A model for diffracting elements to reduce traffic noise

Y.H. Wijnant
Faculty of Engineering Technology, University of Twente, Enschede, the Netherlands.
4Silence, Enschede, the Netherlands.

J. Hooghwerff
M+P Consulting Engineers, Vught, the Netherlands.

Summary
There is a great demand for noise reducing measures to fulfill noise legislation and to reduce the annoyance of traffic noise. Silent pavements and noise barriers are well-known, but are there other alternatives? Recently, a new sound diffracting element was developed; a concrete block with cavities that is placed on ground level in close proximity of the road. These diffracting element deflect tyre-road noise in an upward direction, creating a zone of noise reduction behind the elements. It can therefore act as a complement to existing noise-reducing measures and can be optimized for maximum noise reduction. Based on a finite element model, it is possible to assess the performance of the diffracting elements. However, these models are impractically large for optimization purposes. We therefore developed a very fast, semi-numerical model, which makes optimization of the geometry feasible. In this paper we present this so-called piston-model and report the validation of its correctness by comparison with a finite element model. In an accompanying paper of J. Hooghwerff, a measurement setup and actual experimental results for the diffracting elements will be presented, showing that this innovation can increase noise reduction up to 4 dB.

PACS no. 43.20.+g, 43.40.+s

1. Introduction

Tyre-road noise has been a major problem for a long time and will still be for decades to come. Over the years, two options have been available to reduce it: silent pavements and noise barriers. Silent pavements have been developed (and are still being developed) and applied regularly. If the reduction obtained with these silent pavements is insufficient, noise barriers can be used to obtain higher levels of reduction. Silent pavements can reduce tyre-road noise by 4-7 dB, but it should be mentioned that these reduction values usually decrease over time. Noise barriers retain their high value of reduction of 5-10 dB but can be quite expensive.

Recently, a third option to reduce traffic noise has been introduced; diffracting elements. Diffracting elements, an invention of 4Silence, are concrete blocks with cavities placed alongside the road. The cavities are acoustically tuned to resonate at frequencies which need to be reduced. As a result of this tuning, the grazing incident sound waves emitted by the tyre, are

Figure 1. Diffracting element alongside the road near Soesterberg (N413), the Netherlands.

deflected in an upward direction and a reduction is obtained behind the diffracting elements. If a sufficient amount of cavities are used, a good overall reduction of up to 4 dB can be obtained.

One type of diffracting element is shown in figure 1. Apart from the acoustical feature of diffraction, obviously, safety, drainage, robustness and maintenance are aspects that need to be taken into account. For instance, the wedge shaped extensions to the concrete ribs are a safety feature which also allows motor cyclists to drive safely over the diffracting elements. These aspects however are beyond the scope of this paper.

The acoustic effect of diffracting elements on the sound pressure levels behind the diffracting elements can be analyzed by means of a (2 or 3 dimensional) finite element model. The advantage of such a model is the versatility and ease by which various designs can be analyzed. However, especially for a 3 dimensional model, the large model size needed (a large domain needs to be simulated) and the large number of frequencies required to analyze the performance of the resonators, make these models impractically large and slow. Although one might assess a certain design, an optimization using a finite element model can be considered infeasible.

To reduce the model size and computational efforts, we developed a, so-called, piston-model. In this model all resonators are modeled as pistons, which oscillate at a certain velocity. Given a sound source (and a mirror sound source), the velocity of the pistons can be calculated by 'matching' the impedance at the pistons surface with the impedance of a tube resonator. A system of equations can then be derived for the unknown piston velocities, which can easily be solved numerically. The resulting model is very fast (the number of degrees of freedom is equal to the number of resonators) and surprisingly accurate.

In this paper, a general introduction is given to why resonators, theoretically, are efficient to reduce tyre-road noise. Secondly, the piston-model is introduced and its solution is compared to the full finite element solution. In an accompanying paper by J. Hooghwerff et al. [1], we also experimentally show that resonators do reduce tyre-road noise; in pilot projects overall reduction values in the order of 2-4 dB were measured using a method similar to the statistically pass-by method (SPB).

