

Homogenization of sound propagation in a deformable porous material based on microscopic viscous-thermal effects

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Summary

Porous materials like acoustic foams can be used for shielding and their absorption abilities depend on the interaction of the acoustic wave and the complex microstructure. In this paper, a homogenization model is proposed to investigate the relation between the microstructure and the macroscopic properties. A numerical experiment is performed in the form of simulations of sound absorption tests on a porous material made from polyurethane. For simplicity, an idealized partially open cubic microstructure is adopted. The homogenization results are evaluated by comparison with Direct Numerical Simulations (DNS), showing a good performance of the approach for the studied porous material. By comparing the results, it is found that Biot's model with the parameters obtained from the homogenization approach predict a higher resonance frequency than the DNS, whereas a full homogenization modification improves the prediction due to the incorporation of the microscopic fluctuation.

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1. Introduction

Passive sound absorbing porous materials, such as acoustic foams, can be applied in acoustic shielding covers to improve the sound absorption performance. This paper focuses on modelling acoustic porous materials in order to optimally exploit their sound absorption functionalities.

Models of acoustic porous materials can be classified as equivalent fluid models for a motionless solid skeleton and fluid-solid coupling models. In the equivalent fluid model, acoustic properties depend on the effective density and the effective bulk modulus of the fluid in the porous material. One of the widely accepted models of this type is a semi-phenomenological model usually referred as the Johnson-Champoux-Allard-Lafarge (JCAL) model which gives analytical expressions of the effective fluid density and the effective bulk modulus [1]. The non-acoustical parameters required by the JCAL model can be calculated numerically based on a representative unit cell [2]. Further-

more, a more rigorous equivalent fluid model can be obtained by applying the asymptotic homogenization method to the fluid domain in the porous material [3].

An intrinsic limitation of the equivalent fluid models is the loss of the solid motion which is important in vibroacoustic problems [4]. Furthermore, although the sound absorbing behavior is often believed to be mainly governed by the local visco-thermal dissipations of the fluid, in particular for some partially-reticulated foams, the vibration of the pore membranes is observed to have a non-ignorable influence [5] and the consideration of their elastic properties can improve the agreement with experimental measurements [6]. Therefore, a fluid-solid coupling model should be considered when the solid motion is non-ignorable. The most famous coupling model is probably Biot's model based on Biot's poroelastic theory [7], describing the coupling between the macroscopic fluid and solid displacement fields with effective parameters that are dependent on the corresponding microstructure. Biot's model includes the effects of the microstructure implicitly through the effective parameters: the viscous coefficient and the added density are mainly determined by the effec-

tive fluid density; the elastic coefficients were found to be related to the effective bulk modulus of the solid skeleton and the fluid for isotropic porous materials. There are also many fluid-solid coupling models taking into account the microstructure explicitly. For example, the asymptotic homogenization method has been applied to a porous material including an elastic solid skeleton and a compressible viscous gaseous fluid with each a linearized behavior [8, 9]. By considering the porous material as a mixture of the solid and the fluid, a set of macroscopic thermodynamically consistent constitutive equations can be obtained while applying the volume integration to the microscopic mass and momentum conservation equations [10].

Recently, by assuming Biot's theory to be applicable to the macroscopic problem and considering isothermal conditions, the authors proposed a homogenization approach to obtain the effective parameters based on energy consistency [11]. In this paper, this homogenization approach is further developed: the macroscopic problem is described with a general formulation instead of Biot's theory and a non-uniform thermal field is considered in the microscopic problem. This enhanced homogenization model is still compatible with Biot's poroelastic theory when the microscopic fluctuations of the solid are ignorable. In the end, a numerical example of a simple RVE is discussed and the homogenization approach is evaluated based on the results of Direct Numerical Simulations (DNS).

2. Homogenization framework

The objective of this section is to present a homogenization framework for a general acoustic problem in porous materials. In this paper, the problem is studied in the frequency domain and the time derivative $\partial/\partial t$ is replaced by $j\omega$ with ω the angular frequency and j the imaginary unit. The macroscopic quantities are indicated by the subscript M and the microscopic ones are denoted by the subscript m .

