Calculation of internal powers for anisotropic porous materials within multilayered structures based on plane wave approximation

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Summary
This paper presents a method for calculating internal powers of multilayered configurations with anisotropic porous materials. The methodology takes a plane wave solution for anisotropic multilayered structures as a mathematical base for the derivation of the corresponding integral expressions. Different physical phenomena within the assessed structure can be then studied in detail as a whole or in terms of partial contributions to the total power balance. In the paper, an analysis of the dissipation in an anisotropic porous material as a function of its material coordinate orientation is performed.

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1. Introduction

Poroelastic materials (or sound absorbing materials) dissipate energy through different physical mechanisms (viscous interaction, thermal exchanges, structural damping). Together with their relatively low weight, it has made them interesting candidates in the design of structural components in aeronautical, automotive or even architectural applications.

The Biot theory [1–5] describes the mechanical behaviour of isotropic and anisotropic homogeneous media, allowing these type of materials to be studied in a common modelling frame. In addition, the inherent anisotropic characteristics of poroelastic media, induced by the manufacturing processes, have been shown to significantly influence the overall behaviour of multilayered structures including such materials. The physical modelling of anisotropic poroelastic materials has been the subject of considerable research in the last decades [6–9]. The efforts related to the modelling have in parallel been complemented by advanced methods for the characterisation and estimation of their material properties [10–14].

The physical response behaviour of anisotropic poroelastic materials is considerably more complex than the corresponding isotropic materials. To better understand the phenomena involved in the propagation of waves in anisotropic media, there is a need for methods that enable a detailed analysis of the transfer of mechanical energy between the different phases of the materials, correlating it to the response of different parts of the system. This could, for example, be studied in terms of the internal power balances. Similar works by Carcione [15], have served as an inspiration for the current work, which aims to extend the power analysis to the study of the vibroacoustic response of a multilayered structure modelled by plane wave solutions of an anisotropic media of finite thickness.

The objective of this paper is to propose an analysis method that allows the total and partial power contributions to be calculated from a plane wave solution of the acoustic behaviour analysis of multilayered structures [16]. The plane wave solution is extended to include anisotropic anelastic porous media under arbitrary plane wave excitation. One of the critical contributions provided by this method,

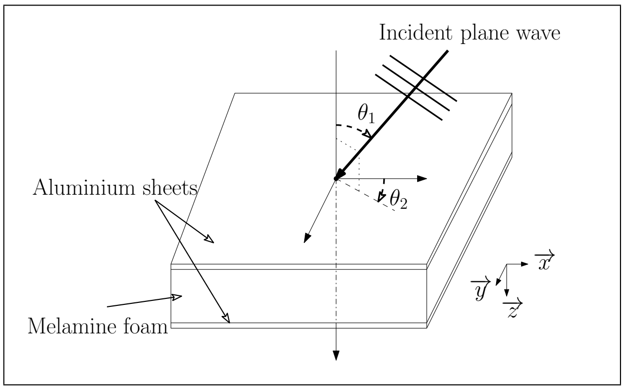

Figure 1. Multilayered system of interest

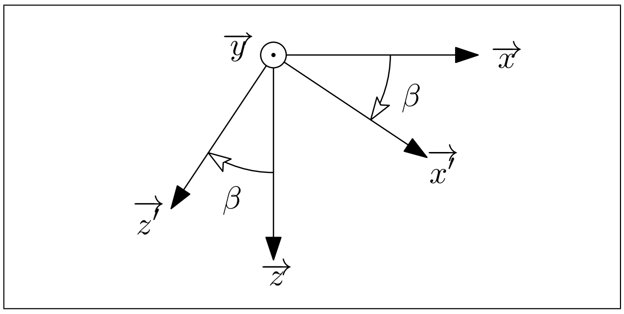

Figure 2. Rotations of the porous material coordinate system with respect to the global coordinate system

that differentiates it from a classical plane wave approach, is that the unknowns field variables are expressed in terms of the amplitudes of the waves propagating through the poroelastic material.

