Predicted and measured anisotropic acoustic and elastic properties for open cell porous material

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Summary

Multilayer panels, as used in train floors, walls and roofs must be designed to fulfil many requirements, including acoustic and structural demands derived from global vehicle requirements. In order to maintain or increase the acoustic performance with reduced weight, porous materials are often used as a part of the panels. The acoustic and elastic properties of the open cell foam are determined by its micro geometry. Modelling the full actual geometry for the foam is still computationally too demanding, and therefore a representative unit cell on micro level can be used to describe the overall behavior in the long wavelength region, provided the micro-geometry can capture the most important characteristics of the foam. The flow resistivity is calculated from a micro geometry and compared to measured anisotropic results for melamine foam. Even though the model is very simple, it can predict the right order of magnitude of flow resistivity, and can also be adapted to anisotropic foams.

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1. Introduction

Multilayer panels, as used in train floors, walls and roofs must be designed to fulfil many requirements, including acoustic and structural demands derived from global vehicle requirements. In order to maintain or increase the acoustic performance with reduced weight, porous materials are often used as a part of the panels. The acoustic and elastic properties of the open cell foams are determined by its micro geometry, but are traditionally characterized from measurements on macro-level of a sample of a porous material. The most common properties for foams are the flow resistivity or the (dynamic) permeability, the porosity (or density) and sometimes tortuosity. The macro level properties are interdependent since they are all related to the actual micro structure. Thus, to design the macro level acoustic properties for a required performance, physically relevant models linking these to the micro structure would be necessary. Several such methods exist [1]-[3}, many giving good results, in particular for materials that can be regarded as reasonably isotropic. However, for materials deviating from this assumption, problems do arise if an isotropic model is to be fitted. As shown by Van der Kelen certain types of porous materials exhibit directionally dependent macro level properties, both with regard to elasticity and acoustics [4][5]. There is however a lack of knowledge as to what extent the anisotropy on micro level of the porous material affects the elasto-acoustic properties on macro level and the vibro-acoustic performance of the material. In addition, for those situations where the load carrying ability as well as the vibration transmission must be taken into account, the elastic properties the static and anelastic (dynamic and frequency-dependent) Hooke's matrices are also required.

Simulations using numerical models of the actual, disordered, geometry on micro-scale are still computationally very demanding. An alternative approach, which also is the main scope of the present work, would be if the micro-structure of the open cell porous material is modelled as an idealized, periodic structure, allowing anisotropic properties on the micro-scale using a long wavelength assumption. The basic assumptions for the modelling are that the wavelength is much longer than the microstructural dimensions, and that the relations both for acoustics and elasticity are linear. From similar, simplified analytical descriptions of the acoustic parameters and their dependence on the micro-structure properties, the microstructural properties can be linked to averaged macroscopic properties, including anisotropy. Several authors have pointed out that not only fibrous materials, such as glass wool, have anisotropic acoustic flow resistivity [4][6] but also open cell porous foams. In spite of this fact, measured parameters are normally not determined for more than one direction and the anisotropy is not explicitly used in the modelling.

In the present work the degree of anisotropy will be characterized by the ration between height and width of the unit cell, as well as by the angle θ describing the shape of the hexagon. A regular reference unit cell has a height to width ratio of one and a regular hexagonal shape in the plane, defined by the angle θ=π/3. There are examples of polyurethane foams with anisotropy of 1.2 and 1.4 and even as much as 1.75[1][16]. In recent work [4] it was shown that the flow resistivity of a melamine foam could vary 10-20% between the principal directions of the material and the Young's modulus for the different directions could vary more than a factor of three [5].

Further, geometrically more advanced micro models using Kelvin cells have been successfully implemented for isotropic foams based on frothing processes producing spheroidal cells, such as polyurethane (PU) foam [2][3]. In contrast to PU foam micro structures, melamine type foams grow with long slender struts, without the presence of partially reticulated membranes, making other types of micro models possible for this type of materials. Melamine has been shown to exhibit an elastic behavior with negative Poisson's ratio, as well as pronounced anisotropic properties [4][5].

2. Micro-scale model for melamine foam

The micro model should be simple enough to allow approximate analytical solutions in order to increase an understanding of the parameters in the model. It should also for future work be one that can be adapted for calculating elasticity, and in doing so not only include bending deformation. The choice has been to look at a hexagonal honeycomb. The structure is widely used in for instance sandwich design, where it is mainly carrying shear loads. Here the in-plane properties are the primary interest, and to start with the flow resistivity that can be achieved by such a structure when modelling an open porous material, such as melamine.

2.1. Model for micro geometry

The micro model geometry is shown in Figure 1.