The fundamental assumption in the homogenization method is that the scale separation principle which requires that the macroscopic characteristic length L_M^* is much larger than the microscopic characteristic length R is satisfied. By doing so, it is possible to separate the multiscale problem for porous materials into a macroscopic problem and a microscopic one. Furthermore, the acoustic wavelength λ should not be smaller than the macroscopic characteristic length. To summarize, the scale requirement in the problem is

$$\lambda \geq L_M^* \gg R. \quad (1)$$

2.1. Macroscopic problem

In the macroscopic problem, there are two coupled phases: a solid skeleton and a gaseous fluid. Firstly, it is assumed that the deformation is small. Then, the porosity ϕ , defined as the volume fraction of the

fluid, remains constant and the density variations of the solid and the fluid are both ignored compared to the static densities ρ_0^s and ρ_0^f . The gaseous fluid is considered to be the air and the sound propagating process is assumed to be isentropic. Hence, the solid displacement $\mathbf{u}_M^s$ and the fluid pressure P_M^f are the macroscopic field variables. The macroscopic governing equations are assumed to be

$$\begin{aligned} \mathbf{f}_M^s - \nabla_M \cdot (\boldsymbol{\sigma}_M^s)^T &= \mathbf{0} \\ \epsilon_M^f - \nabla_M \cdot \mathbf{u}_M^f &= 0. \end{aligned} \quad (2)$$

Here the operator ∇_M represents the spatial gradient at the macroscopic scale. The first equation in Eq. (2) represents the conservation of momentum of the solid, where $\boldsymbol{\sigma}_M^s$ is the macroscopic Cauchy stress of the solid and $\mathbf{f}_M^s$ is the external force exerted on the solid. The second equation represents the mass conservation of the fluid, where ϵ_M^f is the macroscopic volumetric change of the fluid and $\mathbf{u}_M^f$ is the fluid displacement. According to energy conservation, the macroscopic energy variation per unit of volume of a macroscopic point is

$$\begin{aligned} \delta E_M &= (1 - \phi) \boldsymbol{\sigma}_M^s : \delta(\nabla_M \mathbf{u}_M^s) - \phi \epsilon_M^f \delta p_M^f \\ &+ (1 - \phi) \mathbf{f}_M^s \cdot \delta \mathbf{u}_M^s - \phi \mathbf{u}_M^f \cdot \delta(\nabla_M p_M^f). \end{aligned} \quad (3)$$

2.2. Microscopic boundary value problem

A microscopic RVE under the isothermal condition has been discussed in [11]. This paper focuses on the implementation of a non-uniform thermal field. By ignoring the viscous dissipation term, the linearized energy conservation equation in the frequency domain is written as

$$\rho_0^f C_p^f j\omega \theta_m^f = j\omega p_m^f + \nabla_m \cdot \mathbf{q}_m^f, \quad (4)$$

where ρ_0^f is the static density, C_p^f is the heat capacity at constant pressure, θ_m^f is the temperature difference with the ambient temperature, p_m^f is the pressure difference and $\mathbf{q}_m^f$ is the thermal flux. Fourier's law is adopted i.e. $\mathbf{q}_m^f = -k^f \nabla_m \theta_m^f$ with k^f the thermal conductivity of the fluid. Moreover, a linearized state equation is adopted:

$$\frac{p_m^f}{P_0} = \frac{\theta_m^f}{T_0} - \nabla_m \cdot \mathbf{u}_m^f. \quad (5)$$

For the solid, thermal expansion is ignored and the linear isotropic elastic constitutive law of the isothermal case can still be adopted [11].

Analogous to the boundary conditions used under the isothermal condition in [11], a periodic boundary condition for the solid displacement is applied on the solid surface S_e^s (where one corner point is constrained):

$$\mathbf{u}_m^{s+} - \mathbf{u}_m^{s-} = (\nabla_M \mathbf{u}_M^s)^T \cdot (\mathbf{x}_m^{s+} - \mathbf{x}_m^{s-}) \text{ on } S_e^s \quad (6)$$

and a prescribed traction is defined by ignoring the viscous stress on the fluid surface S_e^f :

$$\mathbf{n} \cdot \boldsymbol{\sigma}_m^f = - \left(p_M^f + \nabla_M p_M^f \cdot \mathbf{x}_m^f \right) \mathbf{n} \text{ on } S_e^f. \quad (7)$$