2. System of interest

The setup studied is a sandwich panel composed of two face sheets and a core layer, as illustrated in Fig. 1. The isotropic face sheets are of 1 mm-thick aluminium layers, and the core material is a 88 mm anisotropic and anelastic melamine foam. The multilayered system is excited by a plane wave incident upon one of the face sheets, and the transmission through the panel is given from the computed pressure radiated from the opposite face into an infinite fluid media. Through a fictitious rotation of the anisotropic porous material properties, the degree of compressional and/or shear deformation will vary in the core as one of the face sheets is excited by a plane wave with an arbitrary angle incidence.

To demonstrate the influence of the anisotropy of the porous material and its relative alignment, a comparative study of various states of rotation of the porous material has been performed. Here, the transformations are applied to rotate the foam's material reference system, as opposed to rotating the coordinate system. As seen in Fig. 2, the angle β corresponds to the rotation around the $(0y)$ axis. In the unrotated state, the material system is aligned with the reference coordinate system.

3. Governing equations

In the following, plane wave expansions are considered with a harmonic excitation at the circular frequency ω. Thus, all the scalar physical fields (e.g. displacements, forces, etc.) may be written as:

$$\hat{\chi}(x, y, z, t) = \chi(z) e^{\imath(\omega t - k_x x - k_y y)}. \tag{1}$$

The cartesian coordinate system is defined in Fig. 1. In the considered configuration, the spatial dependence with respect to x and y is common to all fields and imposed by the source, together with the time dependence, it will be omitted in the following. Let $\chi(z)$ denote the amplitude of the physical field, and k_x and k_y denote the wavenumber components of the incident plane wave.

The equations governing the behaviour of the porous material are expressed in terms of the displacement of the solid phase and the total displacement [17], $\{\mathbf{u}^s(z), \mathbf{u}^t(z)\}$. The expressions for anisotropic open-cell porous media are based on the formulations in Hörlin *et al.* [9],

$$\nabla \cdot \boldsymbol{\sigma}^s(z) = -\omega^2 \tilde{\rho}_{\mathbf{s}} \mathbf{u}^s(z) - \omega^2 \tilde{\rho}_{eq} \tilde{\gamma} \mathbf{u}^t(z), \tag{2}$$

$$-\nabla p(z) = -\omega^2 \tilde{\rho}_{eq} \tilde{\gamma} \mathbf{u}^s(z) - \omega^2 \tilde{\rho}_{eq} \mathbf{u}^t(z), \tag{3}$$

$$\boldsymbol{\sigma}^s(z) = \hat{\mathbf{H}}^s \boldsymbol{\epsilon}^s(z), \tag{4}$$

$$p(z) = -\tilde{K}_{eq} \nabla \mathbf{u}^t(z), \tag{5}$$

where $\mathbf{u}^s(z)$ is the displacement vector of the porous solid phase, $\boldsymbol{\epsilon}^s(z)$ is the porous solid strain vector and $\mathbf{u}^t(z)$ is the total displacement in the porous medium.

The expressions for the different porous parameters may be found in the references [7, 9, 18] for the isotropic media. The porous materials are here considered fully anisotropic, thus the terms $\tilde{\rho}_{\mathbf{s}}$, $\tilde{\rho}_{\mathbf{eq}}$ and $\tilde{\gamma}$ are second order tensors. The anisotropy of the poroelastic materials is also reflected in the flow resistivity $\boldsymbol{\sigma}_{\text{flow}}$ which here also is a second order tensor, and the fourth order Hooke's tensor $\hat{\mathbf{H}}^s(\omega)$ which governs the relations between stresses and strains.

All the material parameters used for the porous melamine foam are taken from the characterisation works by Van der Kelen *et al.* [13, 14] and Cuenca *et al.* [11, 12].

