Investigations of an impedance tube technique to determine the transmission loss of materials under angular incidence

Marie Le Bourlès, Emeline Sadoulet-Reboul, Morvan Ouisse
Applied Mechanics departement - FEMTO-ST Institute, 24 rue de l'épitaphe, 25000 Besançon, France

Olivier Doutres
École de Technologie Supérieure, Department of mechanical engineering, 1100 rue Notre-Dame, Ouest, Montréal (QC), Canada, H3C 1K3

Kévin Verdière, Raymond Panneton
GAUS, Sherbrooke University, Department of Mechanical Engineering, Sherbrooke, AB, Canada QC-J1K2R

Summary

The most popular technique for transmission loss characterization of porous materials is undoubtedly the impedance tube method based on the transfer matrix method (described in ASTM E2611) which is valid for frequencies of a few kilo hertz, depending on the tube's size. A mono dimensional harmonic plane wave is created at one tube's extremity; this wave propagates inside the tube and is partially transmitted through an absorbing material to the opposite side of the tube. Using proper boundary conditions and measuring the sound pressure yields to the estimation of the transmission loss after a few algebraic manipulations. All measurement systems developed until now are based on a normal sound incidence. Yet, some modeling methods used for these materials are based on excitations with variable incidence angle and thus the validation of the developed models requires to use a facility system for which the characterization can be done using a variable wave incident angle on the material. The work proposed here details the design of such an experimental system. This system is based on a new design of the classical impedance tube: a numerical study is carried out to identify the range of validity of the proposed technique. The possibilities that this new system offers as well as its limits in terms of incident angle or frequency are illustrated on specific samples.

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1. Introduction

The impedance tube method is a standard method to measure the normal transmission loss of various materials and in particular of porous materials widely used in acoustics. Measurements are based on the transfer matrix method using four microphones. One main disadvantage of the technique is that it is not adapted to an oblique incidence but limited to normal incidence which can not be considered as totally representative depending on the material use. The two-microphone free field method is adapted to such a case but requires quite large samples sometimes difficult to produce [2]. An adaptation of a classical impedance tube for oblique incidence is studied in this paper. A design is proposed and a parametric analysis is carried out to determine the most sensitive design parameters and evaluate the efficiency of the proposed design depending on the material samples properties. To the author's knowledge, no investigation on the development of such system is presented in open literature. However, the paper by Heyl et al. [1] should be mentioned as perhaps the first attempt for the design of a set-up for the characterization of sound absorption at oblique angle of incidence. This paper does not investigate the transmission loss properties of samples, which is the aim of this study.

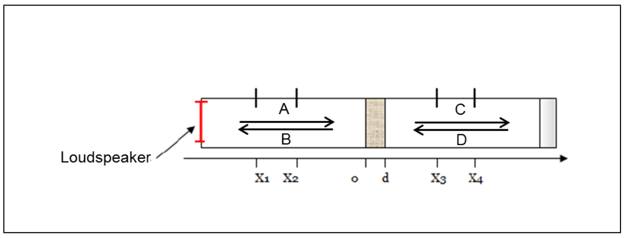


Figure 1. Scheme of an impedance tube equipped with four microphones

2. Transmission loss characterization with the impedance tube method

2.1. Calculation of the normal incidence transmission loss factor

Let us consider an impedance tube equipped with four microphones at positions x_1, x_2 x_3 and x_4 as presented in Figure 1. This tube can be used to measure the transmission loss of a d-thick material sample placed in its middle below its cut-off frequency where the propagation of the waves is assumed to be plane.

