Uncertainty in sound diffusion and scattering coefficients measurement

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Summary
The sound scattering properties of material structures’ surfaces are determined experimentally based on standardized procedures and are divided into two fundamental parameters: scattering and diffusion coefficient. They specify the characteristics of sound wave reflected from the tested surface. Procedure described in ISO 17497-1 for measurement of the sound scattering coefficient states that the uncertainty of the measurement can be calculated based on an error propagation. Only the uncertainty Type A, linked to the test results was determined, while uncertainty of the measuring execution itself according to definition of sound scattering coefficient was neglected. Moreover, the assumption that the input quantities are not correlated was made.

In this paper the attempt to determine the influence of chosen geometric parameters of test setup on obtained results of sound scattering and diffusion coefficient values was made. Comparison between scattering coefficient measurement uncertainty calculated based on error propagation with results obtained with Monte Carlo method was also performed.

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1. Introduction
Room acoustics describes qualities of structures with two parameters – sound absorption coefficient, defined in ISO 354 standard [1], and sound scattering coefficients (ISO 17497-1 and ISO 17497-2, [2-3]). The first parameter directly affects the reverberation time and other acoustic parameters of the interior, while the knowledge about second one allows to consciously avoid acoustical defects and to create numerical models in computer software like Catt-acoustic or Odeon. Sound absorption coefficient was a subject of many interlaboratory researches [4], which allowed to determine approximate value of uncertainty that appears with every measurement. Regarding the sound scattering coefficient, studies like that were not conducted and the analysis of scattering coefficient measurement uncertainty described in ISO standard is based on error propagation of measurements. As has been shown in [5] presumption of lack of correlation between input quantities is not satisfied, therefore results obtained with this method have a margin of error. Regarding the uncertainty of sound diffusion coefficient measurements – research has not been conducted yet.

In this paper the attempt to determine the influence of chosen geometric parameters of test setup on obtained results of sound scattering and diffusion coefficient values was made. Comparison between scattering coefficient measurement uncertainty calculated based on error propagation with results obtained with Monte Carlo method was also performed.

2. Methodology of conducted research

1.1. Sound scattering coefficient

Sound scattering coefficient describes the amount of energy dispersed from specular reflection directions:

\[ s = 1 - \frac{E_{\text{spec}}}{E_{\text{total}}} = \frac{\alpha_{\text{spec}} - \alpha}{1 - \alpha} \] (1)

where: \( E_{\text{spec}} \) – energy reflected specularly, \( E_{\text{total}} \) – total amount of reflected energy, \( \alpha_{\text{spec}} \) – apparent specular absorption coefficient, \( \alpha \) – sound absorption coefficient.

Measurement is performed in reverberation chamber and requires determining the reverberation time for four different situations. \( T_1 \) and \( T_2 \) mean respectively empty chamber and...
chamber with test sample inside, hence, same as for the sound absorption coefficient measurement:

\[
\alpha = 55.3 \frac{V}{S} \left( \frac{1}{c_i T_i} - \frac{1}{c_s T_s} \right) - \frac{4V}{S} (m_s - m_i) \quad (2)
\]

where: \( V \) – volume of the reverberation chamber, \( S \) – area of tested specimen, \( c_i \) – speed of sound in the air, \( T_i \) – reverberation time, \( m_i \) – energy attenuation coefficient of air (calculated using the temperature and relative humidity during the measurement).

Determining the apparent specular absorption coefficient is possible after carrying additional measurements during which the rotating table (\( T_4 \)) or rotating table with sample present (\( T_4 \)) is turning during performing measurement:

\[
\alpha_{\text{spec}} = 55.3 \frac{V}{S} \left( \frac{1}{c_i T_4} - \frac{1}{c_s T_3} \right) - \frac{4V}{S} (m_4 - m_3) \quad (3)
\]

Additionally, the impact of measured reverberation times \( T_{1-4} \) and their uncertainties while determining scattering coefficient was tested, subsequently, the uncertainty values obtained according to ISO standard were compared with results calculated from simplified formulas [5] along with ones obtained from Monte Carlo simulation.

