Application of the Transfer Matrix and Finite Surface Size Correction to Room Acoustics Simulation

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Summary
Most large scale acoustic simulations rely on user input of a Sabine or Eyring-based absorption coefficient in order to describe surface materials. However, the use of these coefficients assumes a locally reacting absorption pattern (equal absorption for waves incident from all directions). In reality, even locally absorbing finish assemblies often absorb in non-locally reacting ways when installed because of finite sample size and other conditions. Using the Transfer Matrix Method and a variation on the Thomasson finite size correction, a method for predicting the absorption of a specified finish from any location of incidence is presented.

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1. Introduction

Geometrical acoustics simulation has been widely used in architectural acoustics. While the data provided by these programs is useful, they can be further improved using more specific data about room finishes. A series of assumptions which are not always correct are employed by the representation of absorption. In the typical energy-based geometrical acoustics simulation, the intensity I reflected from a surface is calculated by the well-known equation:

$$I = I_0(1-\alpha)(1-s)(1-\tau) \qquad (1)$$

where I_0 is the incident intensity, α is the absorption coefficient, s is the scattering coefficient, and τ is the transmission coefficient. The term $1-\alpha$ yields an unphysical result when the absorption coefficient exceeds unity. Unfortunately, it is the case that samples in the reverberation chamber often absorb more energy than their area would permit if their absorption conformed to unity. Acousticians have responded to this in a number of ways. One is to round absorption coefficients greater than 1 down to 1 (i.e. α_p, the practical absorption coefficient).[1] Another approach is to convert Sabine-based coefficients to Millington or Eyring-based coefficients, which always conform to Unity.[2] Unfortunately, neither of these approaches address or explain the phenomenon that the absorber has a greater absorption area than its physical area when tested in a reverberation chamber.

There are other assumptions espoused by geometrical acoustics simulation relating to absorption. One is that all finishes are locally reacting (absorb equally regardless of angle of incidence). In the case of multi-layered materials, this is often not the case. It can be shown that an absorber of infinite extent often becomes non-locally reacting once it is E-mounted according to ASTM standards. Another assumption is that absorption is uniform over the surface of an absorber. However, the edge effect has been widely recognized, and somewhat understood for many decades. It would be cumbersome to enter directional data by octave band for each absorptive material, not to mention accounting for surface size and shape. Fortunately, methods for calculating impedance and absorption based on specifications data (such as static flow resistivity, thickness, percentage free area, etc.) have been developed. It seems feasible to begin exploring a new approach to specification of absorption. By providing 2 numbers - the flow resistivity and depth - an analyst (or indeed a consultant) can provide a far more specific criterion for the absorption of a finish than they could by specifying 6 octave band absorption coefficients.

2. Theory

Finishes with one or more layers can be modelled for the infinite case using the Transfer Matrix Method.[3] In 1980, Thomasson presented a technique for calculating the absorption added due to sample size.[5] This

paper presents a technique utilizing both the transfer matrix method and Thomasson's finite size correction to improve the Geometrical Acoustics representation of room finishes.

2.1. Angle Dependence from the Transfer Matrix Method

An impedance transfer matrix can be used to calculate the acoustical properties of a room finish incorporating multiple layers of building materials and air. By tracing angle of incidence in each layer according to Snell's law of refraction, the wave number in the z-dimension (perpendicular to the absorber) is found for any angle from normal to grazing incidence, and the theoretical absorption at various angles of incidence to an infinite extent of material can be calculated. At normal incidence, this figure would correspond to the impedance values measured in an impedance tube. A state matrix is iteratively 'transferred' once for each layer from substrate to surface, as shown below:

$$\begin{bmatrix} p_n \\ u_n \end{bmatrix} = [T] \begin{bmatrix} p_{n+1} \\ u_{n+1} \end{bmatrix} \tag{2}$$

where p is pressure, and u is velocity at the boundary of a layer. When layers are represented as fluids, as is the case with the Delany/Bazley [6] or Miki [7] representations of porous absorbers, the transfer matrix [T] is:

$$[T] = \begin{bmatrix} cos(kd) & j\frac{\omega\rho}{k}sin(kd) \\ j\frac{k}{\omega\rho}sin(kd) & cos(kd) \end{bmatrix} \tag{3}$$

where k is the wave number normal to the layer boundary, d is the depth of the layer, ω is the angular frequency ($2\pi f$), and ρ is the effective density of the medium. By setting the velocity and pressure at the bottom of the last layer (a rigid mounting point) equal to zero and one, respectively, the impedance transfer termination condition for a rigid mounting condition is found:

$$Z_n = -jZ_c cot(kd) \tag{4}$$

where Z_c is the characteristic impedance of the medium, and Z_n is the impedance at layer boundary n.

