Identification of a Rotating Sound Source in a Duct with High Spatial Resolution

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Summary

To identify the major sources of noise radiated from in-duct fluid machines, one needs to observe the source parameters such as pressure and velocity fields at the source plane. For this purpose, inverse estimation methods are generally used starting from measured pressures and a sound propagation model in the duct. In this paper, a technique is suggested for identifying the rotating noise source in a wide duct. For the detailed observation of sources with high spatial resolution, the evanescent wave is considered in the modeling based on the modal summation method. Also, the Doppler effect caused by rotation of the noise source is considered in the modeling. The validation experiment is conducted on the duct system excited by a rotating loudspeaker radiating a tonal sound in the absence of flow. The measured near-field pressure precisely shows spectral peaks at shifted frequencies due to the Doppler effect. The modal amplitude set related with the rotation of the loudspeaker is estimated to investigate the source parameters. The pressure in the near-fields very close to the source is regenerated by using the estimated modal amplitudes, and the maximum error is found to be less than -10 dB. The pressure and velocity fields at the source plane are restored by the estimated modal amplitudes, and the result clearly indicate the rotating loudspeaker as the main noise radiator within the rotating reference frame.

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1. Introduction

Acoustic source characteristics of fluid machines connected to a duct need to be identified to investigate the fluid-borne noise generation mechanism. The identified source parameters and their spatial distribution over the major machine components are essential for low-noise design of machines.

In estimating the acoustic source parameters at the source plane of a fluid machine in a duct, one often assumes a stationary operation conditions even for the rotating sound sources. If the major characteristics of in-duct noise are identified in this condition, one can predict the discharge noise spectrum and/or optimally design the silencers. To this end, constant source parameters over the duct section has been usually regarded as the sufficient information. However, when the acoustical refinement of the rotating fluid machines is of concern, it is required to identify the detailed source parameter distribution on an irregular source surface. For the axial fan example, the complicated source contour comprising boundaries of multiple curved blades, which are rotating, and a shroud are the main noise generating source. Therefore, it is difficult to spot the strongly radiating areas. To resolve the problems related to the rotating source within a wide duct, an inverse identification technique resulting in a frozen source image is needed.

Conventional source identification methods based on the modal summation method [1, 2] is not able to observe the source with high spatial resolution, and it does not consider the rotation of the source in the analysis. This work proposes an inverse source identification technique using the near-field acoustic pressure data radiated from an in-duct rotating sound source based on the modal summation method. The Doppler effect due to the rotation of the source and the effects of high order acoustic modes are considered in the whole procedure. The suggested theoretical and

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experimental methods are validated with a test example of a rotating sound source in a duct in the absence of mean flow.

2. Sound propagation from rotating source

In this work, the modal summation method is employed to include the effects of source rotation and wave actions among propagating and evanescent modes. The suggested modeling methods relates the source parameters in the *rotating reference frame*, which rotates at the same speed as the source, to the measured pressure in the *stationary reference frame*. To obtain a fine spatial resolution in the source plane, evanescent modes are included in the modelling together with propagating modes different from in the conventional modal summation method.

When a rotating source emits sound in the rotating reference frame, the Doppler shift effect happens. The radiated sound from the rotating sound source propagates in a duct as the superposed form of many in-duct acoustic modes with various circumferential mode order m and radial mode order n. In particular, the order of the circumferential mode order m and the rotating speed of the source Ω determine the shifted frequency of the radiated pressure. The pressure field with a specific circumferential mode order m in the stationary reference frame $p_m(\mathbf{x_s}, \omega_s)$ can be written as

$$p_m(\mathbf{x_s}, \omega_s) = \sum_{n=0}^{N_m} p_{mn}^{i+}(\mathbf{x_r}, \omega) + p_{mn}^{i-}(\mathbf{x_r}, \omega) \; (i = 1,2). \quad (1)$$

where $\mathbf{x}$ denotes the position vector, ω the angular frequency in which the source radiates in the rotating reference frame, and ω_s the shifted frequency due to the rotation of the source, which is $\omega_s = \omega_0 + m\Omega$ for $i = 1$ or $\omega_s = \omega_0 - m\Omega$ for $i = 2$ depending on the rotating direction of modes. The superscript i indicate the rotating direction of each mode in a cross-section: 1 for the counter-clockwise (CCW) direction and 2 for the clockwise (CW) direction. N_m is the number of modes with the given circumferential mode order m. The subscripts s and r denote the stationary reference frame and the rotating reference frame, respectively. The superscript $\pm$ is the axial propagating direction. Between two position vectors in the stationary and rotating reference frame, the following relation holds

$$\mathbf{x_r} = (r_r, \theta_r, z_r), \; \mathbf{x_s} = (r_s, \theta_s, z_s) = (r_r, \theta_r + \Omega t, z_r). \quad (2)$$

Here, r is the radial position, θ the angular position, and z the axial position.