2. Theory

2.1. Introduction

Why resonators should be used at all to obtain optimal reduction is sometimes questioned. One could reason that, based on linear acoustic theory, no acoustic energy would actually be dissipated and therefore no

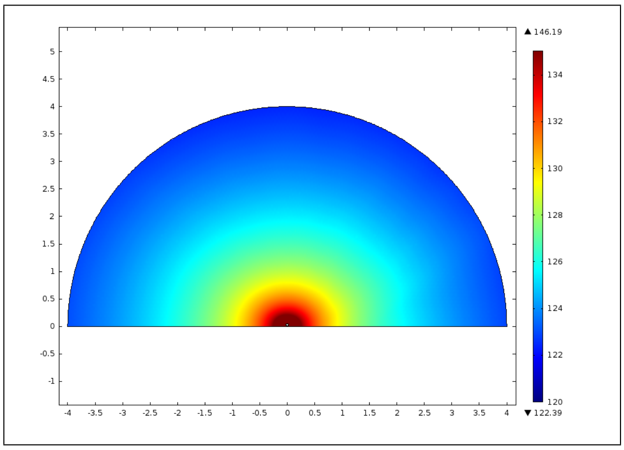

Figure 2. Sound pressure level due to a cylindrical source at $1000 Hz$. The ground impedance of the $1[m]$ strip at $0.75[m]$ from the source is $Z = 100\rho c$.

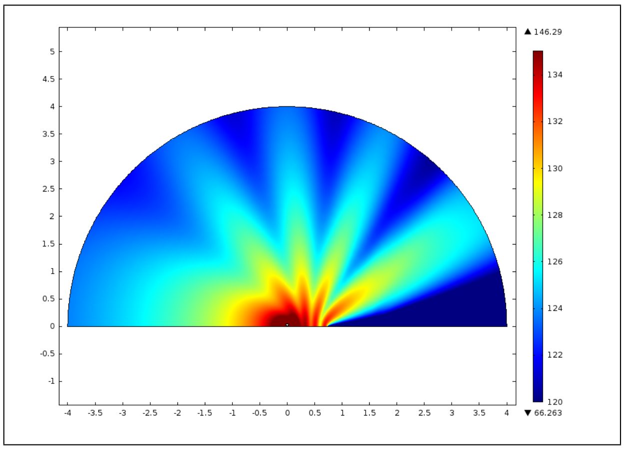

Figure 3. Sound pressure level due to a cylindrical source at $1000 Hz$. The ground impedance of the $1[m]$ strip at $0.75[m]$ from the source is $Z = 0.01\rho c$.

reduction could be achieved. This reasoning is however erroneous; the fact that resonators do provide optimal reduction can be illustrated as follows.

Consider a 2 dimensional model of a semi-cylinder, shown in figure 2 and 3. Close to the center of the semi-cylinder, a simple pulsating cylindrical sound source is placed (shown as a small circle in figure 2 and 3). The flat surface (the ground) is assumed to be fully reflective, apart from a $1[m]$ wide section at $0.75[m]$ to the right of the sound source. This surface has been given an specified normal impedance $Z_n = P/U_n$, where P denotes the acoustic (complex) pressure and U_n is the acoustic normal velocity. Hence, if the impedance is high, a fully reflective surface is obtained. If the impedance equals the characteristic impedance ($Z = \rho c$), the surface fully absorbs *normal incident* sound waves. If the impedance is $Z = 0$, a pressure release surface is obtained. The remaining boundaries on the semi-cylinder are assumed to fully absorb all normal incident waves.