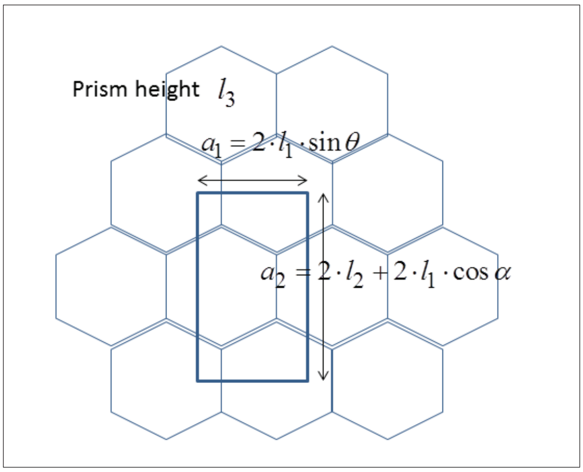

Figure 1. Micro model geometry in the plane x_1-x_2 for a periodic unit cell. The choice of unit cell is shown in the figure, together with the dimensions of the unit cell. The height of the unit cell is l_3.

For this geometry there are several possible unit cells, but the rectangular is chosen for a simpler derivation the analytical flow resistivity.

2.2. Linking micro-scale properties to macro scale properties

The link between the macro scale and the micro scale is done through the porosity Φ. The porosity is measured on a macro scale, and can also be derived on the micro-scale for a chosen micro geometry. For the unit cell in figure, the porosity gives an expression for the ratio between the thickness and length of a strut (t/l_1).

The general expression for porosity contains 5 parameters for the regular case: $\Phi, l_1, l_2, l_3, \theta$ and t. Φ is assumed to be known, as well as one micro-dimension (say strut length l_1, e.g. estimated from photos). All struts are assumed to have the same thickness. In addition the ratio $R=l_3/l_1$ is estimated or assumed to be given. This allows the ratio (t/l1) to be assessed as

$$\left(\frac{t}{l_1}\right)^2 = (1-\Phi)\frac{8R\cdot\sin\theta(1+\cos\theta)}{\pi(3+4R)} \tag{1}$$

Based on the test procedure given in the norm ISO 9053 [12], the flow resistivity is estimated as [17]

$$\sigma \approx \frac{F_{drag,PUC}}{\mathbf{u}/\phi}\cdot\frac{1}{V_0}\cdot vr \tag{2}$$

where u is the flow velocity, V_0 is the isotropic volume of the PUC and vr is the volume ratio between a regular and non-regular PUC. For the regular case vr equals 1. The drag force F_{drag} on micro-scale inside the foam is calculated for viscous and uncompressible flow around the circular struts, including contributions from both longitudinal and transverse flow, using the formulas derived by Tarnow for long, slender fibers, which is a suitable model for melamine [13].

The drag force due to transverse flow, normal to the strut direction is denoted F_{tij} , where i is flow direction and j is the axis of strut orientation . For example, F_{t13} is the transverse force from flow in the x_1 direction across the l_3 strut. The drag force due to flow longitudinally along the strut is denoted F_{lij}, where l denotes longitudinal, i is the direction of flow and j the strut l_j. . Flow at an angle with the strut results in both a longitudinal and a transverse contribution to the drag force.

$$F_{tmk} = -iu_m\pi\rho_f\omega l_k\left(\frac{t_k}{2}\right)^2 \times \left(1-\frac{4\cdot H_1^{(1)}(\frac{k_{\beta k}t_k}{2})}{(k_{\beta k}(\frac{t_k}{2}))H_0^{(1)}(\frac{k_{\beta k}t_k}{2})}\right) = u_m l_k B_t \tag{3}$$

$$F_{lkk} = -il_k.u_k 2\pi\frac{t_k}{2}k_{\beta k}\mu_f\frac{H_1^{(1)}(\frac{k_{\beta k}t_k}{2})}{H_0^{(1)}(\frac{k_{\beta k}t_k}{2})} \tag{4}$$

$$= u_k l_k B_l$$

where the indices m,k can take the values 1,2 and 3 representing x_1, x_2 and x_3, and where none of B_t or B_l depends on flow velocity u_m or strut length l_k. All of the struts are assumed to have the thickness t.

This method is very straightforward for geometries where the struts are aligned with the coordinate axis, since the flow resistivity is obtained directly in the principal directions. For the hexagonal geometry above, where some of the struts are inclined relative to the coordinate axis, the method is slightly different. The unit cell in the different struts in the x_1 -x_2 plane are shown in figure 2.