By using the same notation with Eq. (1), the pressure vector $\mathbf{p_m^s}$ in the stationary reference frame in the frequency domain with a given mode order m can be written as

$$\mathbf{p_m^s} = \mathbf{p_m^{s1}} + \mathbf{p_m^{s2}} \quad (3)$$

where $\mathbf{p_m^{s1}}$ corresponds to the pressure field rotating in the counter-clockwise direction, and $\mathbf{p_m^{s2}}$ to the pressure field rotating in the clockwise direction. Each pressure vector in Eq. (3) can be expressed as

$$\mathbf{p_m^{si}} = \mathbf{G_{mi}} \mathbf{c_s^i}, \; (i = 1,2) \quad (4)$$

where $\mathbf{G_{mi}}$ represents the transfer matrix between the modal amplitudes and the measured pressure including the modes with the given circumferential mode order m, $\mathbf{c_s^i}$ the modal amplitude vector in the stationary reference frame rotating to counter-clockwise and clockwise direction.

Then, the pressure in the stationary reference frame $\mathbf{p^s}$ in the shifted frequencies $\boldsymbol{\omega}_\mathbf{s} = \{\omega_0 - M\Omega, \omega_0 - (M-1)\Omega, \ldots, \omega_0 - \Omega, \omega_0, \omega_0 + \Omega, \ldots, \omega_0 + (M-1)\Omega, \omega_0 + M\Omega\}$ can be expressed by summing all the pressure vectors defined in Eq. (4) as

$$\mathbf{p^s} = \sum_{m=0}^{M} \mathbf{p_m^{s1}} + \mathbf{p_m^{s2}} \; . \quad (5)$$

When the modal amplitudes in the stationary reference frame are estimated by the measured pressure in the stationary reference frame in each shifted frequency ω_s, the modal amplitude set related with the rotating source in the rotating reference frame in the radiating frequency ω_0 can be collected by using the following relation

$$c_{s,mn}^{i\pm}(\omega_0 + (-1)^{i-1} m\Omega) = c_{r,mn}^{i\pm}(\omega_0) \; (i = 1,2). \quad (6)$$

One can model the pressure field in the rotating reference frame $\mathbf{p_r}$ as

$$\mathbf{p^r} = \mathbf{G_r} \mathbf{c_r}, \quad (7)$$

where $\mathbf{G_r}$ represents the transfer matrix between the collected modal amplitudes and the pressure field in the source plane. The collected modal amplitudes in the rotating reference frame $\mathbf{c_r}$ is substituted to Eq. (7) to inversely estimate the pressure field in the source plane.

3. Inverse estimation of modal amplitudes

Amplitudes of the propagating and evanescent modes are estimated at each shifted frequency by neglecting the propagating and evanescent modes to the left, i.e., backward, axial direction. The modal amplitudes can be determined inversely as

$$\left[\mathbf{c_p^{i+}} \quad \mathbf{c_e^{i+}}\right]_\mathbf{s}^\mathbf{T} = \left[\mathbf{G_{mi}^H} \mathbf{G_{mi}}\right]^{-1} \mathbf{G_{mi}^H} \mathbf{p_m^{si}} \; (i = 1,2). \quad (8)$$

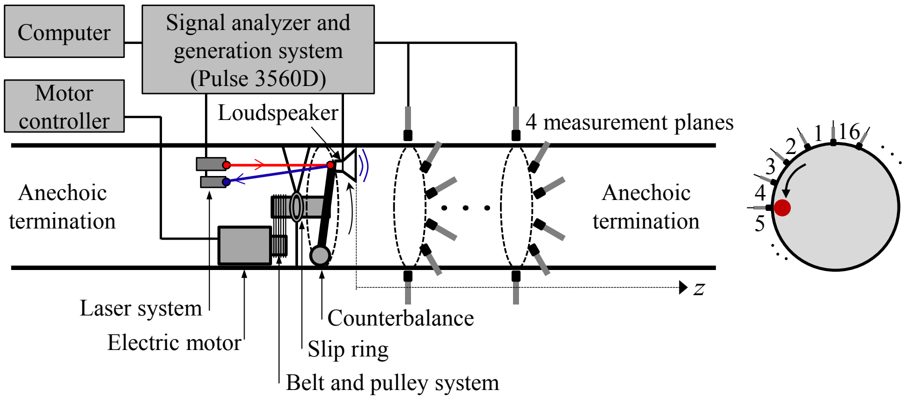

Fig. 1. Test system configuration and instrumentation. The laser system is used for synchronization of data