Here $\mathbf{n}$ is the outward unit normal vector of the surface and p_M^f is the macroscopic pressure difference. The variables $\mathbf{u}_m^{s+}$ and $\mathbf{u}_m^{s-}$ mean the microscopic solid displacement on the two opposite boundaries S^+ and S^- which are geometrically equal in a periodic RVE. A fully continuous condition on the fluid-solid interface is applied including the displacement, the force, the temperature and the thermal flux. Moreover, a periodic boundary condition is used for the thermal flux:

$$\mathbf{q}_m^{\xi+} \cdot \mathbf{n}^+ = -\mathbf{q}_m^{\xi-} \cdot \mathbf{n}^- \text{ on } S_e^\xi, \text{ with } \xi = s, f. \quad (8)$$

According to energy conservation, the microscopic energy variation can be written as

$$\begin{aligned} \int_V \delta E_m dV &= \int_{S_e^s} (\mathbf{n} \cdot \boldsymbol{\sigma}_m^s) dA \cdot \delta \mathbf{u}_M^s \\ &+ \int_{S_e^s} (\mathbf{n} \cdot \boldsymbol{\sigma}_m^s) \mathbf{x}_m^s dA : \delta (\nabla_M \mathbf{u}_M^s) \\ &- \delta p_M^f \int_{S_e^f} \mathbf{n} \cdot \mathbf{u}_m^f dA \\ &- \delta \nabla_M p_M^f \cdot \int_{S_e^f} \mathbf{x}_m^f (\mathbf{n} \cdot \mathbf{u}_m^f) dA. \end{aligned} \quad (9)$$

2.3. Micro-to-macro relations

Energy consistency requires that the energy variation of a macroscopic point is the average of the total microscopic energy variation of the associated RVE, i.e.

$$\delta E_M = \frac{1}{V} \int_V \delta E_m dV. \quad (10)$$

Substituting Eq. (3) and (9) into this equation results in the following micro-to-macro relations:

$$\begin{aligned} (1 - \phi) \mathbf{f}_M^s &= \frac{1}{V} \int_{S_e^s} (\mathbf{n} \cdot \boldsymbol{\sigma}_m^s) dA, \\ (1 - \phi) \boldsymbol{\sigma}_M^s &= \frac{1}{V} \int_{S_e^s} (\mathbf{n} \cdot \boldsymbol{\sigma}_m^s) \mathbf{x}_m^s dA, \\ \phi \epsilon_M^f &= \frac{1}{V} \int_{S_e^f} \mathbf{n} \cdot \mathbf{u}_m^f dA, \\ \phi \mathbf{u}_M^f &= \frac{1}{V} \int_{S_e^f} (\mathbf{n} \cdot \mathbf{u}_m^f) \mathbf{x}_m^f dA. \end{aligned} \quad (11)$$

2.4. Relation with Biot's poroelastic theory

Considering the linearity of the microscopic problem, the macroscopic quantities calculated from Eq. (11) can be linearly expressed in terms of the macroscopic solid displacement, air pressure and their gradients.

Firstly, the relation involving the solid force and the fluid displacement can be expressed by

$$\begin{aligned} (1 - \phi) \mathbf{f}_M^s &= \omega^2 \boldsymbol{\rho}_c \cdot \mathbf{u}_M^f - \omega^2 (\rho_0^s \boldsymbol{\eta}^s + \boldsymbol{\rho}_c) \cdot \mathbf{u}_M^s \\ - \phi \nabla_M p_M^f &= \omega^2 \boldsymbol{\rho}_c \cdot \mathbf{u}_M^s - \omega^2 (\rho_0^f \boldsymbol{\eta}^f + \boldsymbol{\rho}_c) \cdot \mathbf{u}_M^f, \end{aligned} \quad (12)$$

with $(\boldsymbol{\eta}^s, \boldsymbol{\eta}^f, \boldsymbol{\rho}_c)$ diagonal 2nd-order tensors. Furthermore, when the microscopic fluctuation of the solid displacement can be ignored and the volume-average fluid deformation is small enough, it is reasonable to apply the following approximation:

$$\boldsymbol{\eta}^s \approx (1 - \phi) \mathbf{I} \quad \text{and} \quad \boldsymbol{\eta}^f \approx \phi \mathbf{I}. \quad (13)$$

Then, Eq. (12) leads to the anisotropic Biot's poroelastic equations [7]. In this case, the term $j\omega \boldsymbol{\rho}_c / \phi$ has the same physical meaning as the viscodynamic operator defined in [7].

