A Wave Based Transfer Matrix Method for accurate simulation of acoustic problems with multilayered damping treatment

Stijn Jonckheere, Dirk Vandepitte, Wim Desmet
Dept. of Mechanical Engineering, KU Leuven, Leuven, Belgium.

Summary
Nowadays, engineers strongly rely on damping treatments to improve the NVH properties of their products. These materials are often a complex, multilayered combination of elastic, viscoelastic and poroelastic layers. In order to come to an optimised design, efficient CAE-models are indispensable. Currently, most tools for 3D vibro-acoustic simulation are in practice limited to low-frequency simulations due to the strongly increasing computational cost with frequency to control the errors within the approximations. The modelling of damping materials even further reduces the applicable frequency range.
Therefore, techniques are developed to replace the expensive element based techniques, with no or a limited cost in accuracy. Through its modelling procedure, the Wave Based Method (WBM) has an increased efficiency and is therefore a good candidate to substitute (large parts of) the vibro-acoustic system model. To introduce multilayer models with limited additional computational cost, the Transfer Matrix models can be used. When incorporating these TM models into a classical vibro-acoustic (element based) model, however, one should live with two approximation types; (i) the infinite layer assumption and (ii) the fact that the angle of incidence is not known a priori, such that often values for normal incidence or averaged values are used. Whereas the first class of approximations are innate to the TMM, the second class can be overcome by using the angle-dependent information present in the definition of the approximation functions in the WBM.

PACS no. xx.xx.Nn, xx.xx.Nn

1. Introduction

With the ever growing legislative and commercial importance of the noise and vibration behaviour of their products, engineers in many product industries often rely on multilayered damping treatments to improve its characteristics. Often, these materials are a multilayered combination of acoustic, elastic, viscoelastic and poroelastic layers. An important component of these multilayers are the so-called poroelastic materials [1]. These materials consist of two phases: a solid, elastic phase and a acoustic fluid phase.

To assess and optimise the effect of these layered treatments in an early development stage, design engineers rely on CAE-models. The modelling of the vibro-acoustic behaviour of these treatments is, however, far from trivial. The introduction of these multilayers in a vibro-acoustic system where the structural components and the surrounding acoustic fluid mutually interact with each other, further complicates the problem.

Currently, most CAE tools are based on element based tools such as the Finite Element Method (FEM) [2]. By dividing the problem domain into small elements, in which the solution is approximated using polynomial functions, the most complex problem cases can be treated. However, this element based concept is also the weakness of these element based methods as the number of elements required to control the numerical errors increases more than linearly with frequency [3]. Those methods are thus in practice limited to low-frequency simulations due to the strongly increasing computational cost with frequency. The introduction of damping materials even further reduces the applicable frequency range.

Therefore, a substantial research effort is invested in the development of better suited numerical modelling techniques.

Over the past two decades, the Wave Based Method (WBM) [4, 5] has shown the potential to alleviate some of these frequency limitations. As an alternative deterministic approach to the FEM, it is based

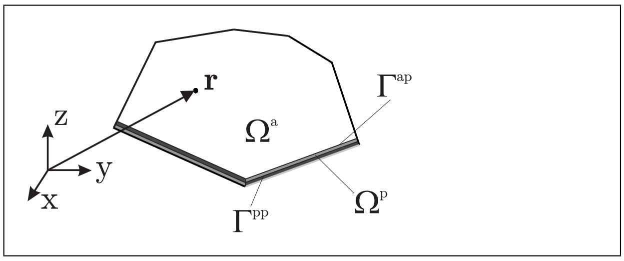

Figure 1. A general 3D vibro-acoustic problem with complex damping layers.

on an indirect Trefftz approach [6] in that it uses a set of *a priori* defined wave functions which are exact solutions of the governing differential equations. Moreover, hybrid techniques have been developed in order to couple with the WBM detailed FE models of damping materials [7].

For the modelling of multilayered damping treatments, the FEM is still often replaced by the Transfer Matrix Method (TMM). This method models the transmission through the thickness of a multilayer analytically, assuming laterally infinite layers. The method, however, requires an estimate for the angle of incidence, information which is not *a priori* available. Therefore, often values for normal or omnidirectional incidence are used.