Here, the index i (= 1, 2) in the superscripts and subscripts represents the counter-clockwise or clockwise rotating modes, respectively in order. The superscript + in the modal amplitude denotes the positive axial direction, and the subscript p and e the propagating and evanescent mode, respectively. The superscript H denotes the Hermitian transpose. For the accurate estimation of modal amplitudes, the number of measurement points should be larger than or at least equal to the number of modes in modelling. In each circumferential mode order *m,* the modal amplitudes are estimated in the stationary reference frame with the measured pressure $\mathbf{p}_{\mathrm{m}}^{\mathrm{s1}}$ in the shifted frequency $\boldsymbol{\omega}_{\mathbf{s}} = \omega_0 + m\Omega$ and with the measured pressure $\mathbf{p}_{\mathrm{m}}^{\mathrm{s2}}$ in the shifted frequency $\boldsymbol{\omega}_{\mathbf{s}} = \omega_0 - m\Omega$.

When the evanescent wave is included in the modelling of the sound propagation in a duct, the matrix equation Eq. (4) relating the measured pressure spectra to the modal amplitudes becomes ill-posed in general. To resolve the ill-posedness in the inversion of the transfer matrix $\mathbf{G}_{\mathrm{mi}}$ due to the evanescent modes, which decay rapidly and can reach the background noise of the measurement [3], regularization is adopted in estimating the modal amplitudes to get physically feasible and mathematically stable solution. For the inverse estimation of the modal amplitudes, Hansen's regularization package is used [4], and Tikhonov regularization is adopted with L-curve function. The modal amplitudes are estimated as

$$\left[\mathbf{c}_{\mathrm{p}}^{\mathrm{i+}} \quad \mathbf{c}_{\mathrm{e}}^{\mathrm{i+}}\right]_{\mathrm{s}}^{\mathrm{T}} = \left[\mathbf{G}_{\mathrm{mi}}^{\mathrm{H}}\mathbf{G}_{\mathrm{mi}} + \lambda \mathrm{I}\right]^{-1} \mathbf{G}_{\mathrm{mi}}^{\mathrm{H}}\mathbf{p}_{\mathrm{m}}^{\mathrm{si}}, \qquad (9)$$

where λ denotes the regularization parameter determined by L-curve function.

The identified propagating and evanescent modal amplitudes with and without regularization are in the stationary reference frame are rearranged for the modal amplitude set in the rotating reference frame by following Eq. (6). Then, by using the obtained modal amplitude set in the rotating reference frame, the pressure field can be restored in the source plane by using Eq. (7).

4. Measurement and analysis

4.1 Measurement setup

The measurement setup and the microphone configuration are illustrated in Fig. 2. In this work, there is no mean flow ($M_f = 0$) in the validation test. The duct diameter D is 250 mm and the thickness of the duct wall is 10 mm. Sixteen 1/4-inch microphones (B&K 4939) as an array are flush mounted for the nearfield measurement, and the nearfield measurement is conducted at 4 different standoff distances, 15, 30, 45, and 60 mm from the source plane. All microphones are calibrated in magnitude and phase.

As a sound source, a small 2 W loudspeaker with a power amplifier (B&K 2716) is used to excite the duct system. As an excitation signal to the loudspeaker, pure tone signal at 500 Hz are given. A signal analyzer (B&K Pulse 3560D) is employed for the generation of the excitation signal, data acquisition, and signal analysis. An electric motor, belt pulley system, and motor controller system are used to rotate the bar with two constant angular speeds, Ω = 300 and 600 rpm. The bar rotates in a counter clockwise direction, and the excitation signal is transmitted through a slip ring. A laser transmitter and a receiver are installed to synchronize the measured data to the rotating speed of the bar, which provides a triggering signal when the loudspeaker passes through a designated microphone. The excitation signal of the loudspeaker and the triggering signal of the laser system are used as the phase reference.