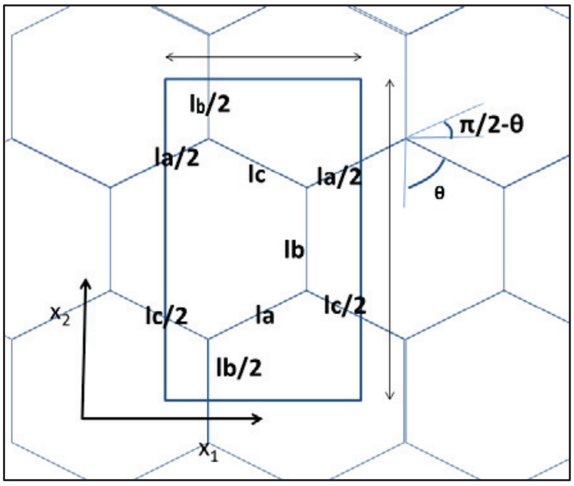

Figure 2. Struts in the unit cell used for calculating the flow resistivity, with notations and orientations. At the junctions a vertical strut is connected, which gives four vertical struts l_d aligned with the x_3 axis in the unit cell.

The purpose is to find an analytical description of the anisotropic flow resistivity of a micro-structure based on hexagonal prism. Figure 3 shows the struts in the unit cell with local coordinate systems.

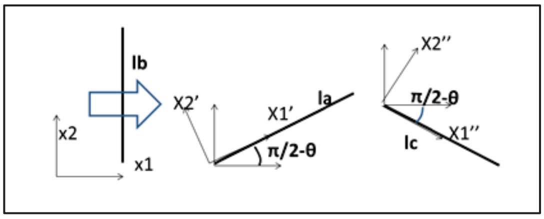

Figure 3. Local coordinate systems for the inclined struts to calculate the flow resistivity.

For the static flow resistivity only one unit cell with 1+1/2+1/2 struts l_b, 1+1/2+1/2 struts l_a and 1+1/2+1/2 struts l_c, and in addition 4 struts l_d aligned with the x_3-axis. In the coordinate system used above two of the three struts in the *x-y* plane are inclined. For a regular honeycomb the angle θ is π/3. To condense the formulas, let

$$K_0 = \Phi \cdot vr / V_0 \tag{5}$$

where *vr* denotes the volume ratio between a non-regular prism and a regular prism, which has equal angles and height equal to width.

The flow resistivity for the *b* strut (aligned with x_2) and strut *d* (vertical strut aligned with x_3) can then be written directly in the global coordinate system as

$$\sigma_b = K_0 \begin{pmatrix} B_t l_2 & 0 & 0 \\ 0 & B_l l_2 & 0 \\ 0 & 0 & B_t l_2 \end{pmatrix} \tag{6}$$

$$\sigma_d = K_0 \begin{pmatrix} l_d B_t & 0 & 0 \\ 0 & l_d B_t & 0 \\ 0 & 0 & l_d B_l \end{pmatrix} \tag{7}$$

In both cases there is a transverse flow creating a drag force over the struts perpendicular to the flow direction and in addition a contribution from the drag longitudinally along the strut when it is aligned with the direction of flow.

To write the flow resistivity for the inclined struts, it is observed that the dissipated power from the drag can be written as

$$P_{diss} = \mathbf{u}^T \boldsymbol{\sigma} \mathbf{u}$$

where **u** and **σ** are defined in the global x_1-x_2 coordinate system [14]. The dissipated power must be independent of the coordinate system in which it is described, and can therefore also be written in a local, rotated, coordinate system aligned with the inclined struts. Since

$$P_{diss,a} = \mathbf{u}_a^{\ T} \boldsymbol{\sigma}_a \mathbf{u}_a \text{ and } P_{diss,c} = \mathbf{u}_c^{\ T} \boldsymbol{\sigma}_c \mathbf{u}_c$$

where $\mathbf{u}_a$ and $\mathbf{u}_c$ are given in rotated coordinate systems where the x_1-axis is parallel to the strut a and strut c respectively. The relation between **u** and $\mathbf{u}_a$ are defined by

$$\mathbf{u} = \mathbf{R}_a \mathbf{u}_a \text{ and } \mathbf{u} = \mathbf{R}_c \mathbf{u}_c \quad . \tag{8}$$

It follows from the orthogonal rotation that $\boldsymbol{\sigma} = \mathbf{R}_c \boldsymbol{\sigma}_c \mathbf{R}_c^T$ and $\boldsymbol{\sigma} = \mathbf{R}_a \boldsymbol{\sigma}_a \mathbf{R}_a^T$ where the rotation matrices describe the rotations shown in figure 2.

$$\sigma_a = K_0 \begin{pmatrix} B_l l_a & 0 & 0 \\ 0 & B_t l_a & 0 \\ 0 & 0 & B_t l_a \end{pmatrix} \tag{9}$$

$$\sigma_c = K_0 \begin{pmatrix} B_t l_c & 0 & 0 \\ 0 & B_l l_c & 0 \\ 0 & 0 & B_t l_c \end{pmatrix} \tag{10}$$

$$R_a = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

$$R_c = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{12}$$

For all struts, the influence of the interaction between struts on the flow resistivity is neglected. The total flow resistivity for the entire unit cell can now be written in the global coordinate system as

$$\sigma = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \tag{13}$$

Now that all the contributions have been transformed to the global coordinate system, they can be added, including the simplification that l_a=l_c=l_b=l_1 and l_d=$2l_3$. With l_a and l_c equal, off diagonal terms cancel out, leaving only the diagonal terms non-zero.