This contribution details a more efficient use of the TMM in a WBM framework by using information about the angles of incidence innate to the wave functions. The paper is organised as follows. Section 2 reviews the problem definition and gives an overview of the mathematical formulations for the considered materials, i.e. acoustic fluids and poroelastic media. Section 3 gives a short overview of the WBM and Section 4 details the introduction of TM based models in a WBM framework. Section 5 shows the improved accuracy of the Wave Based Transfer Matrix Method (WBTMM) as compared to more classical TM approaches. The paper ends with a conclusion of the presented work.

2. Problem definition

Consider a general 3D acoustic problem with a multilayer in Figure 1. The steady-state dynamic behaviour is described by three sets of variables. The first set describes the dynamic pressure distribution inside the acoustic cavity Ω^a. The cavity is in partial contact with a complex damping treatment Ω^p, consisting of poroelastic materials. The dynamic behaviour of these materials, which consists of a porous frame and an interpenetrating acoustic fluid, is described by two additional sets of variables: one set for the frame and one for the fluid. All variables are mutually coupled.

The following section discusses the mathematical models that describe the wave propagation in an acoustic cavity and a poroelastic layer, respectively. Also the coupling conditions between an acoustic fluid and a poroelastic material, and between different poroelastic material layers are discussed.

2.1. Acoustics

The acoustic domain Ω^a is filled with an acoustic fluid, characterised by its speed of sound c_a and its ambient fluid density ρ_a. The steady-state behaviour of the acoustic pressure $p^a(\mathbf{r})$ is described by the inhomogeneous Helmholtz equation [8] under the assumption of a linear, inviscid and adiabatic fluid behaviour:

$$\mathbf{r} \in \Omega^a : \nabla^2 p^a + k_a^2 p^a = \mathcal{Q}_a, \tag{1}$$

where $\nabla^2 \bullet = \frac{\partial^2 \bullet}{\partial x^2} + \frac{\partial^2 \bullet}{\partial y^2} + \frac{\partial^2 \bullet}{\partial z^2}$ is the Laplacian operator, $k_a = \frac{\omega}{c_a}$ the acoustic wave number, and $\mathcal{Q}_a(\mathbf{r})$ an acoustic source term, for example an acoustic point source $\mathcal{Q}_{a,q} = -\mathrm{j}\rho_a q \delta(\mathbf{r}, \mathbf{r_q})$ with $\delta(\bullet, \star)$ the Dirac delta function, $\mathbf{r_q} = [x_q \; y_q \; z_q]^{\mathrm{T}}$ the location of the source and q its amplitude. The imaginary unit is denoted $\mathrm{j}^2 = -1$.

The Helmholtz equation requires one boundary condition imposed at every point on the problem boundary in order to have a unique solution (e.g. imposed pressure, normal velocity or normal impedance, or multi-physical coupling conditions).

2.2. Poroelastic materials

The poroelastic domain Ω^p consists of a porous solid, i.e. a frame of an elastically deformable material which is interpenetrated by a network of pores, saturated with an acoustic fluid. The volume distribution between both is described by the porosity ϕ, i.e. the volume ratio between the open pore volume and the total volume.

The Biot theory [9, 10] applies a homogenised solid and a compressible fluid continuum description on a macroscopic level, which is justified when the characteristic dimensions of the material, e.g. the pore size, are small as compared to characteristic dimensions on the macroscopic level, e.g. the wavelengths of the three different types of propagating waves. Fluid-structure interaction occurs throughout the whole material, and the different wave types can be strongly coupled.