By executing the multiplication of the state matrix and the transfer matrix and using the relation $Z = p/u$, the classical impedance transfer theorem is arrived at:

$$Z_{n-1} = Z_c \frac{-jZ_n cot(kd) + Z_c}{Z_n - jZ_c cot(kd)} \tag{5}$$

In the above equations, Z_c and k may be replaced with $Z_c\frac{k}{k_z}$ and k_z, respectively, in order to calculate the impedance at oblique incidence.[3]

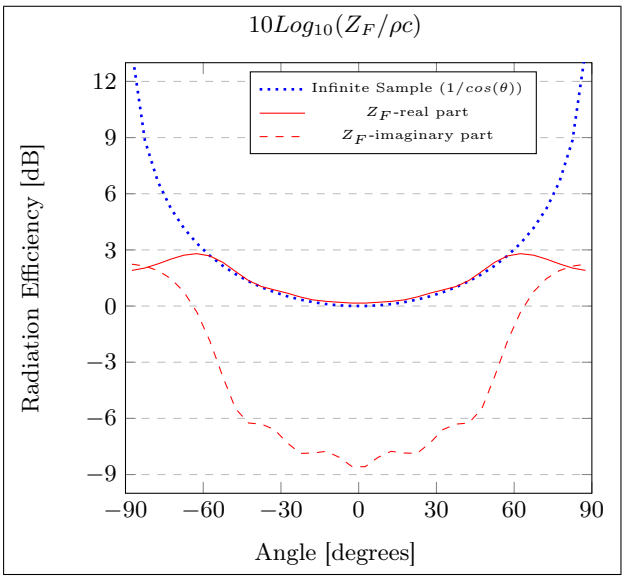

Figure 1. Logarithmic plot of the average normalized radiation impedance $Z_F/\rho c$ at all angles of incidence for 1000 Hz. on a 1m x 1 m sample.

2.2. Finite Field Impedance

Thomasson developed a technique by which the added impedance due to a finite sample size may be calculated. Using the variational approach [4], a quadruple integral is first used to sum the contribution of the entire absorber surface area for each direction individually, for all locations on the absorber.

$$G(x, y||x_0, y_0) = \frac{e^{-jkR}}{2\pi R} \tag{6}$$

$$Z_F = \frac{jk_x}{A} \iiint_S G(x, y||x_0, y_0) \cdot e^{jk(x-x_0)sin\theta cos\phi} \cdot e^{jk(y-y_0)sin\theta sin\phi} dxdydx_0dy_0 \tag{7}$$

where R is the distance between two points on an arbitrarily shaped flat absorber (x, y) and (x_0, y_0), A is the area of the absorber, θ is the vertical angle from normal incidence, and ϕ is the azimuth angle with reference to the direction of impinging sound. The integration limit S refers to the area integral in the x and y domains of the surface.

Thomasson then calculated the absorption coefficient from the field impedance Z_F and the 'normalized' impedance Z_A [1]

$$\alpha_{stat} = 8Re(Z_A) \int_0^{\pi/2} \frac{sin\theta}{|Z_A + Z_F|^2} d\theta \tag{8}$$

[1] $Z_A = Z/\rho c$. Z being the calculated impedance of an infinite sample, ρ being the density of air, and c being the speed of sound in air.