Moreover, the porous medium is considered anelastic with a constitutive model based on an augmented Hooke's law using a fractional order derivative approach. The frequency dependent Hooke's law $\hat{\mathbf{H}}^s(\omega)$ is then given as a superposition of a fully relaxed in-vacuo frequency-independent state $\hat{\mathbf{C}}$, and a complex frequency-dependent relaxation that depends on the fractional derivative order $\hat{\alpha}$, the relaxation frequency $\hat{\beta}$, and the anelastic contribution $\hat{b}$:

$$\hat{\mathbf{H}}^s(\omega) = \hat{\mathbf{C}}\left(1 + \frac{\hat{b}(\imath\omega/\hat{\beta})^{\hat{\alpha}}}{1 + (\imath\omega/\hat{\beta})^{\hat{\alpha}}}\right). \quad (6)$$

All the values of the parameters can be found in the recent publication by the authors [19].

4. Power partitioning and balances

The power densities are physical quantities defined in a material layer, and are functions of the physical fields. The proposed method to evaluate the integrals required to obtain the different power value, is based on the concept of expressing these quantities as a function of the wave amplitudes within the material layer.

The integration is performed over the thickness of the material layer, and gives as a result the power terms related to e.g. stored power by structural means in a solid layer, kinetic energy loss due to phase coupling in a poroelastic layer, etc. To perform the computation of the integral over a quadratic terms, it is necessary to first determine its density, and then to integrate over the width of the layer.

From Eqs. 2,3,4 and 5, a first order system can be established as a function of the vector of physical fields $\mathbf{w}(z)$:

$$\left(\mathbf{R} + \mathbf{A}_z\frac{\partial}{\partial z}\right)\mathbf{w}(z) = \mathbf{0}. \quad (7)$$

The latter can be solved, for example, through the Stroh formalism [20], where the system is rewritten as a function of the subset of physical fields with which the boundary conditions of the problem are expressed, i.e. the state vector $\mathbf{s}(z)$,

$$\frac{\partial \mathbf{s}(z)}{\partial z} = -\boldsymbol{\alpha}\mathbf{s}(z), \quad (8)$$

where $\boldsymbol{\alpha}$ is a variation matrix that only depends on the material parameters and the wavenumbers of the incident plane wave.

In order to assess a given quadratic quantity, the diagonalisation of the physical fields can be used. This way, the amplitude of the propagating waves travelling along the material layers is introduced,

$$\mathbf{s}(z) = \sum_k \phi_{e_k} e^{-\lambda_{e_k} z} q_k. \quad (9)$$

The matrix $\boldsymbol{\phi}_e$ denotes the eigenvectors associated to the travelling waves within the material layer. It reflects the polarisation of each wave projected in each physical field. $\boldsymbol{\lambda}_e$ is the diagonal matrix whose n^{th} term is equal to $\imath\delta_n$, where δ_n is the wavenumber along the z direction associated with the n^{th} wave.

The vector $\mathbf{q}$ is the generalised amplitudes of the waves travelling along both directions of the material. They can be extracted from the projection of the state vector $\mathbf{s}(z)$ on the wave base taken at the origin of the layer, i.e. Eq. 9 at $z = 0$,

$$\mathbf{q} = \boldsymbol{\phi}_e^{-1}\mathbf{w}(0). \quad (10)$$

Through this projection, the integral over a finite thickness d of a quadratic quantity $\mathcal{W}$ may be expressed as

$$\mathcal{W} = \mathbf{q}^*\left(\int_0^d e^{-\boldsymbol{\lambda}_e^* z}\boldsymbol{\Psi}e^{-\boldsymbol{\lambda}_e z}\mathrm{d}z\right)\mathbf{q}. \quad (11)$$

The matrix $\boldsymbol{\Psi}$ is a function of the linear combination of the eigenvectors in Eq. 9, and depends on the nature of the quadratic quantity evaluated.

For example, the kinetic power associated to the deformation of the solid frame of the porous material is analytically expressed as

$$\mathcal{K}^s = \frac{\omega^2}{4}\rho_1 \int_z |\mathbf{u}^s(z)|^* \, |\mathbf{u}^s(z)| \, \mathrm{d}z. \quad (12)$$

To calculate this integral, the relevant physical field $f(z)$ (i.e. $u_x^s(z), \sigma_{yz}^s(z), p(z)$) is extracted from $\mathbf{s}(z)$, through the introduction of a boolean matrix $\mathbf{L}_f$. Thus, if the physical fields involved in the governing equations will yield a vector $\mathbf{s}(z)$ of size $(m \times 1)$ where the field $f(z)$ is on the line m_f, the boolean matrix $\mathbf{L}_f$ will be of size $(1 \times m)$ with a 1 in the row m_f.