The stress-strain relation is

$$\begin{aligned} (1 - \phi) \boldsymbol{\sigma}_M^s &= {}^4\mathbf{D} : \nabla_M \mathbf{u}_M^s + \mathbf{Q} \epsilon_M^f \\ - \phi p_M^f &= \mathbf{Q} : \nabla_M \mathbf{u}_M^s + R \epsilon_M^f. \end{aligned} \quad (14)$$

and is the same as the stress-strain relation in Biot's theory [7]. Instead of the macroscopic measurements required in Biot's theory, the effective material parameters $({}^4\mathbf{D}, \mathbf{Q}, R)$ can be numerically calculated based on the associated microscopic RVE.

To summarize, in the homogenization approach, the effective parameters $(\boldsymbol{\eta}^s, \boldsymbol{\eta}^f, \boldsymbol{\rho}_c)$ and $({}^4\mathbf{D}, \mathbf{Q}, R)$ of the closed macroscopic equations Eq. (2), (12) and (14) are numerically calculated from the microscopic RVE. Besides, under the conditions of ignorance of the solid displacement fluctuation and a small volume average fluid deformation, the macroscopic governing equations agree with Biot's poroelastic theory where the effective material parameters $({}^4\mathbf{D}, \mathbf{Q}, R)$ and the viscodynamic operator $j\omega \boldsymbol{\rho}_c / \phi$ are numerically calculated. In this paper, the full set of closed macroscopic equations with parameters obtained from the microscopic RVE is referred as the Full-Homogenization model and Biot's poroelastic model with parameters obtained from the microscopic RVE is referred to as the Biot-Homogenization model. Obviously, the Full-Homogenization model is more versatile for cases with significant microscopic fluctuations than the Biot-Homogenization model.

3. Simulations

In this section, the numerical results for a cubic RVE with idealized partially open membranes as shown in Figure 1 are discussed. The fluid is considered as air and the solid is a polyurethane (PU) as given in Table I. Simulations of a homogenized macroscopic sound absorption test of a 2-cm porous layer shown in Figure 2 are compared with the DNS, in which the microstructure of the porous material corresponds to the RVE in Figure 1.

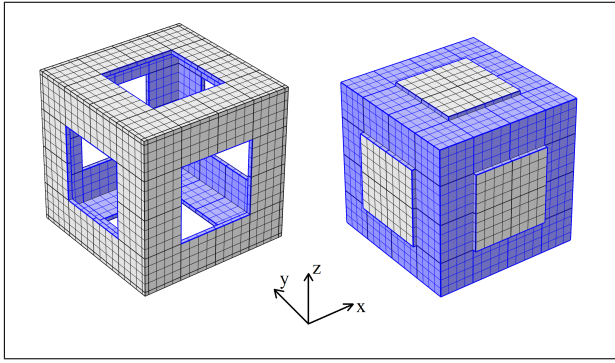


Figure 1. Finite element model of a 1-mm cubic RVE with a porosity of 89.5%, (left) the solid domain and (right) the fluid domain. The membrane thickness is 50 μm , i.e. the wall thickness is 25 μm . The size of the face holes in the RVE is $0.5 \times 0.5 \text{ mm}^2$.

Table I. Material parameters of the polyurethane (PU) used in the simulations.

Density	$\rho_0^s [\text{kg m}^{-3}]$	1100
Bulk modulus	$K^s [\text{GPa}]$	0.22
Shear Modulus	$G^s [\text{GPa}]$	0.083
Heat capacity	$C_p^s [\text{J kg}^{-1} \text{K}^{-1}]$	1800
Thermal diffusivity	$k^s [\text{W m}^{-1} \text{K}^{-1}]$	0.022

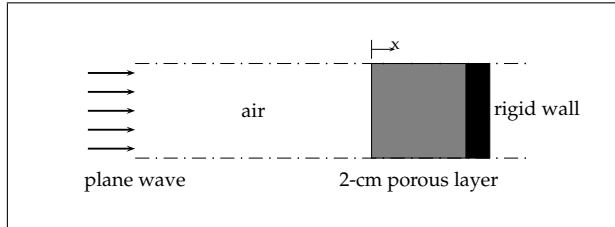


Figure 2. Macroscopic configuration for obtaining normal incident sound absorption coefficients. A symmetry boundary condition is applied on the top and the bottom.