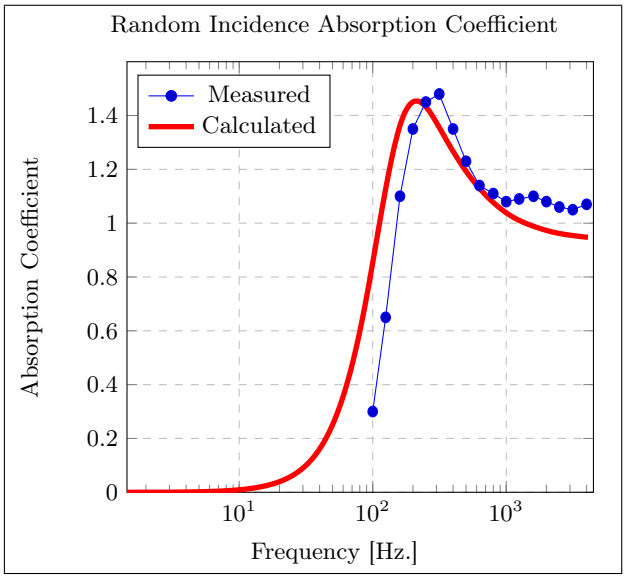

Figure 2. Plot of the random-incidence absorption coefficient, according to equation (9). Data is taken from section 12.3.3 in reference [3]. Calculated sample is a 2.4 m x 2.4 m sample of mineral wool calculated according to Miki's technique, with a flow resisitivity of 15,000 mks-rayls.

In this equation, Z_F is the average of all azimuth angles ϕ. Similarly, Z_A - which is shown outside of the integral- is understood to be the same for all directions. In more recent works (such as [3]) methods to calculate the specific impedance for any discrete angle can be calculated. Because of this, it is possible to re-write equation 8, applying the field impedance for each discrete angle to the specific impedance of the finish for each angle. Figure 2 has been calculated in this way.

$$\alpha_{stat} = \frac{1}{2\pi} \int_0^{\pi/2} \int_0^{2\pi} \frac{4Re(Z_A(\theta))}{|Z_A(\theta) + Z_F(\theta,\phi)|^2} sin\theta d\theta d\phi \quad (9)$$

Some notes on the accuracy of the variational technique as compared with a direct technique are mentioned in appendix F of [8].

3. Application to Direct Simulation

As was previously mentioned, in geometrical acoustics, the movement of sound is calculated either by deterministic means, such as the image source method, or by statistical means, such as the ray-tracing method. Upon incidence with a surface, the energy along a path is reduced by the surface absorption coefficient. In the state of the art, the coefficient is assumed to be the same at all angles of incidence. By using Thomasson's method, a coefficient which varies at all angles can be calculated and referenced according to the direction of an incident ray.

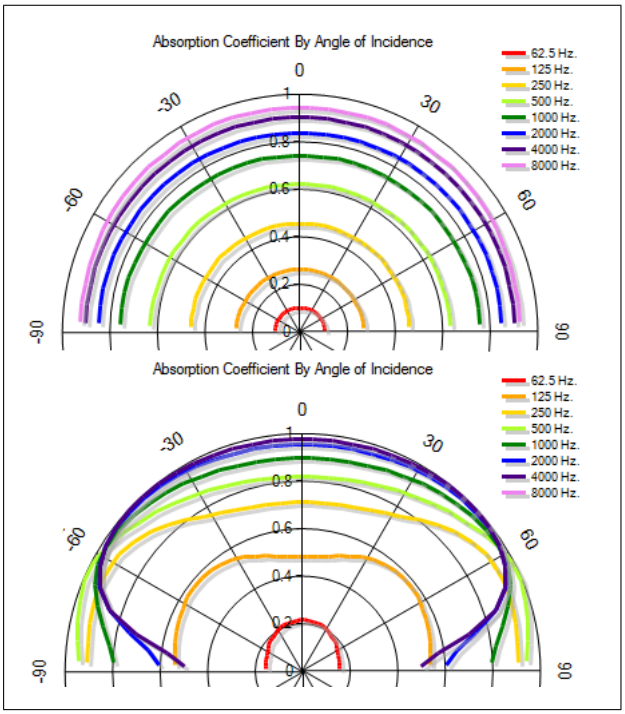

Figure 3. Polar plot of absorption from a 112 mm thick mineral wool, infinite sample (top), and 2.4 meter x 2.4 meter sample using equation (13). (bottom).