For the calculation of the kinetic power defined in Eq. 12, the quadratic matrix $\boldsymbol{\Psi}$ associated with the kinetic power $\mathcal{K}^s$ of the deformation along $i = x, y, z$ is

$$\boldsymbol{\Psi}\left(\mathcal{K}^s\right)_i = \frac{\omega^2}{4}\rho_1 \boldsymbol{\phi}_e^* \mathbf{L}_{u_i^s}^* \mathbf{L}_{u_i^s} \boldsymbol{\phi}_e, \quad (13)$$

where ρ_1 is the density of the solid frame of the porous material.

5. Results

To illustrate the proposed methodology, the power dissipated through different mechanisms and the kinetic power of the solid frame of the porous media within the configuration in Fig. 1 are calculated under normal incidence, i.e. $\theta_1 = \theta_2 = 0°$. As can be observed in Fig. 3, the motion along the z direction governes the kinetic power of the solid phase of the porous media. This is expected due to the perfect coupling between the aluminium face sheets and the porous core. Nonetheless, as the relative alignment of the material reaches an angle of $\beta = \pi/4$ rad with respect to the global coordinate system, the kinetic power contribution from the motion along the x direction increases to the same magnitude as

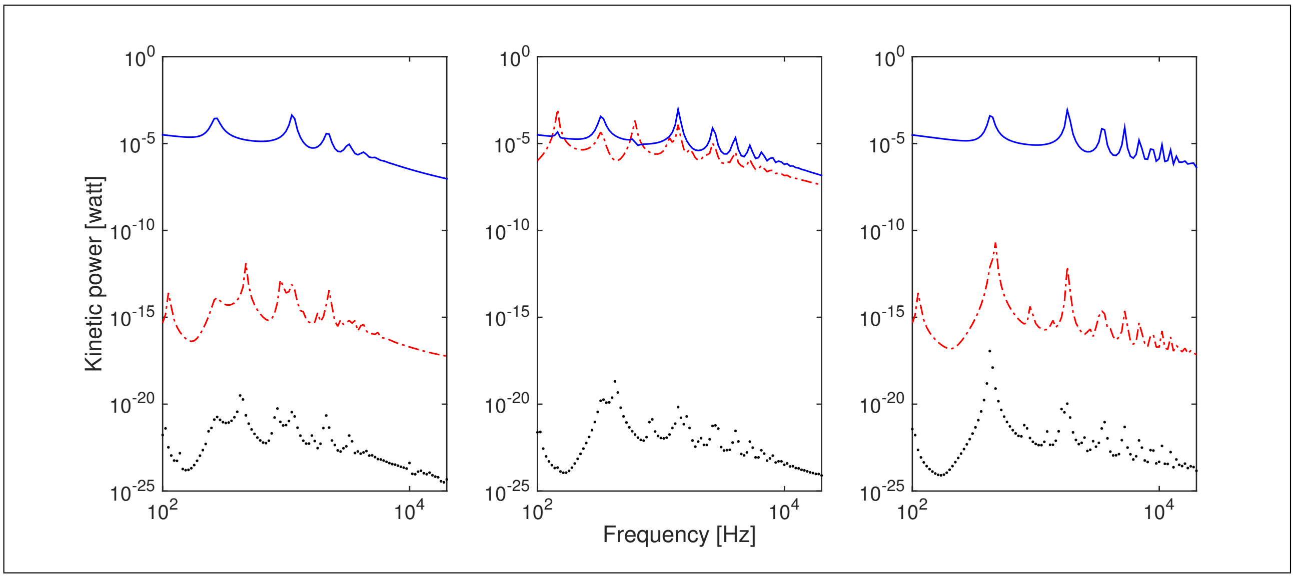

Figure 3. Kinetic power of the solid frame of the porous material associated to the motion along x (dash-dotted line), along y (dotted line), and along z (solid line), in the setup under normal incidence, with respect to the relative alignment of the foam, (left) for $\beta = 0$ rad, (centre) for $\beta = \pi/4$ rad, and (right) for $\beta = \pi/2$ rad.