According to the Biot theory, the behaviour of a poroelastic material is governed by a set of coupled differential equations, called the Biot equations:

$$\begin{aligned} N\nabla^2 \mathbf{u^s} + \nabla[(\lambda + \frac{\tilde{Q}^2}{\tilde{R}} + N)e^s + \tilde{Q}e^f] \\ = -\omega^2(\tilde{\rho}_{11}\mathbf{u^s} + \tilde{\rho}_{12}\mathbf{u^f}), \end{aligned} \tag{2}$$

$$\nabla[\tilde{Q}e^s + \tilde{R}e^f] = -\omega^2(\tilde{\rho}_{12}\mathbf{u^s} + \tilde{\rho}_{22}\mathbf{u^f}), \tag{3}$$

where λ and N are the solid material Lamé constants, $\tilde{Q}$ is a dilatational coupling factor between the fluid stress and the frame dilatation and *vice versa* and $\tilde{R}$ represents the fluid phase bulk stiffness at zero frame dilatation. Both Q and R are related to the bulk modulus of the fluid. The dilatation of phase $\bullet$ is written as $e^{\bullet}(\mathbf{r})$ and $\mathbf{e}^{\mathrm{s}}(\mathbf{r})$ indicates the strain tensor of the solid phase. The displacement vector of phase $\bullet$ is $\mathbf{u}^{\bullet}(\mathbf{r})$. The terms $\tilde{\rho}_{11}$, $\tilde{\rho}_{22}$ and $\tilde{\rho}_{12}$ take into account the inertial effects and viscous energy dissipation caused by the relative motion between both phases [11].

Complemented with the appropriate boundary conditions (e.g. sliding edge boundary conditions or multi-physical coupling terms), the Biot equations state a well-posed problem.

Open pore acoustic-poroelastic interface Assuming the pores of the poroelastic material are open, the acoustic fluid in the pores can directly interact with the surrounding acoustic fluid. This can be expressed as follows:

$$\mathbf{r} \in \Gamma_{op}^{ap}: \begin{cases} v_n^a &= \mathrm{j}\omega(\phi u_n^f + (1-\phi)u_n^s), \\ \sigma_n^s &= -(1-\phi)p^a, \\ \sigma^f &= -\phi p^a, \\ \sigma_s^s &= 0. \end{cases} \tag{4}$$

Open pore, fixed poroelastic-poroelastic interfaces With open pores the fluid phases of both poroelastic layers again directly interact and the flux through the interface is continuous. The solid phase of both materials are rigidly connected to each other:

$$\mathbf{r} \in \Gamma_{fx,op}^{pp}: \begin{cases} u_n^{s_1} &= u_n^{s_2}, \\ \phi_1(u_n^{s_1} - u_n^{f_1}) &= \phi_2(u_n^{s_2} - u_n^{f_2}), \\ u_s^{s_1} &= u_s^{s_2}, \\ \sigma_n^{s_1} + \sigma_n^{f_1} &= \sigma_n^{s_2} + \sigma_n^{f_2}, \\ \frac{\sigma^{f_1}}{\phi_1} &= \frac{\sigma^{f_2}}{\phi_2}, \\ \sigma_s^{s_1} &= \sigma_s^{s_2}. \end{cases} \tag{5}$$

3. Numerical modelling techniques

The Wave Based Method (WBM) forms the core numerical technique of this contribution. Therefore, this section discusses the basic concepts of the WBM for acoustic analysis.

3.1. Wave Based Method (WBM)

The WBM [4, 5] is a deterministic numerical technique for solving a set of (partial) differential equations under given boundary conditions. It follows an indirect Trefftz [6] approach, i.e. it applies exact solutions of the governing differential equations as solution expansion functions. For acoustic problems, these so-called wave functions exactly satisfy the Helmholtz equation (1).

The general modelling procedure of the WBM, as applied to a general 3D bounded acoustic Helmholtz problem, consists of the following four steps:

A. Partitioning of the considered problem domain into convex subdomains Desmet [4] showed that convexity of the considered problem domain is a sufficient condition to ensure the convergence of the method towards the exact solution of the problem. However, often the problem geometry is non-convex. In that case, the problem should be divided into a number of non-overlapping, convex subdomains. To ensure continuity of the acoustic pressure and normal velocity over the interface coupling conditions are applied to both subdomains.

