Unit-cell variability and micro-macro modeling of polyurethane acoustic foams

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Summary

This paper investigates the impact of the irregular microgeometry of polyurethane acoustic foam on the macroscopic acoustic behavior predicted by a unit-cell model. Two semi-empirical unit-cell models coupled to a rigorous sensitivity analysis technique are used for this purpose. In these models, the porous material is idealized as a packing of a periodic unit-cell (PUC) representative of the disordered network that constitutes the porous frame. The non-acoustic parameters involved in the classical Johnson-Champoux-Allard model are derived from characteristic properties of the PUC and semi-empirical relationships. However, due to the large complexity of the foam microgeometry, the measurements of the main unit-cell properties can be subjected to an important variability mainly related to bulk inhomogeneity, microstructural irregularities, and limitations of the used measurement methods. A global sensitivity analysis is performed on these two models in order to investigate how the variability associated with the measured PUC characteristics affects the models outputs. This allows identification of the possible limitations of a unit-cell micro-macro approach. The sensitivity analysis mainly shows that for moderately and highly reticulated polyurethane foams, the strut length parameter is the key parameter since it greatly impacts three important non-acoustic parameters and causes large uncertainty on the sound absorption coefficient even if its measurement variability is moderate. For foams with a slight inhomogeneity and anisotropy, a micro-macro model associated to cell size measurements should be preferred.

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1. Introduction

Porous materials are heterogeneous materials composed of solid and fluid phases. According to the homogenization theory [1], the heterogeneous porous material can be considered as homogeneous if the characteristic dimensions (i.e., macroscopic wavelengths) are large compared to the size of the inhomogeneities (i.e., pore size). The wave properties at the microscopic scale can then be described according to their mean value observed at the macroscopic scale within a representative elementary volume (REV). This important consideration justifies the description of the porous media as an equivalent fluid characterized by a frequency-dependent effective density $\rho(\omega)$ accounting for inertial and viscous effects, and a frequency-dependent effective Bulk modulus $K(\omega)$ accounting for thermal effects. The well-known Johnson-Champoux-Allard (JCA) semi-phenomenological model [2, 3, 4] is used in this paper to predict the frequency behavior of the two aforementioned functions and requires the following macroscopic characteristic of the REV, also known as non-acoustic parameters: porosity ϕ, airflow resistivity σ, tortuosity α_∞, thermal characteristic length Λ' and viscous characteristic length Λ.

The JCA semi-phenomenological model is found successful to simulate the acoustical behavior of porous materials with different microstructures (e.g., foams, fibrous, granular,...). However, when used directly, this approach is not useful for microstructure optimization since by definition, it is blind to realistic microgeometrical details. To circumvent this limitation, numerous works have been proposed to link the characteristic microstructure properties to non-acoustic parameters and acoustic properties. In the particular case of acoustic foams, the three main approaches are (i) analytical [5, 4] based on simplified models of the microstructure and wave propagation inside the material (also known as scaling laws) [4], (ii) empirical [6] and semi-empirical [7, 8], and (iii) numerical [9, 10, 11]. The last two aforementioned approaches (i.e., semi-empirical and numerical) are the most promising since they allow investigation of the impact of the main microstructure parameters on the wave properties without a complex description of the real microgeometry at the meso- or macro- scales and at the same time, without an excessive simplification of the idealized microgeometry. In the particular case of polyurethane acoustic foams, both approaches are based on a periodic unit-cell (called PUC) with a tetrakaidecahedral morphology representative of the complex internal structure [12, 13]; the PUC is considered to be the REV in the homogenization theory. However, identifying a representative PUC of such complex and disordered 3D structure is not straightforward since most of the cells can differ from an idealized tetrakaidecahedron [14, 13] and cell windows (i.e., pores) can be randomly closed or partially closed by thin membranes. Measurements of the main PUC properties can thus be subjected to an important variability related to bulk inhomogeneity, microstructural irregularities and limitations of the used measurement methods.

The objective of this paper is thus to investigate how the variability associated with the microstructure input characteristics of the PUC affects the macroscopic quantities (i.e, non-acoustic parameters, sound absorption coefficient) using a rigorous sensitivity analysis technique [15].