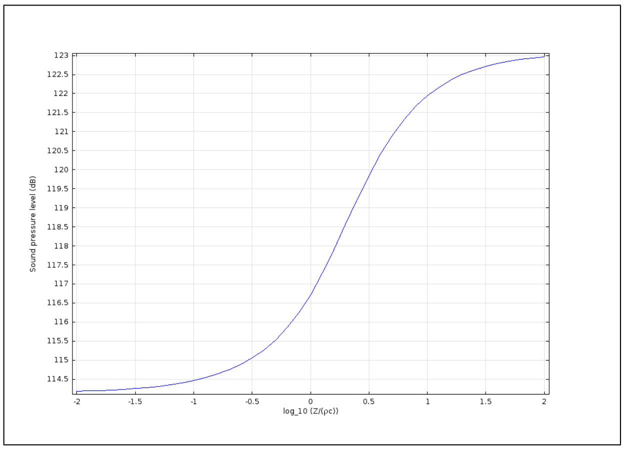

Figure 4. Sound pressure level in the right end corner as a function of $log_{10}(Z/(\rho c))$

A finite element analysis results in the sound pressure levels given in figure 2 and 3 for, respectively, an impedance of the $1[m]$ strip of $100\rho c$ and $0.01\rho c$. From the figures, it is clear that reducing the normal impedance of the $1[m]$ strip between a source and a receiver behind the strip, results in a reduction of the sound pressure levels behind the strip. A 'piece of pie'-shaped reduction region is observed, starting at the impedance strip. The reduction comes at the expense of an increase in sound pressure level at higher positions. The strip thus 'bends' the acoustic wave in an upward direction, explaining the term diffracting element.

The sound pressure levels in the right end corner of the semi-cylinder (at $(4, 0)[m]$), as a function of the impedance $log_{10}(Z/(\rho c))$, is shown in figure 4. It is seen that the largest reduction is actually obtained if the impedance is minimized. Note that for a surface which fully absorbs normal incident sound waves ($log_{10}(Z/(\rho c)) = 0$), the sound pressure level is still more than $2.5[dB]$ larger than if the impedance of the $1[m]$ strip is close to zero. Hence, the general objective to reduce grazing incident sound waves (for a ground surface alongside the road) is thus to minimize the acoustic impedance! Ground surfaces that optimally absorb normal incident sound waves do not provide optimal sound pressure level reduction behind that surface.

Practically, the minimization of the impedance implies the use of resonating elements; for a specific frequency or frequency range, resonating elements have a minimal impedance at the resonators opening.

2.2. The piston-model

As was explained in the introduction, a model describing the effect of the resonators on grazing incident sound waves, needs to be both accurate and fast if it is to be used in an optimization algorithm. This is accomplished by means of a so-called piston-model,

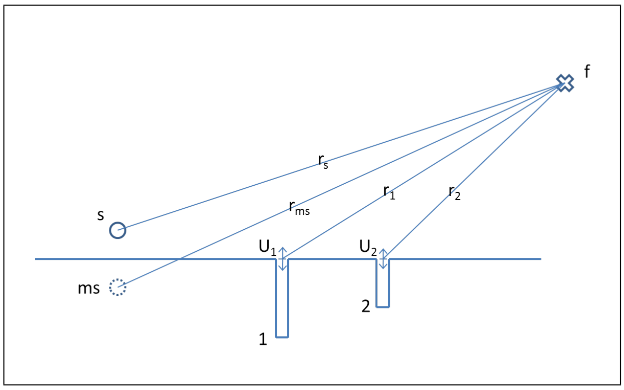

Figure 5. The piston-model (general).

where we describe the effect of the resonator by vibrating pistons. The pistons will start to oscillate due to the pressure wave induced by a sound source. In addition, the vibration of a piston also induces a pressure on the piston itself and on all other pistons. The velocity of the pistons are obtained numerically from a linear set of equations, as explained in this section. Once the velocity of the pistons is known, the sound pressures in any point can be evaluated analytically.

We explain the piston-model, illustrated in figure 5, by means of a pulsating cylindrical sound source. We use this simple source, but any source can be used as long as the full reflection from the ground of that source is accounted for. Hence, we assume that the pressure, measured in any field point f, is due to a given pulsating cylinder at s, its mirror sound source at ms and a number of 'vibrating pistons,' located at each of the resonator openings. These pistons vibrate with a velocity U_i.

