The influence of finite sample size on surface impedance determination of materials with low sound absorption at low frequencies

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Summary
The most common noise reducing measure is to add sound absorbing material on the domain boundaries. The boundaries covered by the material may in simulations be represented by the surface impedance of the material. The impedance can be either modeled or determined experimentally. The experimental determination can be done by the well known standing wave tube method or by a free field method. These free field methods enable impedance determination at any angle of incidence for bulk reacting materials, as opposed to the standing wave tube method that is restricted to normal incidence or locally reacting materials. The method prescribes a point source above the surface and measurements in two points close to the sample surface. From this, the surface impedance can be deduced through the known sound field formulation. Among other things, the impact on the accuracy of the method from the field formulation, signal conditioning and sensor type have been studied in previous work. One major concern is the finite size of the material sample, and its influence on the measurement accuracy. This has previously been investigated for highly absorbing materials and it was shown to be a low frequency problem. Therefore, we focus on the impact of the finite sample in frequencies below 2 kHz. In particular, we relate the magnitude of the impact to the properties of the tested material. Also, the influence of the mounting of the material is analyzed. The study is made through analyzing numerical simulations of the experiment for a variety of setups and materials. Theoretical discussion is provided for deeper understanding of the results. The impact of the finite sample is seen to depend on the material properties, not only the setup as previously shown. Materials with high absorption are shown to be more sensitive to these errors.

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1. Introduction

Sound absorbing materials are used in a wide range of applications to attenuate sound. The sound attenuation obtained in a specific domain from an absorbing material on the domain boundaries can be determined through simulations where the material is represented by its surface impedance as a boundary condition. The surface impedance of an absorbing material can either be modeled or determined experimentally. Developing existing and new experimental methods to determine the surface impedance is of importance in order to provide accurate input to the numerical simulations, especially for complex and bulk reacting materials. The accuracy of one of these measurement methods is of interest in this paper.

Experimental determination of the surface impedance of the material can be performed either in free field [1]-[8] or in a standing wave tube. The standing wave tube method is based on normally incident plane waves [9] and is well established and standardized, however, it suffer from some drawbacks related to e.g. cut outs of samples [10]. It is also limited to locally reacting samples when arbitrary angles of incidence is of interest (annex F [9]). The free field methods on the other hand allow determination of the surface impedance at any angle of incidence for both locally and bulk reacting materials avoiding the limitations of the standardized method. In the free field methods, a point source is placed a certain distance above the sample and the acoustic pressure is measured close to the material

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surface. The impedance of the surface is then deduced from the pressure measurements, assuming that the formulation of the sound field above the impedance surface is know. During the experiment and in the post-processing, there are several possible sources of error that in turn introduce errors in the final result for the impedance. Studies have been performed to understand and quantify the impact of the different sources of error [11]. Among other things, errors associated with the probe type [12][13], with the theoretical description of the sound field [6][14] and sensitivity analysis [15] have been investigated. These sources of error are not investigated further in this paper.

A correct formulation of a sound field above a surface is essential for an accurate deduction of the surface impedance from the field measurements. The total field in the measurement in an ideal case comprises the direct spherical sound waves from the point source and the waves reflected on the impedance boundary. The reflected waves for bulk and locally reacting surfaces has been formulated in many ways [5][8][16]-[19] and the accuracy of the different models have been investigated thoroughly both through simulations and measurements [6][12][14][20].

The sensitivity of the measurement result to unwanted reflections from,.e.g. the walls in the measurement room, have been studied and means to minimize the impact of these errors in the final results have been suggested [21][22][23]. Last but not least, the errors associated with the finite size of the sample have been studied [12][14][24][25]. The finite sample causes three main phenomena leading to errors in the measurement results; The field description is derived for an infinite surface, the sound field in the sample contains several reflections and the edges causes diffracted waves in the field above the material. Numerous studies comparing experimental results and numerical simulations in BEM have been performed to understand the impact of the finite sample size at normal incidence [12][14][24][25]. This problem is most evident when measurements are performed in the centre of a square sample [12] and techniques to reduce the impact have been proposed [12][24][25]. It is concluded that the finite sample size is a problem, however, no real explanation has to the authors' knowledge yet been proposed as to why these relatively weak edge-diffracted waves has such a large impact on the results. Neither has a thorough investigation on in which frequency range the impact is negligible related to the material properties and the measurement setup. The error analysis has also been focused on the normal incidence cases, and not the more general oblique incidence case. The question of the mounting of the sample has not yet been analyzed either.