The microstructrue based modeling, together with the description of the semi-empirical models, can be found in [16]. The two considered semi-empirical models estimate the non-acoustic parameters through geometrical, physical and empirical considerations. The initial 3-parameter model uses strut's length l, strut's thickness t and reticulation rate R_w as input parameters. They are all identified from SEM measurements on a large number of pictures. The 2-parameter model considers the ratio $B = l/t$ as a constant, hence it uses only the cell size C_s and reticulation rate R_w as input parameters. B being fixed, l and t can be geometrically determined from the knowledge of C_s.

A so-called global sensitivity analysis method is used in this work. Detailed calculations related to the method have been omitted for conciseness and clarity. For more information, the reader is referred to the following references [15, 17, 18, 19, 20, 21]. A global sensitivity analysis method is able to estimate the sensitivity of a model to large variations of input parameters using a variance decomposition and identify input parameters cross coupling effects. More particularly, the Fourier Amplitude Sensitivity Test (FAST) has been applied, it is an efficient technique that can be used to estimate the "main effect" SI (also named first order term) and the "Total Sensitivity Indexes" (TSI). The use of normalized indexes allows efficient ranking of parameters, but the variance level should not be forgotten during the analysis of the results. For proper analysis of the sensitivity issues, each time that SI or TSI indexes are presented, we will also give the value of the normalized standard deviation NSD (ratio between the standard deviation and the mean value of the quantity of interest), as a measure of the level of variability of the feature of interest.

2. Materials

The objective of this work is to show how the macroscopic acoustic properties of PU foams are impacted by the PUC variability. Before going into the details of the analysis, it should be emphasized that the sensitivity analysis results of considered foams must be interpreted regarding the knowledge of inputs, which includes geometrical irregularities effects in the foams (i.e., effect of cell anisotropy, inhomogeneity within the bulk volume), together with uncertainties due to measurement procedures. In particular, it should be understood that a parameter could be classified as almost insensitive due to the fact that (i) it has been identified in a very precise manner experimentally, (ii) this parameter is barely affected by bulk inhomogeneities or (iii) simply because the model is not much sensitive to this parameter.

The sensitivity analysis is applied to four foams provided by the Woodbridge Group©. The microstructure of these foams is characterized following the process described in ref. [16]. Foam P1 is a fully reticulated foam with a small cell size ($C_s = 673\ \mu m$). Materials M10 and P2 are partially reticulated foams with a small cell size ($C_s \approx 650\ \mu m$) and a moderate and low reticulation rate, respectively ($R_w \approx 70$ % for M10 and $R_w \approx 30$ % for P2). Finally, foam P3 is partially reticulated with a very large cell size ($C_s \approx 1700\ \mu m$) and a very low reticulation rate ($R_w = 5\%$). It is worth mentioning that an anisotropy has been observed for the reticulation rate parameter of material P3: i.e., a reticulation rate of 5 % is observed in the longitudinal direction (SEM pictures taken in the plane perpendicular to the wave propagation) and of 35 % in the transverse direction (SEM pictures taken in the plane parallel to the wave propagation). Only the coefficient measured in the plane perpendicular to the wave propagation is considered since it is the one that mainly impacts the wave propagation (the value of 5 % has been validated in ref.