The pressure in any point in the field (x_f, y_f), due to a pulsating cylinder located at (x_s, y_s) is known, see [2], to be equal to

$$P_s = \frac{i\rho c u_s H_0^2(kr_s)}{H_1^2(ka_s)}, \tag{1}$$

where $i = \sqrt{-1}$, ρ denotes the density, c the speed of sound, u_s, the surface velocity of the pulsating cylinder, $H_0^2(z)$ the zero'th order Hankel function, $k = \omega/c$ the wave number, $r_s = \sqrt{(x_f - x_s)^2 + (y_f - y_s)^2}$ the distance from the cylinder to the field point f and a_s the radius of the cylinder. To account for the (full) reflection of the road, we add a pulsating cylinder at $(x_{ms}, y_{ms}) = (x_s, -y_s)$. The pressure in the field point due to this mirror source is thus:

$$P_{ms} = \frac{i\rho c u_s H_0^2(kr_{ms})}{H_1^2(ka_s)}, \tag{2}$$

where $r_{ms} = \sqrt{(x_f - x_{ms})^2 + (y_f - y_{ms})^2}$. The pressure induced by a vibrating piston in a baffle at $(x_p, 0)$ follows from [2]:

$$P_p = \rho c k a_p H_0^2(kr_p) U_p, \tag{3}$$

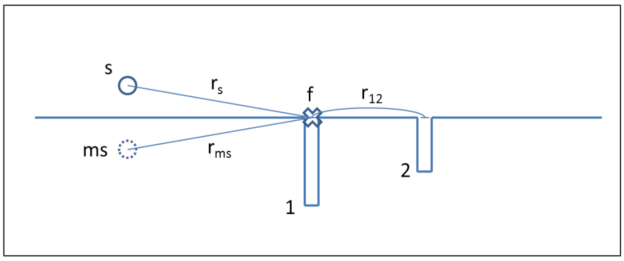

Figure 6. The piston-model (field point on the piston).

where a_p is half of the width of the piston, $r_p = \sqrt{(x_f - x_p)^2 + (y_f)^2}$ and U_p is the, yet unknown, piston velocity. As the system is linear, we can add the pressure due to additional vibrating pistons in the baffle, to the overall sound field.

The unknown piston velocities can be solved from a scattering analysis by matching the impedance on the location of each piston by the impedance of a tube resonator, as is shown in figure 6. That is, we know that the impedance Z_p of a tube resonator p of length L_p at the opening of the resonator equals:

$$Z_p = \frac{P_p}{U_p} = i\rho c \cot(kL_i), \tag{4}$$

and thus $P_p = Z_p U_p$. In addition, the sound source and its mirror source, as well as all vibrating pistons, *except* for piston p itself!, do not induce any normal velocity at piston p. Hence, we can use the expressions given above, evaluate them at the location of each piston and set up a system of equations for the unknown piston velocities U_p.

Due to the singularity of expression 3 for $r_p = 0$, it can not be used to calculate the pressure at piston p due to the velocity of piston p. Assuming the piston size to be small compared to the wavelength, we can however use the expression for the radiation impedance of an infinite strip given by Lipshitz [3]:

$$Z_{pp} = \rho c k a_p \left\{1 + i\frac{2}{\pi}\left[\frac{3}{2} - \gamma - ln(ka_p)\right]\right\}, \tag{5}$$

where γ is Euler's constant (≈ 0.577). The pressure on piston p due to the piston velocity U_p thus equals $Z_{pp}U_p$. Adding up all mentioned contributions, results in the system of equations for the unknown piston velocities.

As an illustration, the system of equations for 2 resonators, thus becomes

$$Z_1 U_1 = P_{s1} + P_{ms1} + Z_{11}U_1 + Z_{12}U_2 \tag{6}$$
$$Z_2 U_2 = P_{s2} + P_{ms2} + Z_{21}U_1 + Z_{22}U_2, \tag{7}$$

where P_{sp} is the pressure on piston p due to the cylindrical sound source, P_{msp} is the pressure on piston p due to the mirror source, $Z_{ij} = \rho c k a_j H_0^2(kr_{ij})$ and r_{ij} the distance between piston i and piston j.