B. Selection of a suitable set of wave functions for each subdomain The steady-state acoustic pressure field $p^a(\mathbf{r})$ in each of the subdomains is approximated, according to the WBM modelling principle, by a solution expansion $\hat{p}^a$ in terms of wave functions $\mathbf{\Phi_a}$ and their corresponding weights $\mathbf{w_a}$:

$$p^a(\mathbf{r}) \approx \hat{p}^a(\mathbf{r}) = \mathbf{\Phi_a}\mathbf{w_a} + \hat{p}_q^a(\mathbf{r}), \tag{6}$$

In accordance with the Trefftz principle, the wave functions $\mathbf{\Phi_a}$ exactly satisfy the homogeneous part of the associated governing Helmholtz equation (1) and thus consist of a set of propagating and evanescent wave functions:

$$\begin{cases} \Phi_{a_1} &= \cos(k_{a_1,x}x)\cos(k_{a_1,y}y)e^{-\mathrm{j}k_{a_1,z}z} \\ \Phi_{a_2} &= \cos(k_{a_2,x}x)e^{-\mathrm{j}k_{a_2,y}y}\cos(k_{a_2,z}z) \\ \Phi_{a_3} &= e^{-\mathrm{j}k_{a_3,x}x}\cos(k_{a_3,y}y)\cos(k_{a_3,z}z) \end{cases}, \tag{7}$$

The only requirement for the wave functions to be a solution of the Helmholtz equation, is that they satisfy the acoustic dispersion relation:

$$k_a^2 = k_{a_\bullet,x}^2 + k_{a_\bullet,y}^2 + k_{a_\bullet,z}^2. \tag{8}$$

By choosing the wave number components for the cosines based on a circumscribing box (Figure 2):

$$k_{a_\bullet,\star} = \frac{\pi n_\bullet}{L_\star}, \tag{9}$$

and the wave number components such that the total wavenumber vector satisfies the dispersion relation (8), a converging set is obtained. This infinite set of wave functions is truncated such that all wave functions with a wavelength larger than or equal to $1/T$ times the physical wavelength at the studied frequency are included.

The expansion is completed by a particular solution function $\hat{p}_q^a(\mathbf{r})$ for a volume point source.

$$\hat{p}_q^a(\mathbf{r}) = \frac{j\rho_a\omega q}{4\pi}\frac{e^{-jk_a d(\mathbf{r},\mathbf{r_q})}}{d(\mathbf{r},\mathbf{r_q})}, \tag{10}$$

where $d(\mathbf{r},\mathbf{r_q}) = \sqrt{(x-x_q)^2 + (y-y_q)^2 + (z-z_q)^2}$.

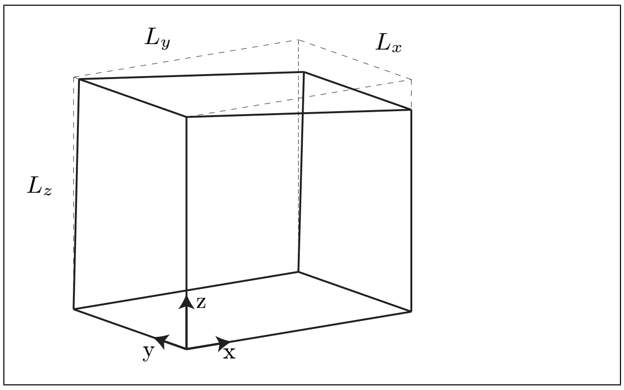

Figure 2. Circumscribing rectangular prism.

C. Construction of the WB system matrices via a weighted residual formulation of the boundary and interface conditions Within each subdomain, the proposed solution expansion always exactly satisfies the Helmholtz equation, irrespective of the values of the unknown contribution factors. However, the resulting dynamic field(s) may violate the imposed boundary conditions and the interface conditions. By minimising these errors by applying a Galerkin weighted residual approach, the system of equations is constructed.

D. Solution of the system of equations factors and postprocessing of the dynamic variables In a final step, the system of equations is solved for the unknown wave function contribution factors. The backsubstitution of these values in the field variable expansions leads to the approximation of the field variables. Derivative quantities, such as acoustic velocities and acoustic intensity can be obtained without any loss of spatial resolution.

4. Wave Based Transfer Matrix method

The Transfer Matrix Method [1, 12] is widely used to predict the dynamic behaviour of multilayer structures. It models the propagation of an incoming plane wave at an angle θ through a flat, infinitely extending layer (see Figure 3). The dynamic behaviour of each layer can be modeled by two propagating waves in the x_3-direction. This makes that the TMM is considered a 1,5D approach.

