the acoustic absorption coefficient was computed for both cases for the porous layer 106 mm thick. These numerical results are in Figure 6 compared with the experimental curve. It is easy to notice that all curves are similar in character. In fact, the absorption peaks of the experimental curve are placed between the corresponding peaks obtained for the $SC_{42\%}$ and $FCC_{42\%}$ cases, which may be considered as two opposing limit cases. As a matter of fact, the results obtained for the $FCC_{42\%}$ are closer to the experimental curve, which seems to be reasonable, since in this RVE the spheres are *not* overlapping.

3.3. Foam with spherical pores

An algorithm for random generation of periodic microstructures for foams with spherical pores was proposed by Zielinski [15, 16]. The algorithm is based on a simulation of dynamic mixing of rigid spheres inside a shrinking cube of representative cell. In order to ensure the periodicity of the representative cell, each pore is represented by an assembly of eight spheres set in the vertices of a cube with the edge equal to the current edge of the shrinking representative cell. Figure 7 illustrates this idea with three visualisations of such random mixing of spheres – for legibility – in a two-dimensional realisation of the procedure. A practical final result of the three-dimensional realisation is also shown in Figure 7 in the form of some periodic arrangement of spheres representing five pores in a cubic cell.

In the proposed approach the generated random, periodic arrangements of pores are characterised by a few features, namely: the open porosity, the average size of pores and its standard deviation, and the typical size of windows linking the pores. Except the last one, those feature are directly controlled. The porosity is set as the stopping condition for the mixing algorithm (i.e., when the desired porosity is reached the mixing and cube shrinking is accomplished), whereas the required average size of pores is ensured by applying an appropriate scaling for the final geometry of representative cell (i.e., in all finite element analyses the edge length is assumed in the way that ensures the required average diameter of pores in the cell). The standard deviation of pores is set by choosing appropriate (initial) sizes of spheres. On the other hand, the typical size of windows linking the pores is only *indirectly* controlled by the so-called maximal mutual penetration factor defined as ζ in Figure 8. By setting its value one approximately controls the size of windows, since for two identical spheres (i.e., $R_1 = R_2 = R_\mathrm{p}$) the maximal widow radius R_w is related to the pore radius R_p in the following way:

$$\frac{R_\mathrm{w}}{R_\mathrm{p}} = \sqrt{\zeta - \frac{\zeta^2}{4}}\,.$$

Figure 9 shows a Representative Volume Element (see also the periodic assembly of overlapping spheres shown in Figure 7) for an open foam with porosity 70%. The periodic representative cell contains five spherical pores. The finite element mesh of the fluid domain is also presented in Figure 9. It served in the microstructural analyses to calculate the transport parameters for such porous medium. Then, those parameters were used by the JCAPL model to determine the effective density and speed of sound. Finally, the acoustic absorption coefficient was estimated for layers of such porous foam with various thickness, namely: 20 mm, 30 mm, and 40 mm. Figure 10 presents these results in the frequency range from 200 Hz to 5.2 kHz. The random distribution of pores makes these estimations reliable provided that the average design features are fully representative for the real foam.

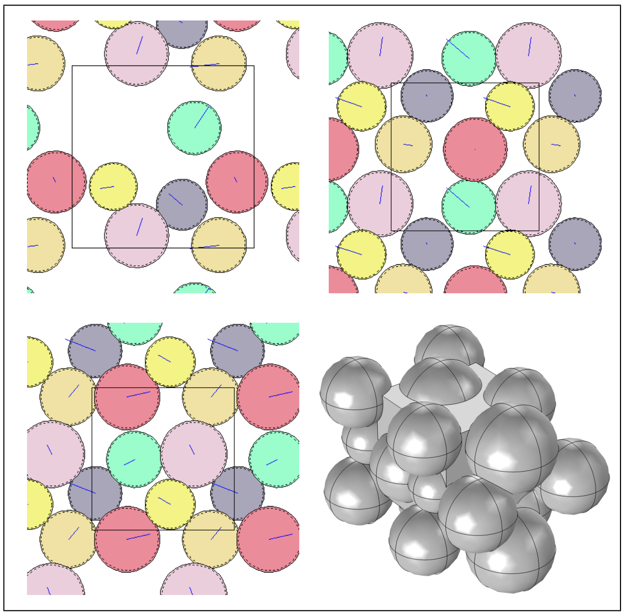

Figure 7. Dynamic packing of periodic arrangements of bubbles (pores) in a shrinking cell, and the final periodic arrangement of pores in a representative cube.

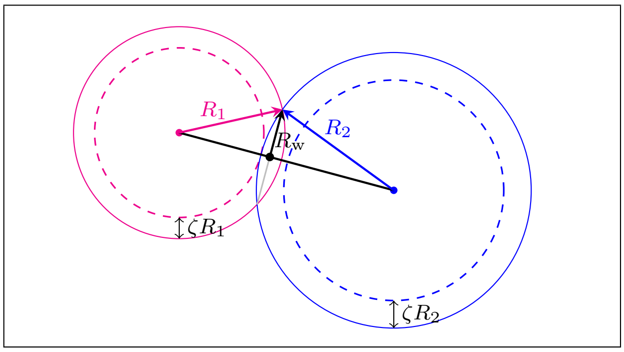

Figure 8. Two mutually penetrating spheres (pores).

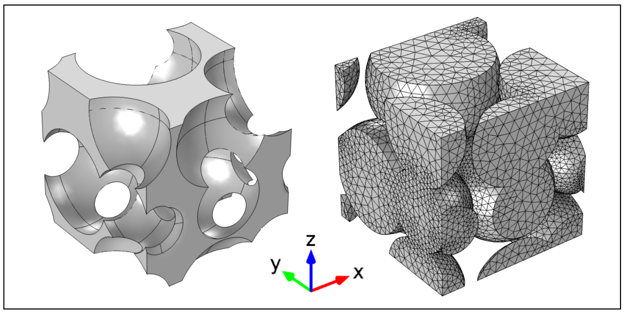

Figure 9. Periodic skeleton of foam with porosity 70% and the corresponding meshed fluid domain.