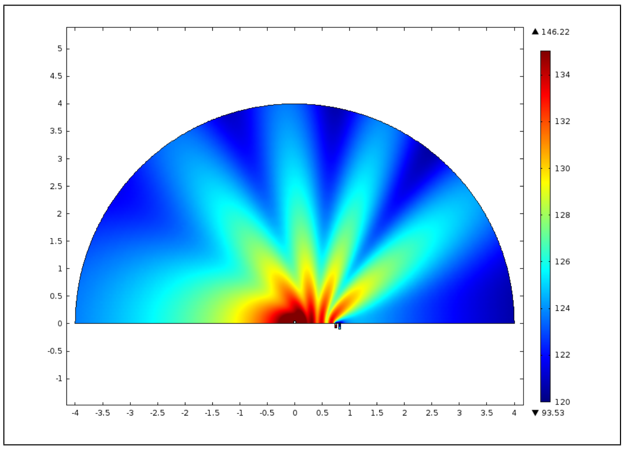

Figure 7. Sound pressure level (fem) at 1000$[Hz]$

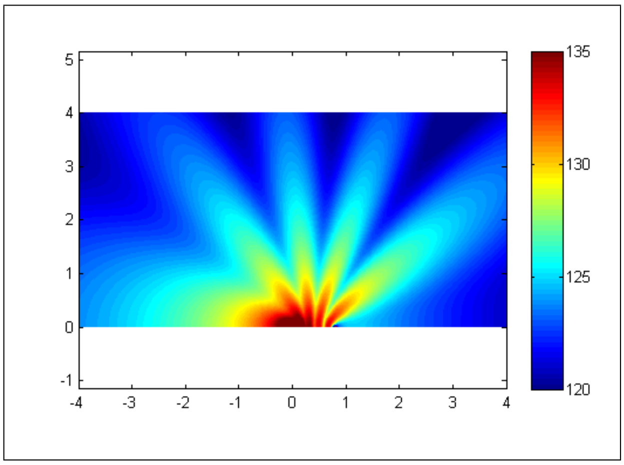

Figure 8. Sound pressure level (piston-model) at 1000$[Hz]$

Generalized to any number of resonators, the system of equations can be rewritten to the linear system of equations:

$$\mathbf{ZU} = \mathbf{P}, \tag{8}$$

where:

$$Z_{mm} = i\rho c \cot(kL_m) + \tag{9}$$
$$-\rho c k a_m \left\{1 + i\frac{2}{\pi}\left[\frac{3}{2} - \gamma - ln(ka_m)\right]\right\}$$

and

$$Z_{mn} = -\rho c k a_p H_0^2(kr_{mn}) \tag{10}$$

if $m \neq n$. This system is easily solved numerically.

3. Results

3.1. Validation

As an illustration of the accuracy of the piston-model, we compare the results of the piston-model with a

(Comsol) finite element solution for a 2-resonator configuration shown in figure 7. The frequency used in the simulation is $1000[Hz]$. In the figure, one can see the pulsating cylindrical of radius $a_s = 0.02[m]$ at $(x_s, y_s) = (0, 0.03)[m]$ and the two resonators of width $2a_1 = 2a_2 = 0.03[m]$ placed at $(x_1, y_1) = (0.75, 0)[m]$ and $(x_2, y_2) = (0.82, 0)[m]$. The length of the resonator on the left is $0.08[m]$. The length of the resonator on the right is $0.1[m]$. From the solution, one cleary observes the diffraction caused by the resonators, i.e. the sound pressure level reduces behind the resonators at the expense of an increase in sound pressure level at higher points. This is similar to the minimized impedance surface shown above.

Keeping all parameters identical to the finite element model, the sound pressure level obtained by the piston-model is shown in figure 8. One observes that, although the number of degrees of freedom in this piston-model is 2! compared to the nearly 35000 degrees of freedom in the finite element model, the piston-model solution is both qualitatively and quantitatively almost identical to the finite element solution. The computation time and model size of the piston-model is however negligible.