3.1. Local Finite Absorption

Thomasson's formula is developed from first principles using the variational approach. As such, when the purpose of each term is understood, it can be unpacked to calculate the local condition. [5]

$$Z_F(x_0, y_0, \theta, \phi) = \frac{jk_x}{A} \iint_S G(x, y||x_0, y_0) \cdot e^{jk(x-x_0)sin\theta cos\phi} \cdot e^{jk(y-y_0)sin\theta sin\phi} dxdy \quad (10)$$

$$\alpha(\theta, \phi) = \frac{4Re(Z_A(\theta))sin(\theta)}{|Z_A(\theta) + Z_F(\theta,\phi)|^2} \quad (11)$$

Note that in the above equations, there is no averaging over directions to achieve a statistical coefficient. The traditional use of this technique is to predict the result in a reverberation chamber. Since our intention is to explicitly represent the absorption of a surface with respect to all angles of incidence at any location on the surface, we omit this step.

3.2. Tailoring to Geometrical Acoustics

The equation for the absorption coefficient α is developed partially from the Paris reverberation chamber directivity function:

$$\alpha = 2 \int_0^{2\pi} \alpha(\theta) sin(\theta) cos(\theta) d\theta \quad (12)$$

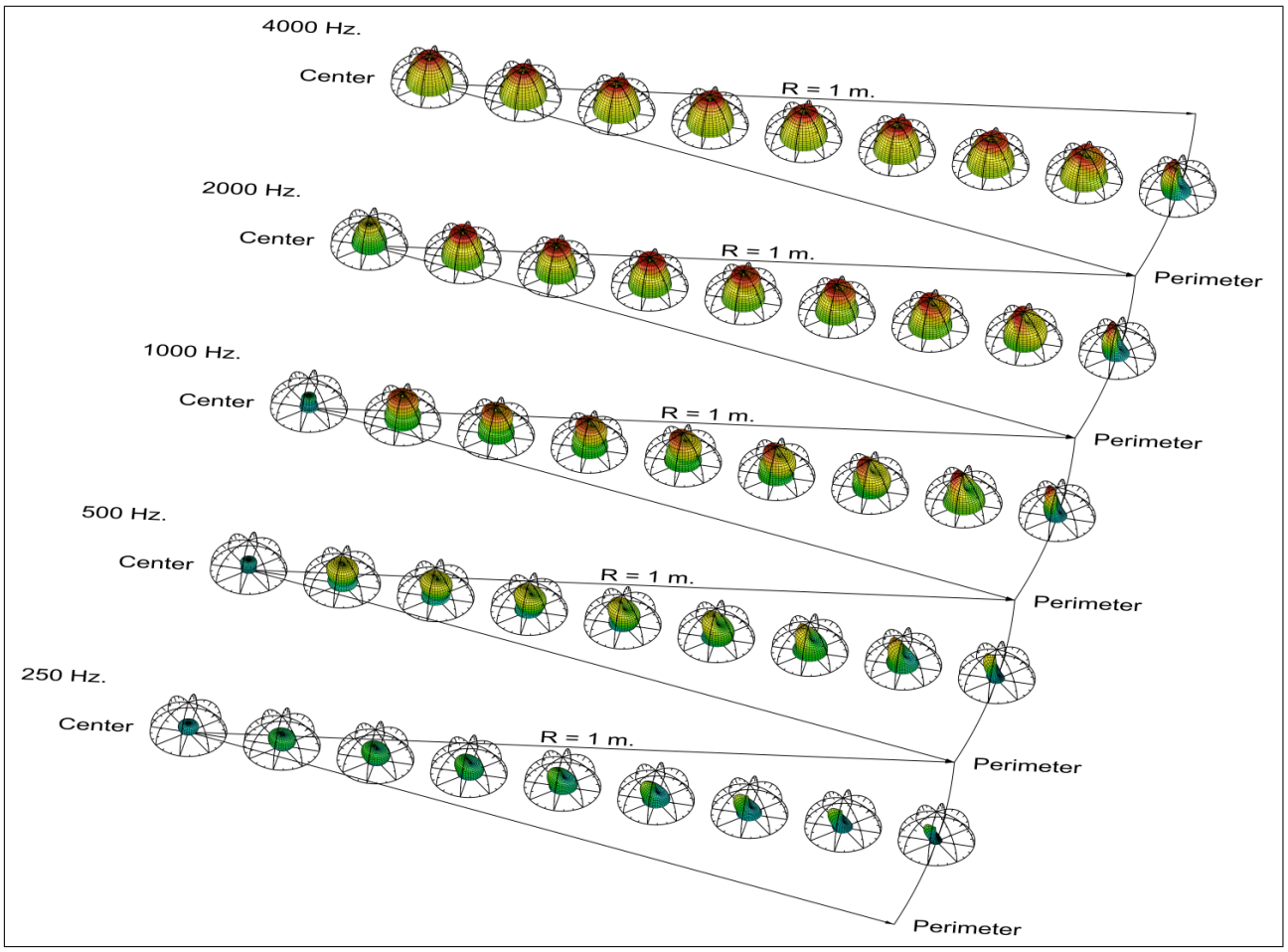

Figure 4. 3d Polar plot of absorption coefficient at various locations on a 2 m. diameter disc of 112 mm thick mineral wool, using equation (13), and averaged over each respective octave band shown.

where θ is the angle of elevation relative to a surface. The term $sin(\theta)cos(\theta)$ is meant to account for the directionality of incident sound power which commonly occurs in a reverberation chamber. Since the directionality of sound is explicitly figured in direct geometrical simulation techniques such as ray-tracing, it is important to remove these directional weights. However, we can not simply divide by the Paris directional term, as this would create a singularity at grazing incidence. Rather, it turns out that the field impedance Z_F approaches the field impedance for an infinite sample (which is $1/cos(\theta)$) as frequency increases. Given this condition, Thomasson resolved the field impedance without the Paris directionality functions in the following way:

$$\alpha'(\theta,\phi) = \frac{4Re(Z_F(\theta,\phi))Re(Z_A(\theta))}{|Z_A(\theta)+Z_F(\theta,\phi)|^2} \qquad (13)$$

While the absorption coefficient resulting from (11) may exceed unity, the coefficient resulting from (13) does not exceed unity, making it acceptable for use in geometrical acoustics simulations without any further modification.

3.3. Using Directional Coefficients