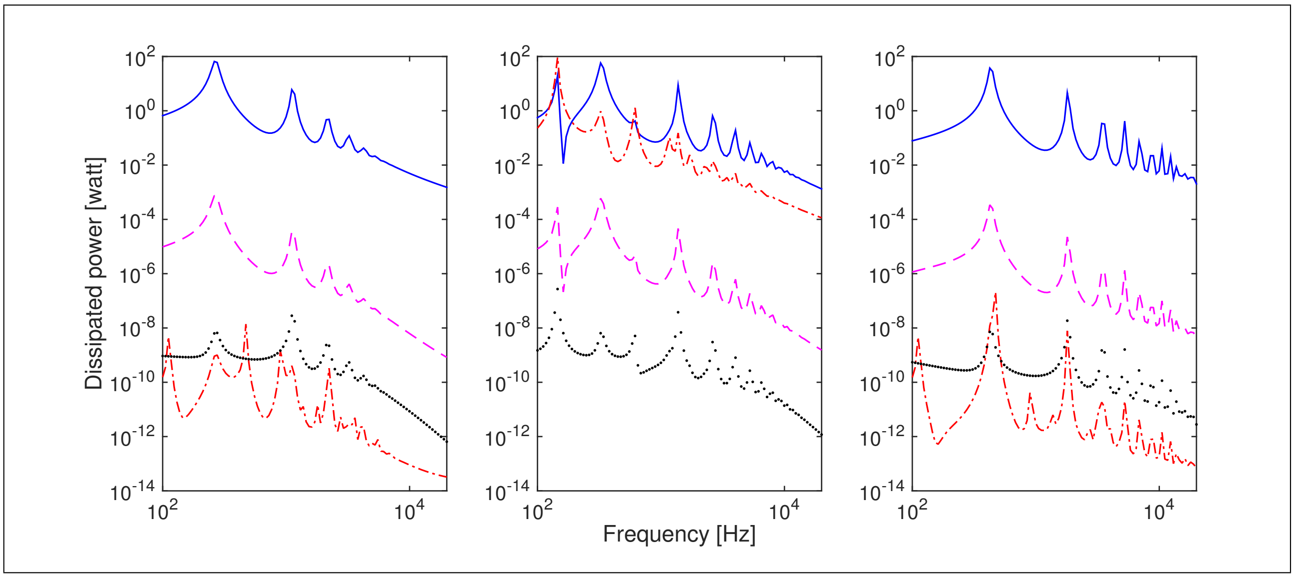

Figure 4. Dissipated powers by thermal losses (dashed line), by viscous losses (dotted line), by structural losses through compression in zz (solid line) and through shear in xz (dash-dotted line) within the porous core layer of the setup under normal incidence, with respect to the relative alignment of the foam, (left) for $\beta = 0$ rad, (centre) for $\beta = \pi/4$ rad, and (right) for $\beta = \pi/2$ rad.

the contribution from the kinetic power resulting from the z direction, and even exceeding it for several resonant frequencies. A similar analysis of the dissipated power due to internal losses in the solid phase of the porous material shows the same trend for the shearing motion in the (xz) plane, see Fig. 4. The dissipated power by the compressional motion in the z direction dominates the global dissipated power. Nonetheless, the dissipated power due to shear in the (xz) plane increases to the same magnitude for a relative alignment of $\beta = \pi/4$ rad. These results suggest that the compression-induced shear motion, specific of anisotropic media, strongly influences the power transmission and dissipation in this setup.

6. Conclusion

A method for calculating the power balance in multilayered structures including anisotropic porous materials was proposed. An investigation of the power contributions related to different deformations, indicate that for a simple multilayered panel with an anisotropic core compression-induced shear motion phenomena are important.

Furthermore, the proposed methodology underlines the importance of the relative alignment of the porous material coordinate system, clearly influencing the energy transmission and dissipation through the structure.

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