### 2.2. Sound diffusion coefficient

Directional sound diffusion coefficient that is being tested in anechoic chamber describes a uniformity of distribution of scattered sound reflected from the diffuser:

\[
d_d = \frac{\sum_{i=1}^{n} 10^{0.1L_i} - \sum_{i=1}^{n} 10^{0.1L_{i,\text{ref}}}}{(n-1)\sum_{i=1}^{n} (10^{0.1L_{i,\text{ref}}})^2} \quad (4)
\]

where: \( n \) – number of receivers, \( L_i \) – sound pressure level in \( i \)’th point.

The measurement consists of determining the directional characteristics of sound wave reflected from the test specimen, with resolution of at least \( 5^\circ \) for test sample (\( d \)) and the flat, sound diffusing reference plate (\( d_{\text{ref}} \)). To eliminate the diffusion caused by the edge of the plate, normalized sound diffusion coefficient is calculated using the following formula:

\[
d_n = \frac{d - d_{\text{ref}}}{1 - d_{\text{ref}}} \quad (5)
\]

Tests were performed on circular 2D Schroeder diffuser based on the N=7 quadratic residue sequence with maximum well depth of 15cm and the diameter equal to rotating table’s diameter being 3m, using sweep sine signal to obtain impulse responses [6]. The reverberation chamber had a total volume of 180.4m\(^3\).

Based on preliminary analysis it was concluded, that if the sample has its outer edge secured with a rigid band along the full height and an insignificant diffusion of the base plate, the substantial impact on the measurement uncertainty will be caused by axial alignment between test specimen and the rotating table. Also, the repeatability of the method was evaluated along with the influence of time between opening/closing the chamber’s doors and starting the measurement on determining the scattering coefficient (Fig. 1).

Figure 1 Tested sound diffuser in reverberation chamber

Tests were conducted using sound diffuser consisting of 30 wells sized 2.5x58cm, which had regulated depth that could be set to any drop from between the 0-60mm range (Fig 2.).

Figure 2 Tested sound diffuser with regulated wells’ depths
Carried tests evaluated the repeatability of method described in ISO standard along with the influence of accurate diffuser’s placement in relation to the sound source and the microphone on determining the directional sound diffusion coefficient. All of the presented results were obtained from measuring 8 various sets of well depths based on different numerical sequences that were chosen in a manner so that they cover a wide range of types of diffusion characteristics that can be achieved using structures of this type. All measurements were made using measurement manipulator [7], that allow to position the microphone in exactly the same position for every measurement [8].

3. Results for scattering coefficient

3.2. Repeatability of the method

To verify repeatability of the procedure for determining scattering coefficient, 4 measurements were performed using the same sample, keeping the same temperature and relative humidity inside the reverberation chamber. Each measurement was performed 15 minutes after opening/closing the chamber’s door to ensure (according to ISO standard) stable conditions inside the chamber.

Analysis was narrowed down to frequencies for which the test sample scattered the sound waves, therefore Tab.I. contains only frequencies from 800Hz to 5000Hz. Obtained results allowed to conclude that repeatability of method described in ISO standard is satisfactory as the coefficient of variation does not exceed 5% of sound scattering coefficient value for any frequency.

3.3. Axial alignment of sample

The diameter of tested diffuser was equal to the diameter of rotating table mounted in the reverberation chamber, therefore its edges were not suppose to protrude outside the base plate and the diffuser’s centre point should overlap the base plate’s centre point.

During the measurements 4 placements of the diffuser were analysed, where the diffuser was shifted by 15mm, 30mm, 40mm and 50mm from the axial placement.

Performing the analysis, average values of scattering coefficient for all frequencies were calculated, along with standard deviations. Using those two parameters the coefficient of variation was obtained.

