Experimental evidence of band gaps in periodic structures

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Summary
The appearance of band gaps in periodic media is a well known phenomenon. In the past 15 years an increasing attention was payed to sonic crystals, as the properties of such arrangements could lead to the design of innovative noise barriers. In fact, sonic crystal barriers exhibit good insulation properties at some frequencies and could be tuned in order to cause stop bands in a frequency range centred at 1 kHz, classic for road traffic noise pollution. In this paper measurements are described that were conducted over an array of cylinders arranged in a square lattice, whose Bragg band gap was centred at about 900 Hz. Measurements were conducted in accordance with EN 1793-6. Together with measurements, FE simulations were carried out in order to address and verify the procedure. In the simulations, cylinders were assumed to be infinitely rigid and the system was investigated in a 2d domain. The first hypothesis is consistent with the high contrast between the acoustic properties of air and PVC. The second one is consistent with the measurement method, as the windowing of the IRs leads to an evaluation of the attenuation due only to the BG without comprising any effect from ground reflections. Simulation and measurements are compared and discussed over a variety of cases.

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1. Introduction

Sonic crystals are arrangements of scatterers in a fluid medium. In particular, much literature has focused on the analysis of rigid cylinders immersed in air, as a 2-D formulation of the scattering phenomenon. When cylinders are arranged in a periodic array, stop band properties emerge due to the properties of wave propagation inside periodic media [1]. The presence of complete and partial band gaps has been thoroughly investigated both theoretically and experimentally. The first experimental measurements that detected sound attenuation from an array of cylinders were performed over the sculpture by Eusebio Sempere at the Juan March Foundation in Madrid [2]. Since then, several works have been devoted to characterise this phenomenon and to extend and tune the band gaps by working on the spatial arrangement of the scatterers and on their mechanical properties [3, 4, 5]. Some prediction methods commonly used for deriving the attenuated field are the Finite Element (FE) method, the Boundary Element method and the Multiple Scattering Theory. Analogue attention has been paid to the extraction of the band structure of the periodic array, using for instance the Finite Element method [6, 7], the Plane Wave Expansion method [8] or the Multiple Scattering Theory [9].

This paper comments sound insulation measurements which were conducted over a sonic crystal made of rigid polyvinyl chloride (PVC) cylinders immersed in air according to the EN-1793-6 standard [10]. First, a description of the measurement setup is provided and the results of the measurements are reported in terms of Sound Insulation Index for sonic crystals made of different numbers of rows of cylinders. Next, the band structures of the unit cell are extracted and the experimental results are matched point-to-point with FE prediction computed both using a plane wave excitation and a point source. The results are finally discussed in the conclusions.

2. Measurement setup and results

The sonic crystal is tested for normal incidence according to EN 1793-6 [10]. Measurements are per-

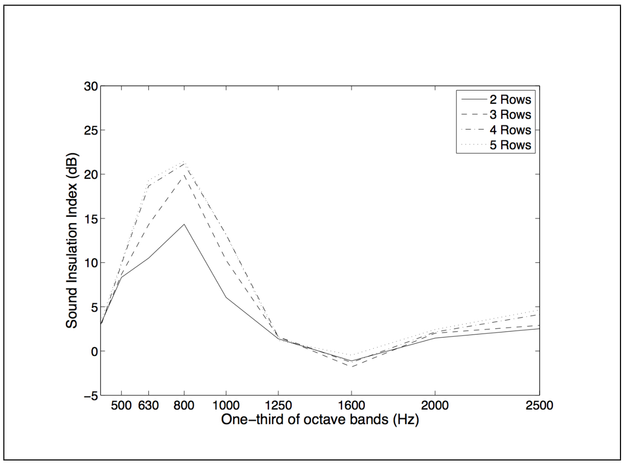

Figure 1. Sound Insulation Index (dB) measured for a sonic crystal made of 2, 3, 4 and 5 rows of cylinders.

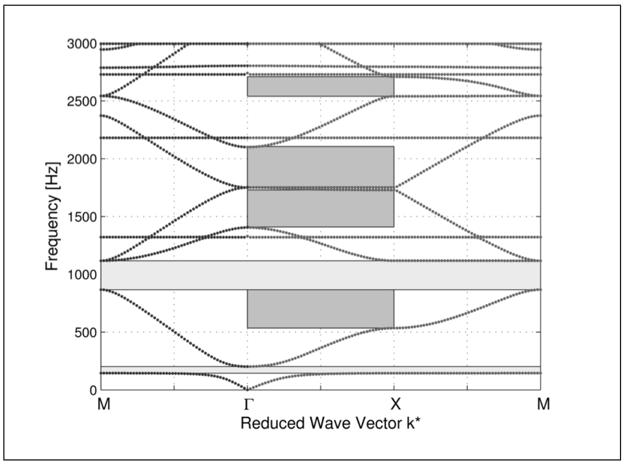

Figure 2. Dispersion curves for a unit cell with $L_c = 0.20$m and $ff = 0.50$.