Once the absorption coefficient has been calculated for all directions at discrete points on the absorber, a series of coefficient banks can be kept, which store these coefficients. Since the function for the field impedance is a continuous function, the coefficient can be interpolated between locations on the absorber, as well as over the directions, allowing fewer coefficients to be stored. An example of the coefficients in a series of balloon plots is shown in figure 4.

4. Conclusions

A technique has been presented which represents absorption taking into account the geometry of the absorptive finish, as well as the direction of incident sound. Although this technique does not yet account for the added absorption caused by finite sample size, many of the assumptions of existing techniques have been accounted for, including the assumption of omnidirectionally uniform absorption. Having taken these steps, it may be possible to better understand and account for the physical means by which an absorptive sample yields coefficients in excess of unity.

4.1. Future Work

The finite size correction provides interesting data for use in energy based simulations. However, when one needs a complex reflection coefficient for pressure based simulations, Thomasson's method falls short, as the phase information is lost using the approach described. A method for calculating a reflection coefficient from a finite sized absorber is still needed. The techniques described here have been implemented as part of the open-source acoustics simulation software Pachyderm Acoustic, as an experimental feature.

In section 1, we mentioned that the usual approaches to tailoring absorption coefficients for use in geometrical acoustics do not account for much of the apparent absorption which leads to absorption coefficients greater than unity in a reverberation chamber test. Because the coefficients calculated in this paper still conform to unity, the added absorption is still not represented. Some further work can be done which explores techniques which endeavor to understand how this added absorption manifests. Another technique is needed in order to supplement the directional absorption coefficient. One can speculate that this may come in the form of allocating absorption to less absorptive surrounding surfaces or nearby zones of air volume, or perhaps ray-tracing schemes which account for the refractive effects of air resulting from field impedance (such as a Hamiltonian ray-tracing technique).

Finally, although the author believes the work shown should yield plausible results, he was unable to find test results to use as a benchmark in order to clearly verify that the technique is indeed an improvement. Further work should be done in order to verify that the technique presented provides realistic, and indeed more precise results than the current state-of-the-art techniques.

Acknowledgement

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