Table I. Repeatability of the results

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>800</th>
<th>1000</th>
<th>1250</th>
<th>1600</th>
<th>2000</th>
<th>2500</th>
<th>3150</th>
<th>4000</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average sound scattering coefficient</td>
<td>0.54</td>
<td>0.80</td>
<td>0.91</td>
<td>0.95</td>
<td>0.87</td>
<td>0.93</td>
<td>1.10</td>
<td>1.04</td>
<td>1.21</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.02</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>4.30%</td>
<td>3.64%</td>
<td>1.24%</td>
<td>1.33%</td>
<td>4.16%</td>
<td>3.76%</td>
<td>2.97%</td>
<td>2.85%</td>
<td>4.78%</td>
</tr>
</tbody>
</table>
Based on obtained results, the table containing correction factors for each frequency was determined. Correction factors were calculated from difference between sound scattering coefficient value measured for diffuser in reference position (displacement equal to 0mm) and value obtained from measurement for shifted placement. Subsequently, the trend line crossing the (0;0) point was fitted for obtained points. For trend line equation being $y=cx$, $c$ was the correction factor for the scattering coefficient.

Finally, corrected value of scattering coefficient should be calculated according to the formula:

$$s_{\text{corr}} = s_{\text{meas}} - c_f p_s$$  \hspace{1cm} (6)

where: $s_{\text{meas}}$ – measured value of scattering coefficient, $c_f$ – correction factor for each frequency (Tab.II), $p_s$ – displacement of tested sample [mm].

### Table II. Correction factors $c_f$ for each frequency

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>315</th>
<th>400</th>
<th>500</th>
<th>630</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction factor</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0008</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0021</td>
<td>0.0035</td>
<td>0.0028</td>
<td>0.0036</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>800</th>
<th>1000</th>
<th>1250</th>
<th>1600</th>
<th>2000</th>
<th>2500</th>
<th>3150</th>
<th>4000</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction factor</td>
<td>0.0008</td>
<td>0.0008</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.0018</td>
<td>0.0055</td>
<td>0.0072</td>
<td>0.0018</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

### 3.4. Stabilization of conditions inside the reverberation chamber

According to ISO 17497-1 measurement should not be performed before waiting 15 minutes for the conditions in reverberation chamber to stabilize. To verify legitimacy of this requirement series of measurements were performed for various time intervals after opening/closing the chamber’s doors and starting the measurement: 1min, 5min, 10min, 15min and 20min.

![Figure 6 Scattering coefficient for different stabilization times](image)

On Fig.6. also the standard deviation markers ($\delta$) for 20min time interval are shown, as it should be the one with the most stable conditions inside the chamber. This was to show that the rest of the measurements also fit into confidence interval $2\delta$, therefore all results are accurate enough, regardless the time interval after closing the doors to the chamber. Additionally the values of $\delta$ were also analyzed but no visible dependency between stabilization time and the accuracy of the result was shown. That questions the requirement included in ISO 17497-1. However it is possible that the differences between air’s temperature and relative humidity in reverberation chamber and the adjacent laboratory rooms were too small to cause significant changes in conditions inside the chamber. Additionally, it is possible that 15 minutes wait is not directly linked to the air properties inside the chamber, but is to ensure that the sample’s temperature and relative humidity will be equate to those of the air.

Based on performed measurements it could be stated that waiting requirement included in ISO standard is unjustified but this matter requires further studies that will confirm of decline the statement.

### 3.5. Monte Carlo simulation

ISO 17497-1 suggest to calculate uncertainty of the measurements according to uncertainty propagation law, supposing, that the input values (reverberation times) are not correlated, and that they come from normal distribution.

First assumption do not have to be fulfilled, if instead of uncertainty propagation, Monte Carlo method is used. Mean values and standard deviations of reverberation times were calculated, next, $10^6$ samples of $T_{1-4}$ from sets with that parameters were drown. As a result, difference between percentile 95 and 5 was compared with results from calculations based on ISO 17497-1 method (Fig. 7). Almost for all frequencies, values obtained from Monte Carlo method were ca. 10% higher.

![Figure 7 Comparison of uncertainty calculated acc. to ISO 17497-1, and Monte Carlo simulation](image)
4. Results for diffusion coefficient

4.1. Repeatability of the method

To test the repeatability of measurement method described in ISO 17497-2, two types of diffusers were used: QRDN7 and QRDN29. They were chosen so that their diffusion characteristics were possibly dissimilar to exclude the influence of diffusion characteristics on repeatability. During tests series of measurements were performed, one after another, without interfering the measurements conditions. Thus, 25 results for QRDN7 diffuser (Fig. 8) and 11 for QRDN29 diffuser (Fig. 9) were obtained. To increase the clarity of presented graphs only the average diffusion coefficient from all measurements is presented, along with minimum and maximum values.