The acoustic pressure at microphones 1 and 2 in the left part of the tube can be expressed as

$$p\,(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)} \tag{1}$$

and at microphones 3 and 4 in the right part of the tube,

$$p\,(x,t) = Ce^{j(\omega t - kx)} + De^{j(\omega t + kx)}. \tag{2}$$

ω denotes the pulsation, k is the wave number, A, C and B, D are respectively the amplitudes of the plane waves travelling forward and backward. One approach to determine the transmission loss of samples in an impedance tube is to use the transfer matrix method [3]. Equations (1) and (2) are used to express the four amplitudes A to D in terms of the pressures at the four microphones such that,

$$A = \frac{j\left(p_1 e^{jkx_2} - p_2 e^{jkx_1}\right)}{2\sin\left(k\left(x_1 - x_2\right)\right)} \tag{3}$$

$$B = \frac{j\left(p_2 e^{-jkx_1} - p_1 e^{-jkx_2}\right)}{2\sin\left(k\left(x_1 - x_2\right)\right)} \tag{4}$$

$$C = \frac{j\left(p_3 e^{jkx_4} - p_4 e^{jkx_3}\right)}{2\sin\left(k\left(x_3 - x_4\right)\right)} \tag{5}$$

$$D = \frac{j\left(p_4 e^{-jkx_3} - p_3 e^{-jkx_4}\right)}{2\sin\left(k\left(x_3 - x_4\right)\right)} \tag{6}$$

where p_i denotes the acoustic pressure at coordinate x_i. These coefficients are used to calculate the sound pressures and velocities at the two faces of the sample extending from $x = 0$ to $x = d$ and the lattest quantities are related to each other by a two-by-two transfer matrix, namely

$$\begin{bmatrix} p \\ v \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p \\ v \end{bmatrix}_{x=d} \tag{7}$$

Two additional equations can be obtained by making a new measurement with a different impedance condition at the downstream section of the impedance tube, applying the so-called two-load method [4]. The coefficients of the T-matrix are thus determined by inverting equation (7). Given the reciprocity of a sample and the system symmetry it follows that

$$T_{11}T_{22} - T_{12}T_{21} = 1 \tag{8}$$

and

$$T_{11} = T_{22}. \tag{9}$$

Finally, using equation (7) along with the conditions (8) and (9), only one single measurement is necessary to compute the elements of the transfer matrix. When these coefficients are known the transmission coefficient can be computed. For instance, when considering the case of a perfectly anechoic termination, the pressure and velocity on both sides of the sample can be written in terms of the absorption coefficient α_a and the transmission coefficient T_a, thus substituting in equation (7) the normal transmission coefficient can be expressed in terms of the coefficients of the transfer matrix as,

$$T_a = \frac{2e^{jkd}}{T_{11} + \frac{T_{12}}{Z_0} + Z_0 T_{21} + T_{22}} \tag{10}$$

where $Z_0 = \rho_0 c$ is the fluid impedance. The transmission loss factor is then written as

$$STL = 10\log\left(\frac{1}{|T_a|^2}\right). \tag{11}$$

2.2. Modelling under a variable incident angle

In the case when the sample is excited by an oblique plane wave, the normal velocities on the sample faces that are involved in equation (7) have to be modified to account for the incident angle (Figure 2). The transmission coefficient can thus be written as

$$T_a = \frac{2e^{jkd}}{T_{11} + \frac{\cos(\theta)T_{12}}{Z_0} + \frac{Z_0 T_{21}}{\cos(\theta)} + T_{22}}. \tag{12}$$

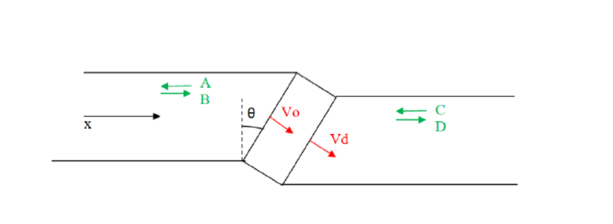


Figure 2. Scheme of the impedance tube with oblique angle of incidence

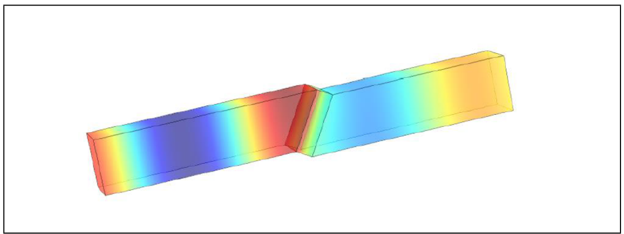


Figure 3. Design of an impedance tube for characterization under incident angle. The colour is the pressure field for one of the tests.