In this paper, the impact of the finite sample size on the measurement accuracy is analyzed. The study is performed using numerical simulations of the measurement setup in Finite Element Method software for square samples of bulk reacting porous materials. The analysis is performed at normal incidence for the free field method using two microphones in the frequency band 0 - 2 k Hz. Three different materials for different setup configurations are used to investigate the impact in the results related to the materials themselves. The frequency ranges where the edge diffracted waves have a significant impact in the measured field related to the material properties and the measurement setup are also identified and discussed.

In the first section of the paper, the measurement method and the impedance deduction procedure is described in brief. Thereafter, the numerical simulation setup and materials are presented. This is followed by the results from the numerical simulations and comments on the results are presented. In the last section, conclusion are drawn and discussion on the impact of the sample size and its mounting on the measured field are presented.

2. The free field method

In this section, the free field measurement method using two microphones is described. The measurement setup, the field formulation and post-processing are also discussed.

2.1. The measurement setup

The method for measuring the surface impedance at normal and oblique incidence was first suggested by Ingard [16] and has since been further developed by for example [1] and [6]. The common setup for these measurements are given in Figure 1.

The measurement position is denoted z_i and in the method used in this paper the pressure is measured in two points. The source is placed a specific distance from the sample with a certain angle of incidence. The pressure in both points are registered as well as the transfer function between the points. The distance in between the microphones determines the valid frequency range [26].

2.2. Field formulation and post processing

The post processing of the measured data is a deduction of the surface impedance from the measured pressures if the sound field from a point source above an infinite impedance surface can be formulated theoretically. The field formulation and hence the impedance deduction can be made in several ways (see for example [8][12][24]). Some of the more recent studies on the accuracy of the measurement method have been focused on the accuracy of the different field formulations. Especially for bulk reacting materials, this is of great importance and several studies have concluded that bulk reaction has to be included in the field description for materials experiencing bulk reaction [20].

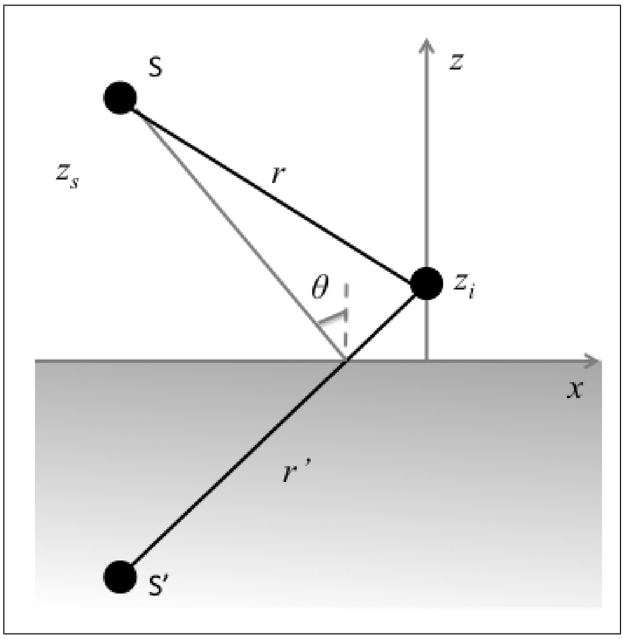

Figure 1. Illustration of the measurement setup for the free field methods. The source S is placed on a height z_s from the material surface forming an incident angle θ on the reflected wave from the mirror source S'. The microphone are placed in positions z_1 and z_2, marked z_i in the figure. The distances are indicated as r and r'.

Independently, every field description includes a direct and a reflected field. The difference is seen in the formulation of the reflected field, which can be formulated with different level of complexity; from a simple mirror source [1][12] to multiple integrals over the material surface [17]-[19]. Common for all formulations though, are that the source is a monopole point source and that the sample is infinite in extent, i.e. the surface impedance is uniform along all the x-axis. One of the simplest formulations, leading to an explicit determination of the surface impedance is given as

$$p(z) = p_{in}\frac{e^{iwt-ikr}}{r} + Rp_{in}\frac{e^{iwt-ikr'}}{r'}, \tag{1}$$

with notation following Figure 1 where k is the wave number in air. Note here that the time dependence $e^{i\omega t}$ is assumed. This formulation is based on spherical spreading of one point source and its mirror source and the pressure in one point is hence the sum the contribution from both sources. If the sample is finite, diffracted waves from the material edges will also be present in the field that will deviate from the field in Equation (1). It is the error due to this that is of interest in this study.