formed over a sample made of 3 m long PVC hollow pipes with an external diameter of $d = 0.16$ m. The cylinders are arranged in a square lattice with lattice constant $L_c = 0.20$ m, resulting in a filling fraction $ff = 0.50$. The total width of the sample is 3 m, i.e. 15 unit cells, while its depth varies from 2 to 3, 4 and 5 rows of cylinders. The sample is installed at the DIN Laboratory of the University of Bologna, an industrial hall with an approximate volume of 5,000 m^3 that allowed to place the sample far away from reflecting surfaces. The loudspeaker is located at a distance of 1 m from the cylinders while the receiver grid is placed beyond the sonic crystal at a distance of 0.25 m or 0.50 m from the last row of cylinders. A 16 channels Analog to Digital converter RME M-16 AD connected to an RME Hammerfall HDSPe MADI was used for the A/D and D/A section. On the grid, 9 Brüel & Kjær 4935 microphones were installed, connected to a 16 channels Brüel & Kjær 2694 preamplifier. As power amplifier a Samson Servo 201A was employed. The sound source was a Zircon, declared compliant to CEN/TS 1793-5:2003 standard specifications by the manufacturer. The measurements were performed using MLS test signals (256K samples with a sampling rate of 44.1 kHz).

The Sound Insulation Index (SI) was derived from the measured IRs according to the definition provided by the EN 1793-6 standard [10]:

$$SI_j = -10\log\left\{\frac{1}{n}\sum_{k=1}^{n}\frac{\int_{\Delta f_j}|F[h_{tk}(t)w_{tk}(t)]|^2 df}{\int_{\Delta f_j}|F[h_{ik}(t)w_{ik}(t)]|^2 df}\right\} \quad (1)$$

where, with reference to the k-th scanning point, $h_{ik}(t)$ is the free-field impulse response, $h_{tk}(t)$ is the impulse response with the barrier in between, $w_{ik}(t)$ and $w_{tk}(t)$ are the Adrienne time windows for the free-field and the transmitted components respectively, F is the symbol of the Fourier transform, j is the index of the j-th one-third octave frequency band, Δf_j is the width of the j-th one-third octave frequency band and $n = 9$ is the number of scanning points. The window length, tuned to the dimensions of the sample, allowed to window out the components due to the edge diffraction as well as the ground reflection, leading to define the transmission response of the sample for normal incidence. The Sound Insulation Index is plotted in Fig. 1 in one-third of octave bands. Positive SI values are found around Bragg frequency, where they reach 22 dB in the one-third of octave band centred at 800 Hz. A positive SI value is achieved also for the sample comprising only two rows of cylinders. Increasing the number of rows of the sample, a slight increase in SI is found, even though less marked than expected. This might be due to problems related to the windowing procedure described above.

3. Extraction of the band structures

Dispersion diagrams are obtained from looping the Acoustic-Structure Module in Comsol Multiphysics© with Matlab© [11] and are reported in Fig. 2 relatively to the three high symmetry directions ΓX (normal incidence), ΓM (oblique incidence at 45°) and XM (incidence at 90°). Given the lattice constant $L_c = 0.20$ m, the first frequencies eligible for Bragg's law are $f_{Bragg,\Gamma X} = 858$ Hz for normal incidence and $f_{Bragg,\Gamma M} = 606$ Hz for an incidence angle of 45°. A complete band gap (light grey shade) is predicted centred at the first Bragg frequency, spanning the frequency range 852-1096 Hz. In the ΓX direction, a wider partial band gap (dark grey shade) is found in the range 503-1096 Hz.

It is interesting to notice that the SI trend in frequency follows closely the predictions from the band structure as it concerns the first Bragg band gap, but at higher frequencies the sound insulation values are close to zero, i.e. no stop band phenomena are detected.

4. FE predictions for a finite sample

Experimental data were matched to simulations performed over a finite size sample and a semi-infinite sample using Comsol Multiphysics©. Since the windowing procedure allows to cut off the ground reflection and the edge diffraction effects, the cylinders were modelled as a bi-dimensional domain constrained on the sides in order to compute the transmitted sound component only.

Predictions were performed using a plane wave source and a point source. The boundaries were modelled differently in the two cases, as shown in Fig. 3. For plane wave simulations, one side of the rectangular domain is the sound source, and the opposite side is modelled as a PML in order to simulate Sommerfeld's radiation conditions. The other two sides are modelled as reflecting surfaces, due to the well-known problems encountered with absorbing boundary condition for a wave propagating parallel to the boundary interface. The choice of a reflecting wall leaves the propagation of the incoming plane wave unchanged but provides lateral reflection which can be thought as generated by mirror sources located beyond the boundary. In this way the sample is characterised as infinite in the direction normal to the wave vector of the incoming plane wave. Predictions with a point source allowed the use of a finite array of cylinders, thus PMLs are applied to all the boundaries. The domains were discretised using a triangular mesh with maximum edge length of 0.01 m in order to get reliable results up to 2,500 Hz.