[8] when comparing sound absorption measurements and micro-macro predictions). Note that these four foams have already been presented in references [7, 8] and are at the limits of the range of microstructure properties used in the first characterization set (i.e., for materials M1 to M15, 500 $\mu m < C_s <$ 1600 μm and 10% $< R_w <$ 100%). All foams are considered quasi-isotropic with a $DA < 1.25$. The measured non-acoustic properties of the foams are given in Table II. Their microstucture properties are summarized in Table II. Mean values are provided together with the uncertainties levels which are related to measurement, anisotropy and heterogeneity of the sample. The standard deviation for the three microstructure properties C_s, l and t is given as a percentage of the mean value. Table II indicates that, the variability measured on strut dimensions is larger than the one measured on cell size and reticulation rate: the variability on C_s and R_w is globally below the threshold of 10 % (except for foam P3) and conversely, the variability on strut dimensions l and t slightly exceeds 10 %. For material P3, the larger variability on cell size can be attributed to the cell anisotropy which is at the limit of the threshold set to 1.25. The sensitivity analysis is applied for the 4 foams and to the 2 micro-macro based models. Strut dimensions and cell size are described by Gamma probability density functions identified from maximum-likelihood estimation using the large set of local measurements available on the various foam samples [21]. Concerning the reticulation rate, the low coupling effects associated to small amount of measurements and unclear physical limits of R_w on the samples of interest led us to use uniform probability density functions for this variable. The reason for this small amount of data is that the reticulation rate is estimated at a higher scale from the SEM pictures (i.e., meso-scale). A deeper analysis of the impact of the choice of probability density functions, can be found in reference [21]. The calculations and measurements presented here correspond to a material thickness of 2 in., except for materials P1, P2 and P3, which are 1 in. thick. This choice will be discussed later.

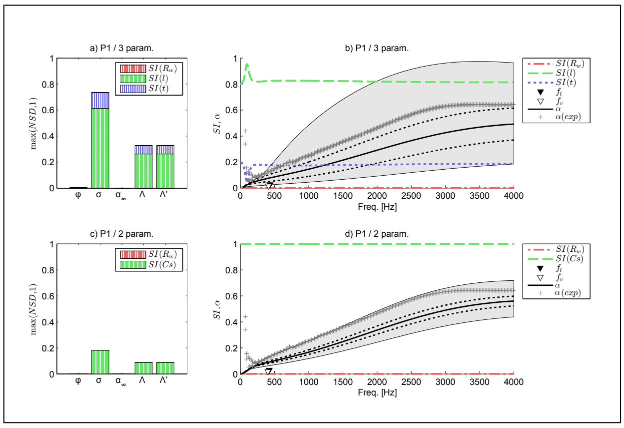

Figure 1. Sensitivity analysis on the foam P1. First line: First-order sensitivity index SI for macroscopic non-acoustic parameters and associated Normalized Standard Deviation NSD. Second line: normal incidence sound absorption (bold line: mean value; dotted line: standard deviation; gray area: extremal bounds) and associated first-order sensitivity index SI. Viscous/inertial and isothermal/adiabatic transition frequencies f_v and f_t are also shown on the plots.

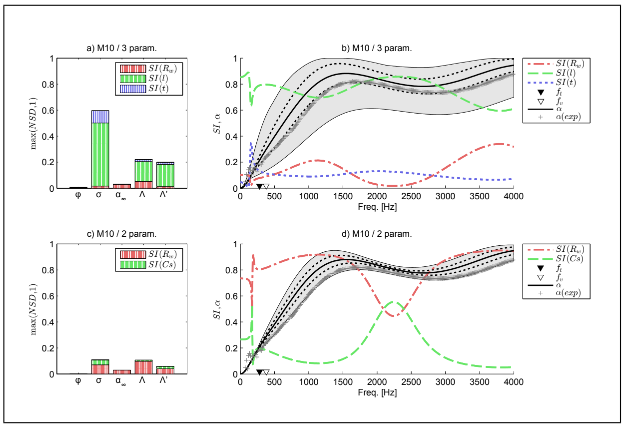

Figure 2. Sensitivity analysis on the foam M10. First line: First-order sensitivity index SI for macroscopic non-acoustic parameters and associated Normalized Standard Deviation NSD. Second line: normal incidence sound absorption (bold line: mean value; dotted line: standard deviation; gray area: extremal bounds) and associated first-order sensitivity index SI. Viscous/inertial and isothermal/adiabatic transition frequencies f_v and f_t are also shown on the plots.