The anechoic termination used for the measurement is constructed by following ISO 5136 [5]. In this measurement setup, the amplitudes of propagating and evanescent modes generated by

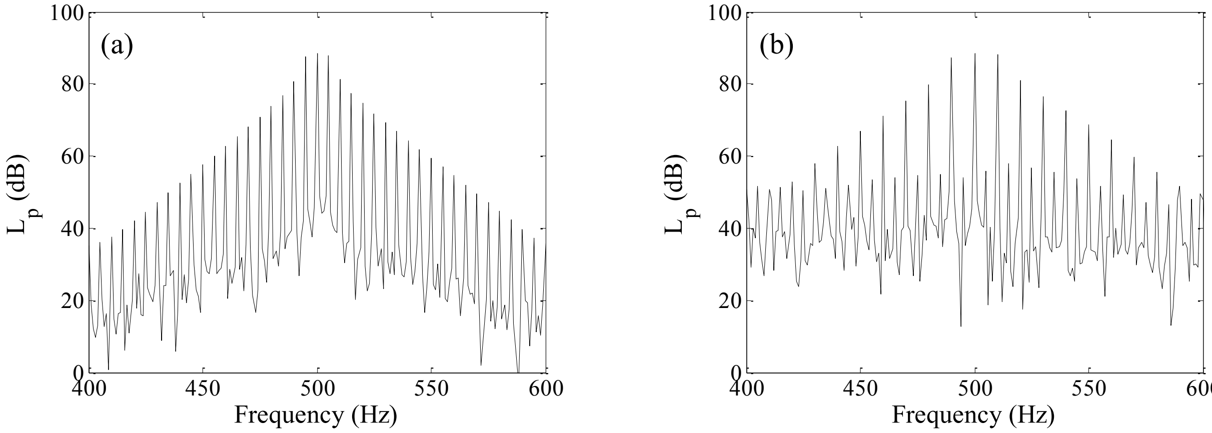

Fig. 2. Measured pressure spectra at mic. #5 in the 1st measurement plane with different rotating speeds (f_e = 500 Hz): (a) Ω = 300 rpm; (b) Ω = 600 rpm.

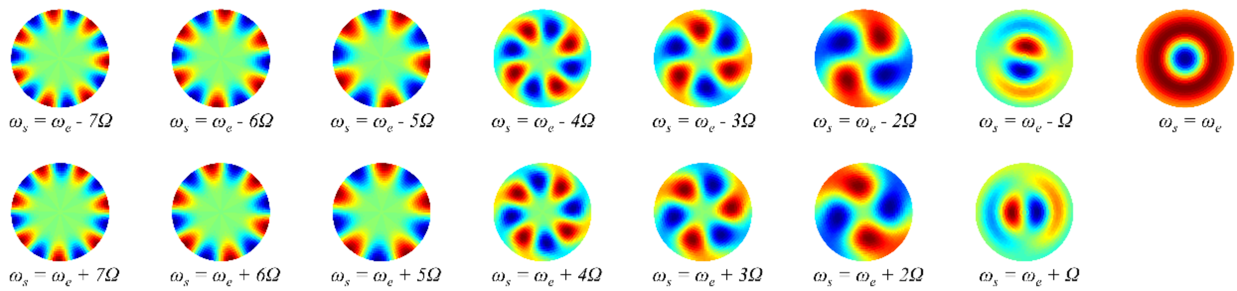

Fig. 3. Restored pressure field of a rotating source at the source plane in the stationary reference frame in the shifted frequencies (f_e = 500 Hz, Ω = 300 rpm).

the sound source can be correctly estimated in the range $0.57 < kR < 9.5$. The total number of modes used for the estimation of source parameters at the source plane is 27, and the reflected propagating and evanescent modes are neglected in the analysis.

4.2 Measured sound pressure in the nearfield

When the loudspeaker rotates, the Doppler effect is observed, and the radiated noise from the rotating source moves to other frequencies depending on the circumferential mode order m and the rotating speed Ω. The measured pressure spectra with a rotating loudspeaker in the near-field are shown in Fig. 2. The frequency spacing Δf between the peaks depends on the rotating speed, and Δf = 5 and Δf = 10 Hz are for the rotating speed Ω = 300 and Ω = 600 rpm, respectively. Due to the counter clockwise rotation of the loudspeaker, the measured pressure level $L_{p,ccw}$ at a frequency of $f_e + m\Omega$ is larger than the measured pressure level $L_{p,cw}$ at $f_e - m\Omega$ due to the convective amplification.