In this paper, the focus is to analyze the impact of the finite size of the sample on the measurement accuracy for different materials and measurement setups. The absolute error of course depends heavily on the choice of field formulation, although the error in the measured data are the same. The affect on the measured data is of main interest in this paper, since this explains more about the edge diffracted waves than the final post processed absorption coefficient. However, the simple post-processing in Equation (1) is used on the data to analyze how the variation in the error in the measured data affects the absorption coefficient. This is hence to see the for which materials and measurement setups the impact on the final data is most pronounced. For example, the increase or decrease of the impact due to a certain parameter is observed in the absorption coefficient with all field formulations. The absolute size of the impact on the other hand can not be uniquely determined. The absorption coefficient will be presented to show how the error behaves and the analysis of the error is done from the measured data, i.e. the pressure in two points unaffected by any post processing.

The reflection coefficient is deduced by formulating the pressure in the two points z_1 and z_2 according to Equation (1), setting the transfer function H to $p(z_1)/p(z_2)$ and rearranging the equations to Equation (2).

$$R = \frac{\frac{e^{-ikr_2}}{r_2} - H\frac{e^{-ikr_1}}{r_1}}{H\frac{e^{-ikr'_1}}{r'_1} - \frac{e^{-ikr'_2}}{r'_2}} \tag{2}$$

The subscripts 1 and 2 refers to position z_1 and z_2, respectively. The absorption coefficient and surface impedance (for normal incidence $\theta = 0$) are related to the reflection coefficient in Equation (3).

$$\alpha = 1 - |R|^2, Z_s = \frac{\rho_0 c_0 - R}{\rho_0 c_0 + R} \tag{3}$$

3. Numerical simulation setup

The sensitivity of the free field method to the finite sample size and the mounting of the sample is studied through numerical simulations in the FEM software COMSOL [27]. The simulations are made in a spherical calculation domain with rigid floor and non-reflecting boundaries above (representing a perfect semi-anechoic room) by means of perfectly matched layers (PML). To reduce computational time the symmetries of the problem are used to reduce the computational domain to a eight of the total sphere. The mesh size is based on the rule of thumb of 6 nodes per wave length. The porous materials are modeled as equivalent fluids [28]. The materials are characterized by their flow resistivity and material A, B and C have flow resistivity 5 000, 25 000 and 55 000 Rayls/m respectively. The choice is based on typical parameters for absorbers used in many applications, both with high and low absorption. The simulations are performed for the frequency interval 10 - 2 000 Hz with a frequency resolution of 10 Hz. The size of the calculations sphere is determined so that the finite sample and the point source are within the computational domain, not impinging on the PML.

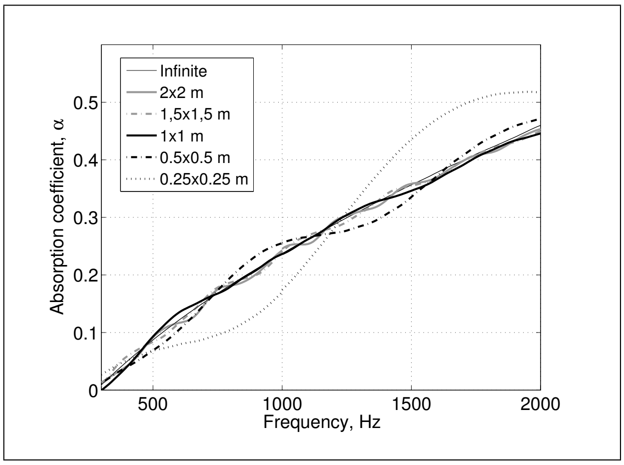

Figure 2. Absorption coefficient for Material A of different sample size. The loudspeaker distance is constant at 1 m.