3. Numerical modelling

3.1. Impedance tube design

The proposed tube design is depicted in figure 3. The design is inspired from [5]. It consists in a square cross-section tube for an easier future building of the system. A parametric study is then conducted to quantify the influence of the geometry design (lateral dimensions,positions of the microphones), as well as the influence of the sample properties (thickness, material properties). Finite element computations are done with Comsol Multiphysics.

3.2. Material modelling

In order to validate the results obtained for the transmission loss factor, an analytical formulation is used as a reference. Two acoustic models are considered [6]:

- the Delany-Bazley-Miki (DBM) model based on the flow resistivity σ;
- the Johnson-Champoux-Allard (JCA) model based on five intrinsic properties of the material, namely the flow resistivity σ, the porosity ϕ, the tortuosity α_∞ , the viscous characteristic length Λ, and the thermal characteristic length Λ'.

The comparison of the results obtained using both approaches is carried out on a 3 cm thick sample and with a 15° incidence. Four materials are tested with properties given in Table I. The analytical results obtained with the two models are compared to the finite element results. The figure 4 presents the STL in dB for a 15° incident angle computed with the finite element model (FEM) and with the analytical models, using DBM and JCA formulations. The tube has a 0.1 m×0.1 m square-section. Figure 5 presents

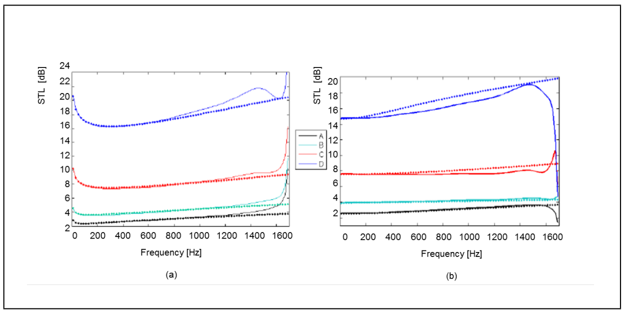


Figure 4. Transmission loss factor computed on four porous material (A, B, C, D) for a 15° incident angle using the finite element model (−) compared to the DBM (a - · · ·) and the JCA (b - · · ·) models.

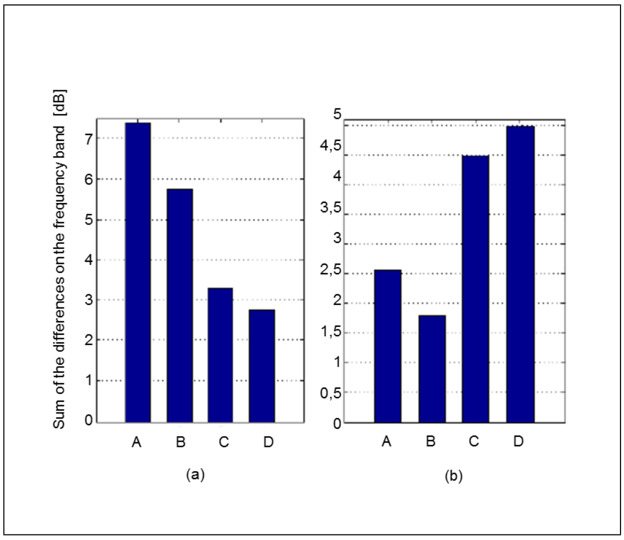


Figure 5. Sum of the differences on the STL between the FEM and the DBM (a) or JCA (b) models for four samples and a 15° incident angle.

the sum of the differences between the different approaches over the frequency band: it appears that discrepancies can be observed in various situations: the use of JCA model gives better results than the DBM model for samples A and B but it is almost the contrary for samples C and D. However, this first analysis gives a trend: the approach seems to be less effective on samples exhibiting high STL. In the following, the DBM model is chosen as a reference for its simplicity, since the material description involves only a single parameter, which renders the sensitivity analysis more simple in this first step.