3. Impact of the PUC variability

The results of the sensitivity analysis are given in Figs. 1-4. The reticulation rate of foam P1 is 100%, hence tortuosity is constant and its normalized standard deviation is null as shown in Fig. 1(a). The porosity is barely affected by the measured variability on l and t and its NSD is close to 0. For the other non-acoustical parameters (σ, Λ and Λ'), the trends are equivalent, namely more than 3/4 of the sensitivity is due to the strut length l while less than 1/4 is due to strut thickness t. The NSD of these non-acoustical parameters is high, particularly for the airflow resistivity. As far as the acoustic features are concerned (see Fig. 1(b)), relative contributions are consistent with the observations made on non-acoustic parameters: the fully reticulated foam P1 exhibits almost constant sensitivity indexes on the frequency range of interest, confirming the fact that the strut length l has much more impact on the variations of the outputs than the strut thickness t. The sound absorption coefficient derived from the 3-parameter model is underestimated compared to impedance tube measurements because the airflow resistivity is underestimated. This fact is already commented in details in

Table I. Non-acoustic properties of the foams and transition frequencies

Material	ϕ (%)	σ (N.s.m^{-4})	α_∞	Λ (μm)	Λ' (μm)	f_v (Hz)	f_t (Hz)
P1	95.6	3 490	1.06	187	250	412	433
M10	98.2	3 670	1.25	240	310	378	280
P2	95.8	17 440	1.73	46	220	1 267	557
P3	97.1	19 360	2.16	24	458	1 142	128

Table II. Microstructure properties of the four foams

Material	C_s		l		t		R_w		Degree of anisotropy DA
	mean (μm)	Stdeva (% of mean)	mean (μm)	Stdeva (% of mean)	mean (μm)	Stdeva (% of mean)	mean (%)	expanded uncertainty (%)	
P1	673	9	208	14	53	14	100	-	1.12
M10	681	3	204	13	62	11	69	9	1.10
P2	637	9	213	10	58	9	32	9	1.14
P3	1751	15	554	12	172	10	5	4	1.25

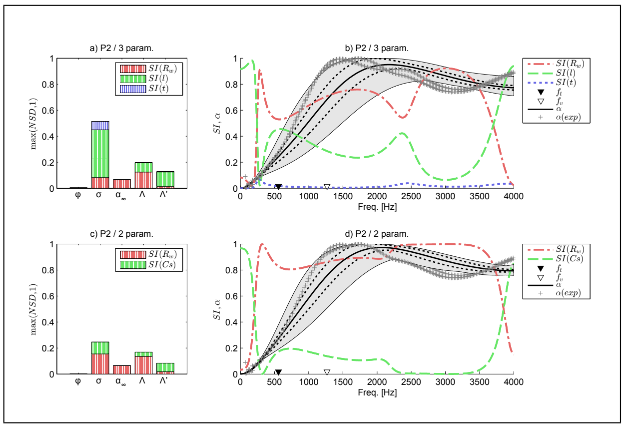

Figure 3. Sensitivity analysis on the foam P2. First line: First-order sensitivity index SI for macroscopic non-acoustic parameters and associated Normalized Standard Deviation NSD. Second line: normal incidence sound absorption (bold line: mean value; dotted line: standard deviation; gray area: extremal bounds) and first-order sensitivity index SI. Viscous/inertial and isothermal/adiabatic transition frequencies f_v and f_t are also shown.

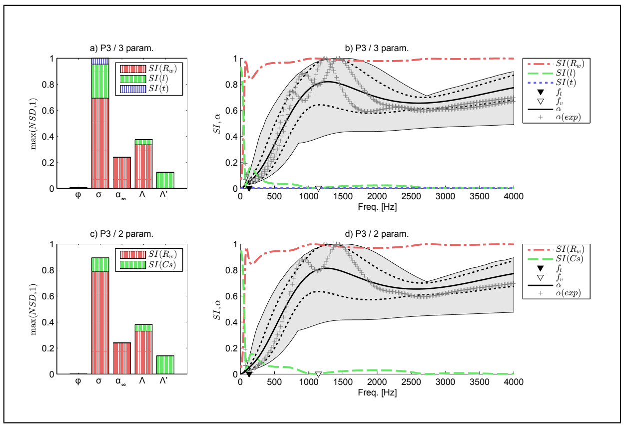

Figure 4. Sensitivity analysis on the foam P3. First line: First-order sensitivity index SI for macroscopic non-acoustic parameters and associated Normalized Standard Deviation NSD. Second line: normal incidence sound absorption (bold line: mean value; dotted line: standard deviation; gray area: extremal bounds) and first-order sensitivity index SI. Viscous/inertial and isothermal/adiabatic transition frequencies f_v and f_t are also shown.