The wave propagation from the point A to the point B through a multilayer material is governed by a transfer matrix $\mathbf{T}$. This transfer matrix can be built up by first assuming 1,5D propagation through each of the separate layers. Through the interface conditions, the separate layers are assembled into a global transfer matrix between point A and point B, which can be condensed into the following form:

$$\begin{Bmatrix} p_B \\ v_A \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} p_A \\ v_B \end{Bmatrix}. \tag{11}$$

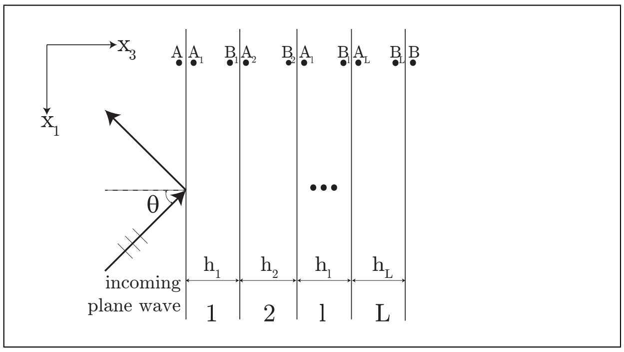

Figure 3. Infinitely extended multilayer with an incoming plane wave at an angle θ.

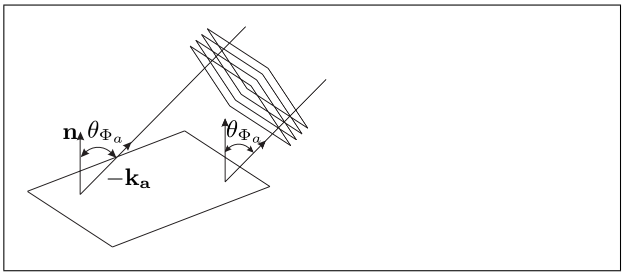

Figure 4. Wave function dependent angle θ_{Φ_a}.

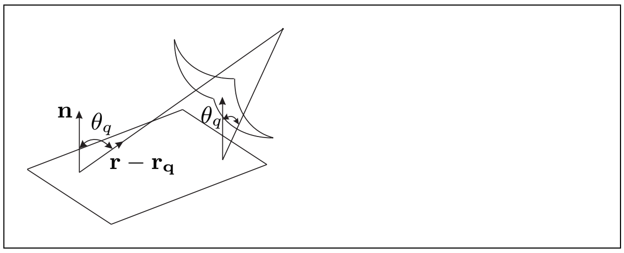

Figure 5. Particular solution dependent angle $\theta_q(\mathbf{r})$.

For the angle θ, two logical choices can be made:

- Normal incidence ($\theta = 0°$) (i.e. ideal Kundt tube measurements),
- Averaged incidence $0° \leq \theta \leq 90°$ (similar to reverberant room measurements).

In both cases the TMM is a mere preprocessing step to the model, as is conventional for the FEM. In this setting, however, the TM model is never fed the correct angles.

The WBM, however, contains additional information on the angle of incidence present of the wave functions and the particular solutions relative to a given flat surface with normal $\mathbf{n}$.

The acoustic wave functions (7) are defined as a set of propagating and evanescent plane waves. Each wave function has a wave number vector $\mathbf{k_a} = [k_{a,x}\ k_{a,y}\ k_{a,z}]^{\mathrm{T}}$ which expresses the propagation direction of the wave. Propagating waves have real values for all three directions of $\mathbf{k_a}$ whereas evanescent waves have a complex value in one direction.

Table I. Material properties of air.

Air		
Specific heat ratio	γ	1.4
Absolute temperature	T	293.15 K
Fluid density	ρ_a	1.205 kg/m^3
Gas constant	R	286.7 J/(kg K)
Fluid kinematic viscosity	ν_f	15.11·10^{-6} m^2/s
Thermal conductivity	k	2.57·10^{-2} W/mK

Through calculation of the angles between each of the wave number vectors $\mathbf{k_a}$ and the surface normal $\mathbf{n}$, the incident angles $\boldsymbol{\theta_{\Phi_a}}$ (see Figure 4) for a given wave function set $\mathbf{\Phi_a}$ can be calculated as follows:

$$\cos\left(\theta_{\Phi_a}\right) = -\frac{\mathbf{k_a}\cdot\mathbf{n}}{\|\mathbf{k_a}\|}, \tag{12}$$

where $\| \bullet \| = \sqrt{\sum \bullet_i^2}$ is the Euclidean norm.