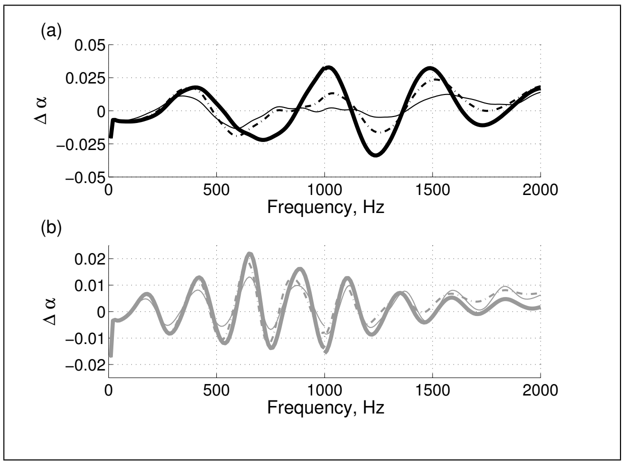

Figure 3. Difference in absorption coefficient for speaker distance 1 m for different material. Material A (thin solid), material B (thick dash-dot) and Material C (Thick solid). Sample side length 1 m (a) and 2 m (b).

4. Results and discussion

Numerical simulations have been performed according to the setup described in section 2.1 and 3 and using the theory in section 2.2 to analyze the impact of the sample size on the obtained absorption coefficient of the analyzed materials (section 4.1). The influence of the mounting of the sample in the room has also been investigated at normal incidence in section 4.2. The different sample side lengths are 0.25, 0.5, 1 and 2 meters and the source is position was varied between 0.2 and 1 meter from the sample. The measurement positions are $z_1 = 2$ cm and $z_2 = 3$ cm.

4.1. Finite sample

The impact on the absorption coefficient of the finite sample size has been investigated on the three porous materials for normal incidence. The size of the finite sample and the distance between the source and sample have been varied for all materials in order to determine the sensitivity of the method for these parameters. The absorption coefficient has been calculated from the numerical simulations of the measurement method. The influence of the sample size is studied by fixing the source distance and varying the sample size for all three materials and studying the impact on the obtained absorption coefficient and measured pressures.

As an example on how the finite size impact differs with the sample side length, the absorption coefficient of material A with the source 1 meter from the sample of varying sample size is shown in figure 2.

The disturbance is seen as oscillations about the correct value, as discussed in previous work [12][24][25], where the period depends on the sample size. The amplitude of the disturbance is larger and the period longer for small samples due to the phase and amplitude difference in the diffracted waves from the sample edge. Making the same analysis for material B and C show the same trends, however, the amplitude of the errors are larger the higher the absorption of the material is. The fact that the size of the impact on the absorption coefficient depends on the material itself has to the authors' knowledge not yet been reported in previous research. In figure 3, the difference in absorption from a finite sample and an infinite sample is shown for all three materials at source distance 1 meter for a sample with side length 1 and 2 meters. This indicates that the impact on the measurement accuracy of the finite sample size depends not only on the measurement setup but also on the material sample measured. The accuracy is reduced to a larger extent due to the sample size when a material with high absorption is used.

The impact is hence larger for 1) a highly absorbing material for 2) a small sample with 3) a large source distance. The two latter observations may be explained by the fact that for small source distance and large samples the edge diffracted waves are significantly smaller than the direct and reflected wave due to the large difference in propagation paths. In addition to that this confirms previous observations [12][24][25], it highlights the fact that the error is dependent on the material properties and not only the setup.

The magnitude of the impact depends on the material, the sample size and the setup and in addition, the post-processing. By studying the measured pressures, the latter parameter is excluded. The absolute value of the pressures in both measurement points are registered for both an infinite and finite sample in order to see the size of the edge-diffracted waves. From the simulations, it is shown that the edge diffracted waves are stronger for materials with higher absorption. The relative difference in pressure is also larger for the highly absorbing materials. This is the reason for the larger sensitivity to the finite sample for the highly absorbing materials. A sensitivity analysis on