4.3 Error analysis

The pressure in the 1st measurement plane in which the effect of evanescent modes is strongest is regenerated and compared with the measured pressure spectra by using the obtained modal amplitudes. To calculate the difference between the regenerated and measured pressure quantitatively, the error e between them is defined as

$$e = 20\log_{10}\left(\frac{1}{N}\sum\left|\frac{p_r - p_m}{p_m}\right|\right), \qquad (9)$$

where p_r is the restored complex pressure, p_m the measured complex pressure in the stationary reference frame at each shifted frequency, and N the number of measurement points. The defined error becomes minus infinity when the regenerated pressure and the measured one coincide. The estimated error for the worst case is observed with the rotating speed Ω = 600 rpm, but the defined error is still below -10 dB. The modal amplitudes are well estimated, and the source parameters are identified by the estimated modal amplitudes.

5. Identification of the rotating source

5.1 A rotating source in the stationary frame

The source identification technique for the stationary source is also applied to the rotating source condition. When the technique is applied to

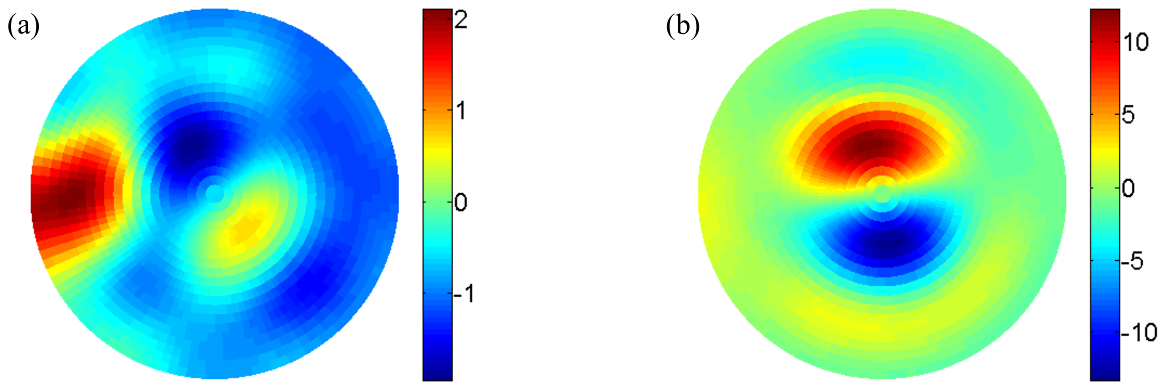

Fig. 4. Regenerated pressure field of the rotating source at the source plane in the rotating reference frame: (a) Ω = 300 rpm; (b) Ω = 600 rpm. Pseudo-inversion is used for the estimation of modal amplitudes.

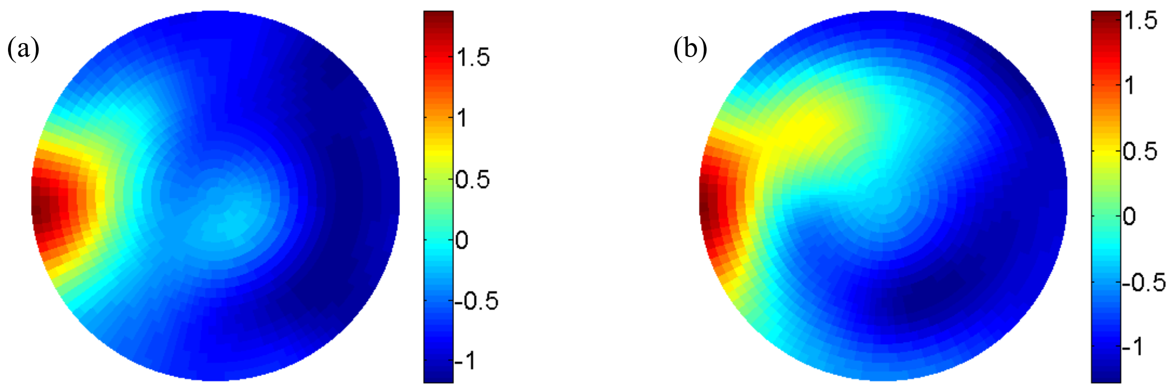

Fig. 5. Restored pressure field of the rotating source at the source plane in the rotating reference frame: (a) Ω = 300 rpm; (b) Ω = 600 rpm. Tikhonov regularization with L-curve function is used for the estimation of modal amplitudes.