In the simulations with the homogenization models, there are 369 cuboid elements in the porous layer. The pure air part is governed by the Helmholtz equation with the sound speed of air at room temperature, i.e. $c_{\text{air}} = 343 \text{ m/s}$, and a given incident plane wave $p_{\text{inc}} = e^{-jkx} [\text{Pa}]$, where k is the wave number and x is the horizontal distance from the air-porous interface with the acoustic-poroelastic coupling condition [13]. The frequency in the simulation ranges from 100 Hz to 5000 Hz with a stepsize of 10 Hz. In the DNS, the porous layer is composed of the fully detailed microstructure including the solid and the fluid domains, i.e., an array of 20 unit cells. The governing equations in the DNS are identical with the simulations of the microscopic RVE.

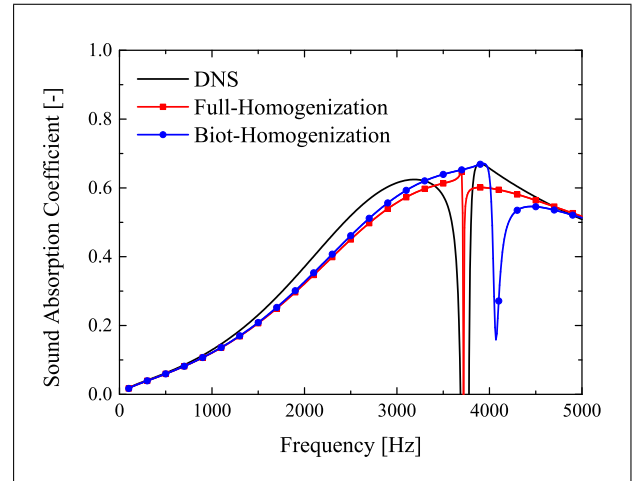


Figure 3. Sound absorption coefficients of the simulations.

The normal incident sound absorption coefficient is calculated by

$$\alpha = 1 - \left| \frac{p_{\text{ref}}}{p_{\text{inc}}} \right|^2, \quad (15)$$

with the incident wave $p_{\text{inc}} = 1 \text{ Pa}$ on the air-porous interface and the reflective wave $p_{\text{ref}} = p - p_{\text{inc}}$ where p is the pressure on the interface. Figure 3 plots the sound absorption coefficients and it shows that the resonance frequencies predicted by the two models are different: compared with the DNS, the Full-Homogenization model improves the resonance behavior relative to the Biot-Homogenization model. This is because the microscopic fluctuation of the solid that is ignored in the Biot-Homogenization model can influence the resonance behavior and results in an increase of the dynamic mass making the resonance frequency lower. It should be noted that the coefficient calculated by Eq. (15) cannot describe the absorbed energy correctly when resonance occurs, because the energy is mainly absorbed by the solid skeleton at the resonance frequency. The difference between the DNS and the Full-Homogenization model is due to the boundary conditions adopted in the homogenization approach such as the ignorance of the boundary viscous stress and the breakdown of the scale separation principle in high frequencies.

4. Conclusions

Acoustic problems of porous materials were investigated by using the homogenization method in this paper. Based on the scale separation principle, the multiscale problem was divided into two separate problems at difference scales: the macroscopic problem was controlled by the macroscopic solid displacement and fluid pressure; the microscopic RVE was described by linearized equilibrium equations and linear constitutive laws. The homogenization framework was

built by applying a periodic solid displacement, a prescribed fluid traction and a periodic thermal flux on the boundary of the microscopic RVE. The micro-to-macro scale transition relations are obtained based on two-scale energy consistency.

The Full-Homogenization model was obtained by considering the linearity of the microscopic problem and the required effective parameters are obtained through numerical simulations of the microscopic RVE. By ignoring the microscopic fluctuation terms, the Biot-Homogenization model corresponding to Biot's poroelastic theory can be derived from the Full-Homogenization model. Practically, by letting the dynamic densities be equal to the corresponding volume fractions, the Full-Homogenization model is simplified into the Biot-Homogenization model. A numerical simulation of a macroscopic sound absorption experiment on a porous material with an idealized cubic microstructure was simulated by using the two models. A DNS result was used as a reference. By comparing the sound absorption coefficients, it was shown that the Full-Homogenization model gives a better prediction of the resonance frequency which is affected by the microscopic fluctuation of the solid.

To summarize, the proposed homogenization approach has been applied to acoustic porous materials in this paper. The new homogenization model can be considered as a modification of Biot's poroelastic model with the consideration of the microscopic viscous-thermal effects. For low frequencies or for materials with very high stiffness, the performance of this model is almost the same as that of Biot's model. However, by comparing with the DNS, this new model gives a better prediction of the resonance frequency because of the consideration of the microscopic fluctuation of the solid.

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