4. Parametric analysis

4.1. Influence of the positions of microphones

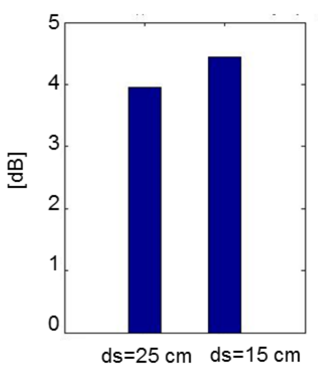
The influence of the vertical and horizontal positions of the microphones, and their relative position is firstly considered. Three vertical configurations are

Table I. Properties of four porous materials used for model validation

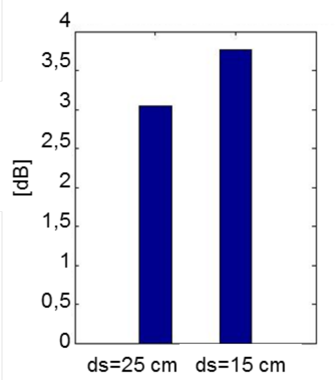
	Material	Φ	σ (N.s.m^{-4})	Λ (μm)	Λ' (μm)	α_∞
A	Melanin	0.999	9724	110	122	1.00
B	Glass wool	0.999	15957	97	530	1.00
C	Rockwool	0.960	41554	92	92	2.59
D	Rockwool	0.940	135000	49	166	2.10

Table II. Sum of differences over the frequency band between finite element and DBM results when changing the distance between the microphones (dm), the length of the tube (L) and the distance between the microphones and the sample (ds)

$L = 1$m

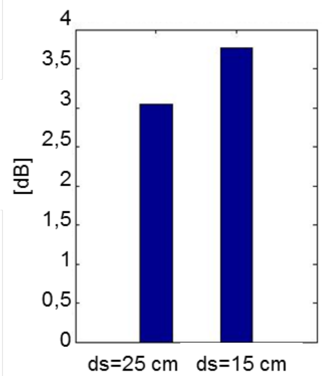


$L = 1.5$m

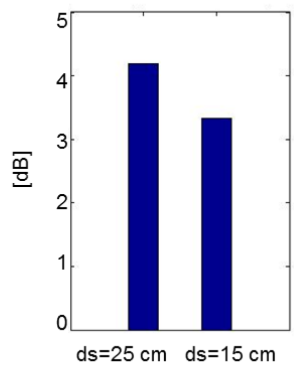


$dm = 10$cm

$L = 1.5$m



$L = 2$m



$dm = 20$cm

studied with microphones (Figure 6), the closest microphone being at a distance of 15 cm from the sample. There is almost no difference between the results processed from the three cases, up to 1150 Hz. Above this frequency, the field is no longer mono directional, and the real position of the microphone (on the edge of the tube) does not provide useful measurement without correction. It should be emphasized that, on this case, the results given by the approach are very close to the reference one up to 650 Hz. Above this frequency (and under 1150 Hz), a maximum difference on STL value of 1 dB is observed. Considering the horizontal position of the microphones, the length of the tube as well as the distance between microphones on each side of the sample are taken into account. Various configurations are tested: microphones placed at respectively 15 and 25 cm from the sample, a distance between microphones of 10 and 20 cm, and a tube length of 1, 1.5 or 2 m. The sum of differences over the frequency band for all these test-cases are presented in Table II. It appears that better results are obtained for microphones placed farer from the sample (ds=25 cm) and for a microphone separation distance of 10 cm. This is coherent with the previous results: a long tube should be used, with microphones far enough from the sample in order to be in zones

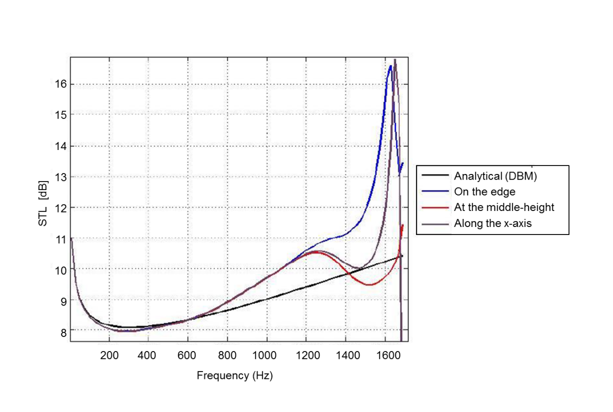


Figure 6. Study of the influence of the vertical positions of microphones on the STL computation