the rotating source at the shifted frequencies $\boldsymbol{\omega}_s$, the modes with the corresponding circumferential order m should be mainly concerned at the source plane. The identified pressure field for the rotating source in the stationary reference frame is shown in Fig. 3, where the excitation frequency is f_e = 500 Hz and the rotating speed is Ω = 300 rpm. In Fig. 3, one can interpret the pressure field depicted for each shifted frequency $\boldsymbol{\omega}_s = \omega_0 \pm m\Omega$ as the existence of many stationary sound sources that mainly excite the modes corresponding to the circumferential mode order m in the frequency domain, although there is one rotating sound source that radiates a pure tone. When the rotating source behavior is observed in the stationary reference frame, the location of major sound radiators cannot be found. Only the equivalent strengths of excited modes can be estimated, and it is difficult to directly relate this variable with the design parameters of the rotating source. In this regard, one needs to restore the rotating source in the rotating reference frame.

5.2 A rotating source in the rotating frame

The pressure field in the source plane with two different rotating speeds are estimated for the rotating loudspeaker. The modal amplitude set obtained by the pseudo inversion operation is used for the reconstruction of source parameters in the source plane, and the result is illustrated in Fig. 4. The color bar and the scale shows the acoustic pressure in Pascal. The angular position of the main sound radiator, i.e., the rotating loudspeaker, is well identified in the rotating reference frame when the rotating speed is Ω = 300 rpm. However, the radial position of the rotating loudspeaker is erroneously obtained with Ω = 600 rpm. The modes with radial mode order $n \geq 1$ tend to be over-amplified in the inverse estimation process when combined with the flush-mounted measurement setup [6]. Therefore, the center of the duct is identified as the main noise radiator when the source rotates with Ω = 600 rpm.

Regularization is adopted in the estimation process of modal amplitudes. The source parameters are reconstructed by the modal amplitude set obtained by Tikhonov regularization in the rotating reference frame in which the L-curve method is used for the selection of regularization parameter. Figure 5 shows the results for different rotating speed, and it reveals that the suggested method can pinpoint the major radiating component of the rotating source with high spatial resolution for two different frequencies.

When the loudspeaker rotates with Ω = 600 rpm, the estimated source radiating area is expanded in the angular direction and compressed in the radial direction compared to the radiating area of the rotating source with Ω = 300 rpm. In general, when the rotating source is observed in the rotating reference frame, the location of the loudspeaker is well identified as the main sound radiating component.

6. Conclusions

In this work, an in-duct source identification technique for a rotating sound source is proposed. The rotation of the source and the high order acoustic modes are considered in the identification process. A validation experiment is conducted by using a duct system excited by a rotating loudspeaker. The modal amplitude set that is related with the rotating loudspeaker is well estimated with a maximum error of -10 dB when compared with the measured pressure. From the results, it is seen that the position of the high pressure level corresponds to the triggering position of the laser system when the loudspeaker rotates while emitting pure tone signal. The technique suggested in this work can be applied to estimate the main sound radiators of a complicated aero-acoustic source with high spatial resolution by including the convection of the medium.

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References

[1] T. Bravo and C. Maury, “Enhancing the reconstruction of in-duct sound sources using a spectral decomposition method,” *J. Acoust. Soc. Am.* **127**, 3538-3547 (2010).

[2] J. Lavrentjev and M. Åbom, “Characterization of fluid machines as acoustic multi-port sources,” *J. Sound Vib.* **197**, 1-16 (1996).

[3] M. R. Bai, J.-G. Ih, and J. Benesty, *Acoustic Array Systems: Theory, Implementation, and Application*, John Wiley & Sons, Singapore, 31-32 (2013).

[4] P. C. Hansen, “Regularization tools: A Matlab package for analysis and solution of discrete ill-posed problems,” *Numer. Algorithms* **6**, 1–35 (1994).

[5] ISO Standard 5136:2003(E), “Acoustics - Determination of sound power radiated into a duct by fans and - In-duct method,” International Organization for Standardization (2003).

[6] Y. -H. Heo, J.-G. Ih, and H. Bodén, “In-duct identification of fluid-borne source with high spatial resolution,” *J. Sound Vib.* **333**, 6074 - 6089 (2014).