Application of the TMM for each angle, leads to a different TM $\mathbf{T_{\Phi_a}}$ for each wave function.

A similar reasoning can be followd for acoustic source terms. The particular solution (10) is defined as the free field solution of an acoustic monopole and describes a spherical wave front originating from a given point in space $\mathbf{r_q}$. By locally linearising the spherical wave front by a plane wave front for each point of the interface, a spatially dependent angle of incidence $\theta_q(\mathbf{r})$ (see Figure 5) can be calculated for each of the particular solution functions:

$$\cos\left(\theta_q(\mathbf{r})\right) = -\frac{(\mathbf{r}-\mathbf{r_q})\cdot\mathbf{n}}{\|\mathbf{r}-\mathbf{r_q}\|} \tag{13}$$

Application of the TMM for each angle, leads to a different, spatially varying TM $\mathbf{T_q}(\mathbf{r})$ for each particular solution.

Using the WBTMM, all available information on the propagation angle of the wave functions, can be exploited in the TM model. This way, as the following validations show, an increased accuracy can be obtained at hardly an increased computational cost, as the calculation of the TM coefficients can be done analytically.

5. Numerical validation

The approach is validated on a convex acoustic cavity (1.15 m × 0.815 m × 0.982 m) with a multilayered damping treatment (Figure 6). The cavity is excited by an acoustic point source with $q = 1$ m^3/s at (1.03, 0.12, 0.3). The air inside the cavity has the properties given in Table I. The non-treated walls of the cavity are considered rigid. The response point at (0.13, 0.72, 0.15) is also indicated in Figure 6.

The 2 cm multilayered damping treatment has sliding edge boundary conditions and consists of 1 cm of Eurocell material on top of 1 cm of Fireflex material. The material properties can be found in Table II. The

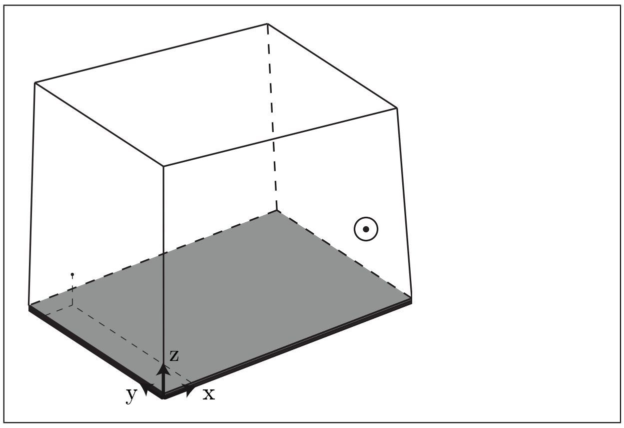

Figure 6. Problem geometry of a convex acoustic cavity with a multilayered damping treatment.

Table II. Biot-JCA material properties of the Eurocell and Fireflex materials [13].

Eurocell [13]		
Bulk density	ρ	126 kg/m^3
Bulk shear modulus	N	154·10^3 + j · 11·10^3 Pa
Bulk Young's modulus	E	372·10^3 + j · 38·10^3 Pa
Porosity	ϕ	0.95
Tortuosity	α_∞	1.07
Viscous length	Λ	19·10^{-6} m
Thermal length	Λ'	38·10^{-6} m
Static flow resistivity	σ	52·10^3 kg/m^{3}s
Fireflex [13]		
Bulk density	ρ	37.3 kg/m^3
Bulk shear modulus	N	66·10^3 + j · 4·10^3 Pa
Bulk Young's modulus	E	148·10^3 + j · 14·10^3 Pa
Porosity	ϕ	0.95
Tortuosity	α_∞	1.17
Viscous length	Λ	179·10^{-6} m
Thermal length	Λ'	359·10^{-6} m
Static flow resistivity	σ	9.2·10^3 kg/m^{3}s

poroelastic-poroelastic interface has open pores and is fixed (5). The pores on the acoustic-poroelastic interface are also open (4).