reference [8]. The sensitivity analysis mainly shows that the strut length parameter is the key parameter for fully reticulated materials since it greatly impacts three important non-acoustic parameters. However, it also induces large uncertainty on the sound absorption coefficient as shown in Fig. 1(b) even if its measurement variability is moderate (i.e., the standard deviation is less than 15% of the mean value). This parameter should be measured with great care and only fully isotropic foams should be considered with a DA as close as possible of 1 in order to minimize the strut length variability. The sensitivity analysis applied to the 2-parameter model (see Figs. 1(c) and 1(d)) shows that both the non-acoustic parameters and the sound absorption coefficient are less impacted by the cell size variability. Indeed, the 2-parameter model is less impacted by the cell size variability than the 3-parameter model by the strut length variability. Furthermore, as far as foam P1 is concerned, the measurement variability on cell size is less important than the one of the strut length. This tends to promote the 2-parameter model. Indeed, the variability on the estimated sound absorption is reduced (see Fig. 1(d)) and the agreement between sound absorption measurements and estimates is improved. However, the model still shows differences compared to measurements since the simple expression considered underestimates the airflow resistivity for material P1 [8] (σ is measured at 3490 N.s.m^{-4} and estimated at 2760 N.s.m^{-4}).

Consider now the two partially reticulated foams M10 and P2 which sensitivity analysis are presented in Figs. 2 and 3 respectively. These two foams share identical microstructure properties both in terms of mean values and standard deviations. The main difference is the amount of open pores; the foam M10 is moderately reticulated with $R_w = 69\%$ and the foam P2 is poorly reticulated with $R_w = 32\%$. However, the expanded uncertainty on R_w measured for both foams is identical and close to 10%. The two materials also slighlty differ by their strut variability; material M10 shows larger variability (i.e., 13 % of the mean value for M10 and 10 % for P2). Furthermore, the sample of material M10 is 2 in. thick whereas the sample of material P2 is 1 in. thick.

For both foams, the sensitivity analysis applied to the 3-parameter model indicates that, once again, the impact of the strut length parameter is important on the three non-acoustic parameters σ, Λ and Λ' (see Figs. 2(a) and 3(a)); the most impacted parameter with the greater NSD being the airflow resistivity σ. Except for the tortuosity, the variability on all non-acoustic parameters (i.e., NSD) is greater in the case of material M10 compared to material P2 most likely due to the larger strut length variability measured for M10. The sound absorption variability of material M10 is thus more important as shown in Figs. 2(b) and 3(b). Indeed, Fig. 2(b) shows that the strut length is the most sensitive parameter on the whole frequency range. The impact of R_w increases logically with the amount of closed pores (see Figs. 2(a) and 3(a)); for example, the reticulation rate explains almost 25% of the sensitivity on the viscous characteristic length Λ for material M10 and 65% for material P2. It also explains 3% of the sensitivity on the airflow resistivity σ for material M10 and 16% for material P2. For both cases, the reticulation rate captures all sensitivity effects on tortuosity α_∞ and the strut length explains almost 90% of the sensitivity on the thermal characteristic length Λ'. Fig. 3(b) indicates that the effect of R_w on the sound absorption coefficient of material P2 is predominant but the impact of the strut length l is non negligible. l even dominates around the first dip in the sound absorption curve, i.e. around 4 kHz in this case considering the given thickness of 1 inch. l also dominates at very low frequencies ($f < 100$ Hz) but it is not relevant since the variability of the sound absorption is null in this frequency range. This is coherent with the known effect of Λ' on α curves documented in the literature [22, 21].