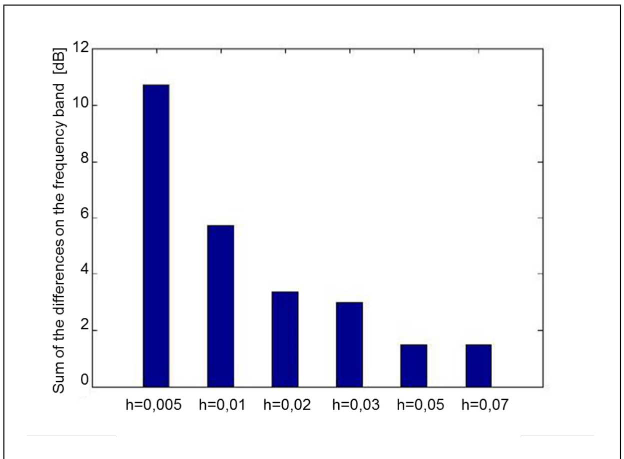


Figure 7. Sum of the differences over the frequency band between finite element and DBM results for different sample thicknesses

where the pressure field is as homogeneous as possible.

4.2. Influence of the sample parameters

4.2.1. Thickness of the sample

The thickness of a sample with a fixed resistivity and a fixed incident angle is changed from 0.005 to 0.07 m to study the impact of this quantity on the validity of results. The sum of the differences over the frequency band between the finite element and the DM results is presented in figure 7 for different sample thickness. It appears that the error globally increases as the thickness reduces. In this case, results obtained with samples having a thickness of 1 cm or less are considered as non valid.

4.2.2. Resistivity

A similar study is carried out on the impact of the sample resistivity : results for a sample with a varying resistivity from 1000 to 200000 $\mathrm{N.S.m^{-4}}$ are compared to the DBM results and presented on figure 8. The sample resistivity has quite a little influence

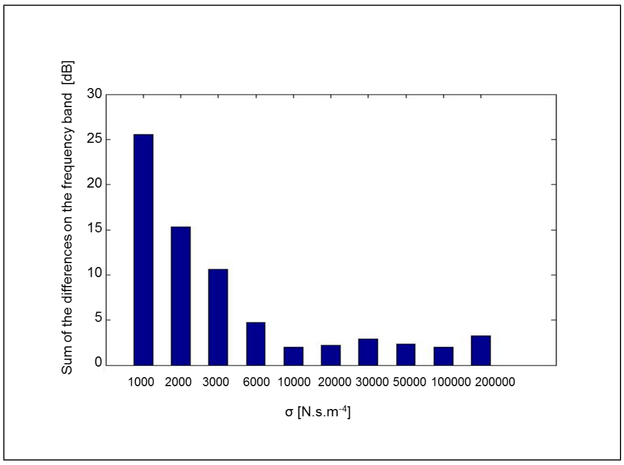


Figure 8. Sum of the differences over the frequency band between finite element and DBM results for different sample thickness

on the results as long as its value is greater than 10000 $\mathrm{N.s.m^{-4}}$.

5. Conclusion

A tube impedance design for characterization of porous samples under oblique incidence is presented in this paper. A numerical study is carried out to evaluate the validity of the proposed technique by comparison to an analytical approach. Two acoustic models have been considered, the Johnson-Champoux-Allard and the Delany-Bazeley-Miki models but without clear conclusions about the precision of the approach. The parametric analysis shows that the system can be used for samples of thickness greater than 1 cm and resistivity higher than 5000 $\mathrm{N.s.m^{-4}}$. The width of the square cross-section is linked to the cut-off frequency and has to be chosen according to the frequency band of interest. By increasing the length of the tube it becomes possible to increase the distance between the microphones and the sample to place in an area where the plane waves are well established. The ongoing work focuses on identification of bounds that will be associated to the STL measurements, together with the real design of the proposed oblique configuration to experimentally validate the numerical results presented in this paper.

6. Acknowledgements

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