$$\begin{aligned} \sigma_1 &= B_t K_0 \cdot (l_1 + 2l_3) + 2K_0 l_1 (B_l \cos^2\alpha + B_t \sin^2\alpha) \\ \sigma_2 &= B_l K_0 l_1 + 2B_t K_0 l_3 + 2K_0 l_1 (B_t \cos^2\alpha + B_l \sin^2\alpha) \\ \sigma_3 &= K_0 (2B_l l_3 \ + \ 3B_t l_1) \end{aligned} \tag{14}$$

The flow resistivity tensor is described by a diagonal matrix with three separate values of flow resistivity in the three principal directions, and is positive definite and symmetric as expected. It is also invariant under rotation of 2π/3, which could be expected from the given geometry. The unit cell geometry is not symmetric with respect to the x_1 and x_2 axis, and therefore σ_1 and σ_2 can not be expected to be symmetric with respect to x_1 and x_2 axis.

3. Results

3.1. Numerical example

There are for the moment no micro photos with scales available for this sample, but based on other references, the strut thickness should fall within the range of 5-7 µm, and the cell size in the range around 100-200 µm [1][15]. The calculation is made with input data according to Table I.

Table I. Model parameters for numerical example.

Model parameter	symbol	value	unit
Foam density	ρ	9.8	kg/m^3
Density of solid melamine	ρ_s	1310	kg/m^3
Strut length in-plane	l_l	80	µm
Prism height	2 l_3	80	µm
Prism width	4 l_1 sinθ	64	µm
Strut thickness	t	4.5	µm

3.2. Comparison to measured results

The calculated anisotropic flow resistivities are compared to measured results from van der Kelen and Göransson [4]. In total 7 seven samples from one and the same block of foam was measured.

As an example the result is compare to Melamine sample 2 with the flow resistivity characterized and measured as shown in Table II.

Table II. Measured data for melamine foam [4].

Density [kg/m^3]	Flow resistivity [Pa s/m^2]	Method of characterisation
9.8	9100	Anisotropic characterization first eigenvalue
10.9	9300	Uni-directional measurement x2 direction
10.5	8700	Uni-directional measurement in x2 direction
9.8	10000	Anisotropic characterization second eigenvalue
9.8	10600	Anisotropic characterization third eigenvalue

In figure 4 the calculated flow resistivity is shown.

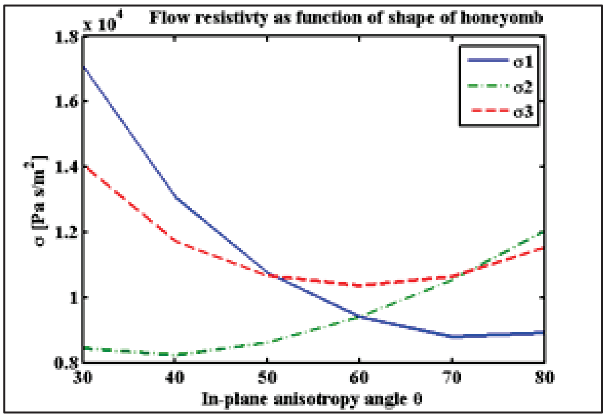

Figure 4. Calculated flow resistivity for a hexagonal micro-cell. The height of the PUC is approximately the same as the width. The average for a regular cell with θ=60 deg is 9.7 kPa s/m^2. The height is 80% of the in-plane strut length.

For a regular hexagonal prism with θ=60 deg, the highest direction have a flow resistivity of 10.3 kPa s/m^2 and the other two a value of 8.4 kPa s/m^2. This is in the right range compared to the measured values, which are 9.1, 10.0 and 10.6 kPa s/m^2 from the anisotropic characterization. The average is also rather close to the measured uni-directional results, which are 9.3 and 8.7 kPa s /m^2.

The anisotropy in the plane changes the flow resistivity. When the hexagonal shape is changed from a regular shape (θ=60 deg) with equal angles and equal side lengths, to a short and wide (θ > 60 deg) or tall and narrow (θ < 60deg the flow resistivity is increased, which can be seen in figure 4.

4. Conclusions

The hexagonal prism micro geometry can be used for modelling the flow resistivity of open cell porous materials. Also anisotropic properties can be captured with the model, for instance when the three principal directions do not have the same flow resistivity, which is the case for anisotropic materials. Based on a few parameters, a flow resistivity of the right order of magnitude can be predicted, in spite of the simplicity of the model.

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