The sensitivity analysis applied to the 2-parameter model (see Figs. 2(c), 2(d), 3(c) and 3(d)) confirms that both the non-acoustic parameters and the sound absorption coefficient are less impacted by the cell size variability. In the case of foam P2, the variability on l and C_s is identical and close to 10% of the mean value; the improvement is thus only due to the low sensitivity of the 2-parameter model to C_s (i.e., the airflow resistivity in the 2-parameter model is a function of C_s^2). As far as foam M10 is concerned, the cell size variability is also reduced compared to strut length variability as indicated in Table II. The reticulation rate R_w thus becomes the most sensitive parameter for all non-acoustic and acoustic properties as presented in Figs. 2(c) and 2(d). The NSD of all non-acoustic properties are also greatly reduced so as the variability on the sound absorption coefficient.

Finally, the sensitivity analysis is applied to the poorly reticulated foam P3 characterized by a very large cell size, a very low reticulation rate $R_w = 5\% \pm 4\%$. The measured uncertainty on R_w is important in comparison with the mean value. The results of the sensitivity analysis is presented in Fig. 4. The airflow resistivity σ, the tortuosity α_∞ and the viscous length Λ greatly vary with input parameters, and their values are mainly driven by the reticulation rate R_w. The NSD of these non-acoustical parameters is very high, particularly for the airflow resistivity. The strut length parameter has a noticeable influence on both σ and Λ' but in this case, its influence on the sound absorption coefficient is poor in the whole frequency range (see Fig. 4(b)). It can be concluded that the thermal characteristic length Λ' has no influence for this class of material; i.e., thermal effects are negligible compared to the viscous ones. The low thermal transition frequency (i.e., 128 Hz) and the high viscous transition frequency (i.e., 1142 Hz) confirm this analysis. The sound absorption variability is very high in this case (see gray area between back dashed curves in Figs. 4(b) and 4(d)). However, it is considered acceptable since it includes measured values which are themselves highly impacted by variability due to frame vibration and sample lateral boundary conditions inside the impedance tube. The sensitivity analysis carried out on foam P3 shows that in this case, only the reticulation rate R_w should be identified in a precise manner.

4. Conclusion

This paper investigated the impact of polyurethane foam microstructure irregularity on the estimation of their macroscopic acoustic ($\alpha(\omega)$) and non-acoustic properties (σ, α_∞,...) using a robust sensitivity analysis performed on two semi-empirical unit-cell models. In these models, the porous material is idealized as a packing of an isotropic tetrakaidecahedra PUC representative of the disordered network that constitutes the porous frame. The PUC microstructure properties are measured directly from SEM micrographs. The main sources of measurement uncertainty on the PUC properties are related to bulk inhomogeneity, microstructural irregularities, natural anisotropy and limitations of the used measurement methods. The sensitivity analysis shows that, depending on the PUC microstructure parameter, the measured variability (due to natural cell anisotropy, inhomogeneity within the bulk volume and measurement bias) may have a significant influence on the model output variability. When the 3-parameter model is considered, all the analyses made on foams show that the impact of measurement variability on strut thickness t is low. Thus, a precise measurement of this microstructure parameter is not required. For moderately and highly reticulated foams (i.e., $R_w > 30\%$), the sensitivity analysis mainly shows that the strut length parameter l is the key parameter since it greatly impacts three important non-acoustic parameters (σ, Λ and Λ'). It also causes large uncertainty on the sound absorption coefficient even if its measurement variability is moderate (e.g., foams P1 and M10). This parameter should be measured with great care and only fully isotropic foams should be considered with a DA as close as possible of 1 in order to minimize the microstructure variability. For this class of PU foam, the impact of R_w variability is low and it can be identified in a less precise manner. In the case of poorly reticulated PU foams (i.e., $R_w < 30\%$), it is concluded that only the reticulation rate R_w should be identified in a precise manner. Indeed, it explains the larger part of the high variability observed of σ, Λ and α_∞ and controls the sound absorption coefficient in the whole frequency range.

For foams with a slight inhomogeneity and anisotropy, the 2-parameter model associated to cell size measurement should be preferred since (i) this model is less affected by cell size variability and (ii) it has been observed that for this type of foams, the cell size measurement is less impacted by bulk variability than the strut length parameter.

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