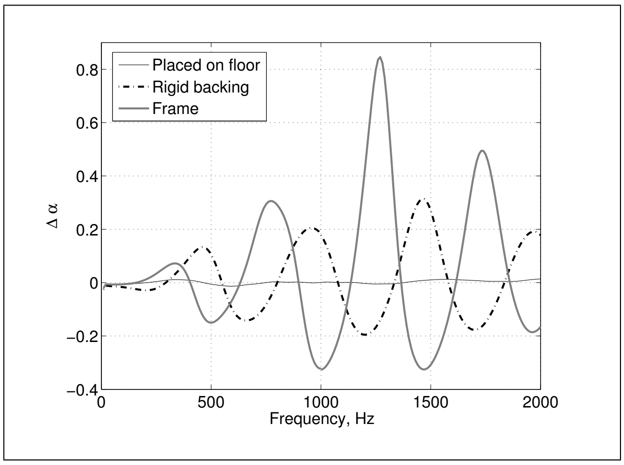

Figure 4. Difference in absorption coefficient for speaker distance 1 m and sample side length 1 m for Material A with different mountings: placed on the floor (thin solid), rigid backing (thick dash-dot) and frame (Thick solid).

the absorption coefficient on the transfer function H (and hence also in $p(z_1)$ and $p(z_2)$) shows that the absorption coefficient is more sensitive to errors in the transfer function in low frequencies. In high frequencies, the error in the transfer function is almost invisible in the absorption coefficient, why the error is less important in high frequencies even though .

In order to explain the sensitivity of the edge diffraction for highly absorbing materials, a formulation of the edge diffracted wave is useful. This part of the sound field can be expressed as an integral of point sources along the entire edge of the sample. These points sources have phase and amplitude difference from the path length from the source and a "source strength" of the edges, dependent on the surface impedance at the edge, see for example [29]. A derivation of this will be provided in future work.

4.2. Mounting of the sample

The impact of the mounting of the sample is investigated in this section. This has to the authors' knowledge not yet been analyzed, although it is of large interest. Especially if the measurements are to be performed in a semi- anechoic room where a rigid backing has to be provided in order to get the surface impedance of the material in front of a wall.

Two different mountings are implemented to observe its impact: A rigid plate to the same size as the sample and a rigid frame under and on the side of the material. The difference in the absorption coefficient to the infinite sample is shown in Figure (4) for the sample placed on the floor without any mounting, mounted on a rigid plate and mounted in a frame.

The result clearly shows the importance of designing the mounting in a careful way if a mounting is needed, for example when measuring in an anechoic room. From the simulations, materials of high absorption is less sensitive to these distrbances.

5. CONCLUSIONS

The sensitivity of the free field measurement method to the finite sample and its mounting has been investigated by numerical simulations on three bulk reacting materials. The sound pressures were evaluated in two points in the field above the sample and the absorption coefficient was determined for different sample sizes, for different source distances for the three materials to see how the impact of the finite sample size is affected by these parameters. Also, the mounting of the material in the measurement room was studied.

The impact of the finite sample is seen as oscillations about the correct absorption coefficient. The period of the oscillations are due to the phase difference between the direct and reflected wave related to the edge diffracted waves, hence the period differs between sample sizes. The amplitude of the oscillations depend on three factors; the material properties, the sample size and the source distance. The amplitude is reduced when the sample size is increased and the source distance is reduced, verifying previous work. More interestingly, it is shown that materials with low air flow resistivity, hence low absorption, are less sensitive to the edge diffracted waves than materials with high absorption. The oscillation amplitude is higher for materials with high absorption, implying the importance of the sample size when measuring highly absorbing materials. This can be explained by looking at the measured pressures. The relative difference in pressure compared to the infinite sample case is larger for the highly absorbing materials and the edge diffracted waves are stronger for these cases.

The impact on measurement accuracy from the mounting of the sample is an issue not previously shown. This is shown to be a important factor when performing measurements, since the impact of this is larger than the edge diffracted waves discussed above. The mounting comes in to play when for example measuring in a measurement room where no rigid backing is naturally there. The impact of edge diffracted waves from a frame or a rigid backing plate on the material is significant, both looking at the measured transfer function and the final absorption coefficient. The impact is, as for the finite sample issue, smaller for large samples and small speaker distances. However, the impact is smaller for highly absorbing materials as opposed to the finite sample issue. This derives from the fact that a highly absorbing material attenuates the incident sound waves to a larger extent before they hit the backing or frame edges hence the refracted waves are weaker.

In future work, the corresponding analysis of the edge diffraction impact on the measurements for oblique incidence will be performed.

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