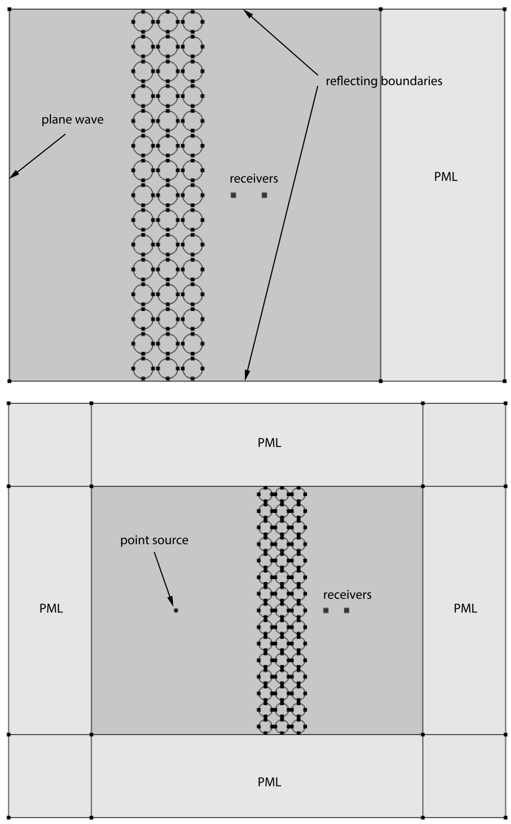

Figure 3. Computational domains and boundary conditions used for the FE modelling with an incident plane wave (top) and with a point source (bottom).

5. Discussion of the results

In order to compare predicted and measured insulation values, the sound insulation was computed following Eq. 1 but without averaging over nine microphones. In fact, only the microphone that faces the source (and located at the same height) was considered. The evaluation of this single-point SI allowed a point-to point comparison with the results of the FE predictions.

The comparison between measured and predicted values of sound insulation is shown in Fig. 4. The continuous line represents the measured SI evaluated on a single point, while the dashed and dotted line represent the FE predictions with point source and plane wave source respectively. The first Bragg band gap is well predicted by both kinds of spreading, while the second one is characteristic only of the plane wave propagation, consistently with band structures calculations. In fact, at twice the Bragg frequency measurements and predictions computed with a point source return negative insulation values, i.e. a constructive interference. Fig. 5 is the analogue to Fig. 4 with the receiver placed at a distance of 0.50 m from the sample. When the receiver is placed further away from

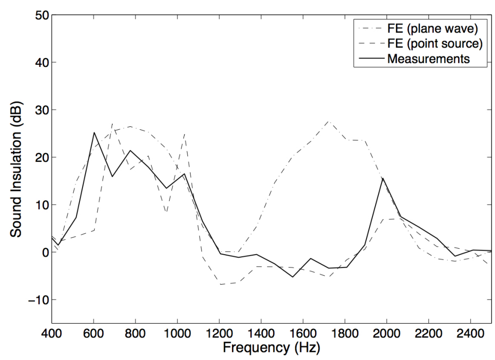

Figure 4. Measured and predicted sound insulation for an array of 15x3 cylinders. The receiver is spaced 0.25 m apart from the sample.

the sample, the FE predictions with the plane wave source remain clearly unchanged, while the FE predictions with the point source excitation and the measurements return well-matching results. At the second

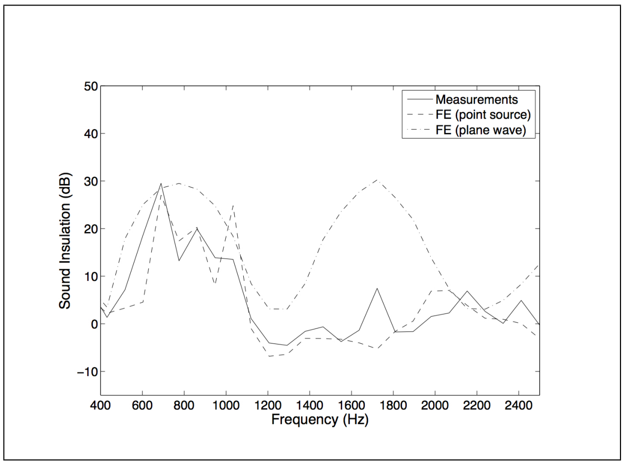

Figure 5. Measured and predicted sound insulation for an array of 15x3 cylinders. The receiver is spaced 0.50 m apart from the sample.

Bragg frequency, slightly negative insulation values are still present, with values smaller than in the case with a spacing of 0.25 m.

6. Conclusions

This paper discussed the first measurements that lead to the characterisation of a sonic crystal according to the EN 1793-6 standard. Significant SI values were found at the first Bragg band gap while at twice the Bragg frequency the sound insulation assumes slightly negative values. The measurements were supported by the calculation of band structures, which revealed band gaps at frequencies which are integer multiples of Bragg frequency. FE predictions were also performed on a finite sample, considering plane wave propagation considering a semi-infinite sonic crystal and a spherical propagation considering a finite sample. The former case detected positive SI values at Bragg frequency, matching the experimental values, and at twice Bragg frequency, not matching the experimental data; the latter case revealed a good match with the measurements.

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