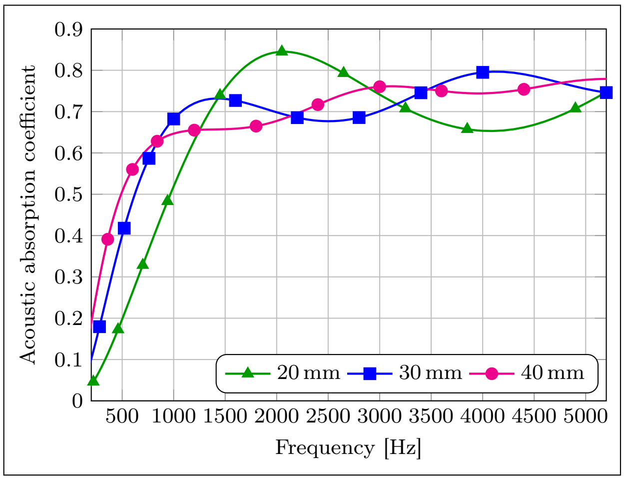

Figure 10. Acoustic absorption calculated for layers of foam with porosity 70% and various thickness

4. Conclusions

- More than a few pores, fibres or grains in a periodic cell (RVE) are necessary to well represent the geometry of real porous or fibrous media.
- However, more pores (grains or fibres) in a representative cell means larger RVE and that would require more computational power.
- Moreover, the size of a large RVE may at higher frequencies become comparable with the wavelengths, which would worsen the accuracy and reliability of estimations, because of a weak separation of scales.
- A few (random) microstructures with the same average features should be generated for one porous medium and the estimations of acoustic wave propagation and absorption should be computed as an average result.

Acknowledgement

Financial support of Structural Funds in the Operational Programme – Innovative Economy (IE OP) financed from the European Regional Development Fund – Project “Modern Material Technologies in Aerospace Industry” (No. POIG.01.01.02-00-015/08-00) – is gratefully acknowledged.

References

[1] J. F. Allard, N. Atalla, Propagation of Sound in Porous Media: Modelling Sound Absorbing Materials, Second Edition, Wiley, 2009.

[2] A. Cortis, D. M. J. Smeulders, J. L. Guermond, D. Lafarge, Influence of pore roughness on high-frequency permeability, Phys. Fluids **15** (6) (2003) 1766–1775.

[3] C. Perrot, R. Panneton, X. Olny, Periodic unit cell reconstruction of porous media: Application to open-cell aluminum foams, J. Appl. Phys. **101** (2007) 113538 (1–11).

[4] C. Perrot, F. Chevillotte, R. Panneton, Dynamic viscous permeability of an open-cell aluminum foam: Computations versus experiments, J. Appl. Phys. **103** (2008) 024909 (1–8).

[5] F. Chevillotte, C. Perrot, R. Panneton, Microstructure based model for sound absorption predictions of perforated closed-cell metallic foams, J. Acoust. Soc. Am. **128** (4) (2010) 1766–1776.

[6] C. Perrot, F. Chevillotte, M. T. Hoang, G. Bonnet, F.-X. Bécot, L. Gautron, A. Duval, Microstructure, transport, and acoustic properties of open-cell foam samples. experiments and three-dimensional numerical simulations, J. Appl. Phys. **111** (2012) 014911 (1–16).

[7] T. G. Zieliński, Inverse identification and microscopic estimation of parameters for models of sound absorption in porous ceramics, in: P. Sas, D. Moens, S. Jonckheere (Eds.), Proceedings of International Conference on Noise and Vibration Engineering (ISMA2012) / International Conference on Uncertainty in Structural Dynamics (USD2012), 2012, pp. 95–108.

[8] T. G. Zieliński, Microstructure-based calculations and experimental results for sound absorbing porous layers of randomly packed rigid spherical beads, J. Appl. Phys. **116** (3) (2014) 034905 (1–17).

[9] ISO 10534-2: Determination of sound absorption coefficient and impedance in impedance tubes (1998).

[10] S. Gasser, F. Paun, Y. Bréchet, Absorptive properties of rigid porous media: Application to face centered cubic sphere packing, J. Acoust. Soc. Am. **117** (4) (2005) 2090–2099.

[11] C.-Y. Lee, M. J. Leamy, J. H. Nadler, Acoustic absorption calculation in irreducible porous media: A unified computational approach, J. Acoust. Soc. Am. **126** (4) (2009) 1862–1870.

[12] T. G. Zieliński, Sound absorption of porous layers of loosely-packed rigid spheres: multiscale modelling and experimental validation, in: Proceedings of the 7th Forum Acusticum 2014, 2014.

[13] C. Boutin, C. Geindreau, Estimates and bounds of dynamic permeability of granular media, J. Acoust. Soc. Am. **124** (6) (2008) 3576–3593.

[14] C. Boutin, C. Geindreau, Periodic homogenization and consistent estimates of transport parameters through sphere and polyhedron packings in the whole porosity range, Phys. Rev. E: Stat., Nonlinear, Soft Matter Phys. **82** (2010) 036313 (1–18).

[15] T. G. Zieliński, Microstructure generation for design of sound absorbing foams with spherical pores, in: SAPEM2014: Symposium on the Acoustics of Poro-Elastic Materials, 2014.

[16] T. G. Zieliński, Generation of random microstructures and prediction of sound velocity and absorption for open foams with spherical pores, Journal of the Acoustical Society of America (revision submitted).