For the acoustic cavity, a single domain WB model is used with $T = 2$, leading to 24-518 wave functions. The multilayered material is modelled using a TM, following three approaches: $\theta = 0°$, averaged between $0° \leq \theta \leq 90°$ and WBTMM. The results are compared to an explicit Hybrid FE-WB reference model using the $(\mathbf{u^s}, p^f)$ formulation with 12296 DOFs in a hybrid FE-WB setting [7].

Figure 7 shows the frequency response of the Sound Pressure Level from 50 Hz to 800 Hz for a point (0.13, 0.72, 0.15) inside the cavity. Because the pores are open, the air inside the pores can directly interact with the air in the acoustic cavity. Moreover, the sliding edge boundary conditions introduce a certain degree of symmetry. As a consequence, mainly the fluid phase contributes to the dynamic behaviour of the poroelastic layer. Therefore, the TMM assump-

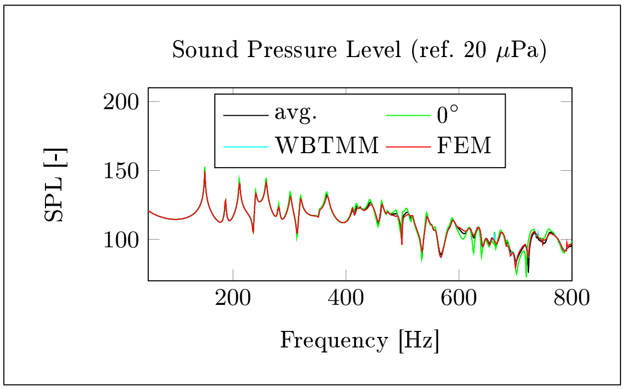

Figure 7. Sound Pressure Level [dB] for a response point (0.13, 0.72, 0.15) from 50 Hz to 800 Hz.

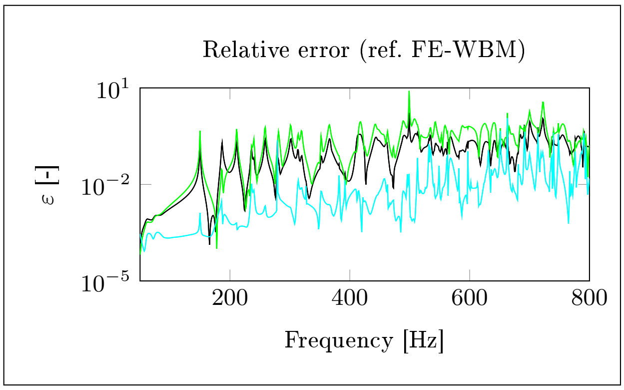

Figure 8. Relative error ε [-] (ref. FE-WBM) for a response point (0.13, 0.72, 0.15) from 50 Hz to 800 Hz.

tion of an infinite layer, which ignores the boundary conditions, is not too harsh.

The main problem is the estimation of the incoming angle for the TM models; the presence of an acoustic point source results in location dependent incoming angles. Moreover, the modal behaviour of the cavity gives, depending on the frequency, also a different incoming angle. It is clear that a normal incidence assumption in this case is not sufficient. The use of an averaged TM, on the other hand, smears out the information over the considered frequency range.

Figure 8 shows, that because the WBTMM uses correct values for the incident angles, it performs up to two orders better than the classical TM schemes, especially in the mid-frequency range.

6. Conclusions

This paper presents a Wave Based Transfer Matrix Method (WBTMM), which allows to use the TMM more efficiently in a WBM framework. Rather than using values for normal or omnidirectional incidence, as done in classical element based approaches, the WBM can feed the TM models with analytical information. By using the *a priori* known information about the propagating wave numbers of the wave functions and particular solution functions, the impact of damping layers on the acoustic cavity can be assessed much more accurately than with the classical assumptions. This was illustrated on an acoustic cavity with a multilayered damping treatment.

Acknowledgments

The Research Fund KU Leuven is gratefully acknowledged for its support.

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