3.2. Example: noise reduction of a diffracting element having 18 resonators

As the piston-model is computationally efficient, it can be used to optimize the resonator dimensions such that diffraction is maximized in a certain frequency range. Although the optimization is beyond the scope of this paper, we illustrate the efficiency of the piston-model, as well as the feasibility of diffracting elements to reduce tyre-road noise, by calculating the sound pressure level at $7.5[m]$ from a sound source, when a diffracting element having 18 resonators is placed at $0.75[m]$ from the source. Because the frequency range in which tyre-road noise is dominant, is approximately between $550[Hz]$ and $1500[Hz]$, the depths of the resonators have been chosen to range evenly between a quarter of the wavelength associated with the largest, respectively, the lowest frequency of interest. Hence, the resonator depths range from $15.5[cm]$ to $5.7[cm]$. The reduction in sound pressure level, i.e. the difference between the sound pressure level without and with a diffracting element, is shown in figure 9.

For this simple sound source at $3[cm]$ above the ground, the model predicts a reduction (below 2 m) in the order of $5[dB]$ between 550 Hz and 1000 Hz, decreasing to values in the order of $2.5[dB]$ at 1400 Hz, after which it increases again to values above $5[dB]$ for frequencies above $1500[Hz]$. The computational time to calculate this graph is negligible. To illustrate the efficiency even more, computation times are the same for evaluation of the reduction at $50[m]$!; a finite element model simulating this domain does not even fit into a normal laptop. The result is shown in figure 10. Note the increased height of the evaluation points in figure 10.

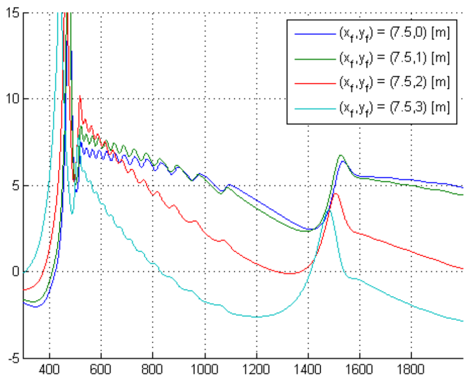

Figure 9. Sound pressure level reduction at $7.5[m]$ away from the source, due to the placement of 18 resonators at $0.75[m]$ from the source at $0[m]$, $1[m]$, $2[m]$ and $3[m]$ high.

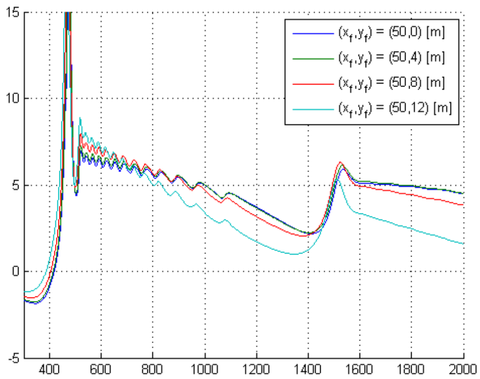

Figure 10. Sound pressure level reduction at $50[m]$ away from the source, due to the placement of 18 resonators at $0.75[m]$ from the source at $0[m]$, $4[m]$, $8[m]$ and $12[m]$ high.

The figure illustrates an additional advantage of the diffracting elements; since the grazing incident wave is diffracted, reduction is seen to be larger at larger distances. At a $50[m]$ distance, even at heights of up to $8[m]$, a good reduction is obtained.

4. Conclusions

In this paper, we introduce a so-called piston-model to assess the performance of diffracting elements to reduce tyre-road noise. The model has been validated by means of a finite element model and good agreement, both qualitatively as quantitatively, has been obtained. The piston-model can be used in an optimization algorithm, as it is both small and fast. The piston-model has been extended to 3D, but this model will be reported elsewhere.

Acknowledgement

I would like to acknowledge former master students Remco Brantjes, Onno de Boer and Mart Bruggink for their help in developing the current method.

References

[1] J. Hooghwerff, H.F. Reinink, W.J.van der Heijden,Y.H. Wijnant: Whisstone, a sound diffractor: does it really affect traffic noise?. Proc. Euronoise 2015.

[2] D.T. Blackstock: Fundamentals of physical acoustics. John Wiley & Sons, 2000, ISBN 0-471-31979-1.

[3] P Stanley Lipshitz: On the acoustic impedance of strip radiators. Proc. Audio Engineering Society convention 1990.