All measurements were performed in scale 1:2.5. Analyzing standard deviation for both diffusers it was confirmed that measurement method is very accurate and the deviation does not exceed 0.03 value for any frequency.

4.2. Inaccuracy of sample’s placement

Two main diffuser’s placement inaccuracies were tested: shift from the centre point on axis going crosswise to the wells (2cm, 4cm, 6cm and 8cm) and tilt of the diffuser in reference to speaker’s position (2°, 4°, 6° and 8°). The reference position was situation when centre point of the diffuser is exactly below the centre point of the speaker, and its axis is perpendicular to the diffuser’s plane.

For running tests, 8 different numerical sequences determining well depths were used to cover wide range of diffusion characteristics. For each diffuser the difference in diffusion coefficient between reference placing and the shifted placing was calculated for each frequency, followed by taking the absolute value of obtained results and performing averaging for all 8 diffusers types. The relationship between sample’s placement and obtained value of diffusion coefficient appeared to have a visible pattern only for low and mid frequencies, therefore only this part is presented in this paper (Fig. 10).

From obtained results it was indicated that the error increases linearly until 40mm shift and then stays relatively on constant level. Using the linear regression, correcting factors were determined for 250Hz – 3150Hz frequencies and can be used to determine the size of an error using formula:

\[ \Delta d = c_p p_d \]  

(7)

where: \( c_p \) – correction factor for given frequency (Tab.III.), \( p_d \) – inaccuracy of sample’s placement [mm].

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>250</th>
<th>315</th>
<th>400</th>
<th>500</th>
<th>630</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction factor</td>
<td>0.0011</td>
<td>0.0018</td>
<td>0.0017</td>
<td>0.0015</td>
<td>0.0013</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>1000</th>
<th>1250</th>
<th>1600</th>
<th>2000</th>
<th>2500</th>
<th>3150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction factor</td>
<td>0.0013</td>
<td>0.0013</td>
<td>0.0010</td>
<td>0.0012</td>
<td>0.0013</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

For shifts greater than 40mm, in formula (7) \( p_d \) is always equal to 40. For the rest of the frequencies....
(4000Hz – 12500Hz) there was no distinguishable dependency that can be easily defined. Similarly as with the diffuser shift, the differences between reference placing (speaker’s axis perpendicular to diffuser’s plane) and the measurement for tilted diffuser were analyzed but again, only for lower frequencies (Fig. 11).

![Figure 11 Average ABS values of differences in diffusion coeff. for various tilts](image)

Analyzing results collected for frequency range 250Hz – 4000Hz the trend line crossing the (0;0) point was fitted for obtained points to determine how fast the diffusion coefficient value increases along with the tilt. Size of an error related to sample’s tilt can be obtained using formula:

$$
\Delta d = c_i t_d
$$

where: $c_i$ – correction factor for each frequency (Tab.IV.), $t_d$ – inaccuracy of sample’s placement [˚].

### Table IV. Correction factors $t_d$ for each frequency

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>250</th>
<th>515</th>
<th>825</th>
<th>1250</th>
<th>2500</th>
<th>3500</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction factor</td>
<td>0.017</td>
<td>0.017</td>
<td>0.022</td>
<td>0.024</td>
<td>0.015</td>
<td>0.018</td>
<td>0.016</td>
</tr>
</tbody>
</table>

For the rest of the frequencies (5000Hz – 12500Hz) it was indicated, that coefficient changes were not linear but it was clear that tilting the diffuser causes increase in diffusion coefficient. Nonetheless determining the actual relationship requires conducting further research.

### 5. Conclusions

In this work the influence of sample's placement during measurement on determining the sound scattering and diffusion coefficient was tested. Along with that, method for establishing the measurement uncertainty suggested by ISO standard was analyzed. For obtained diffusion coefficient values gathered from tests performed for 8 various diffusers, each based on 30-element pseudo-random numerical sequences determining its well's depths, linear dependency between specimen's displacement and measurement uncertainty has been shown. Likewise for scattering coefficient, the proportionality factors between axial displacement of tested sample on the rotating table and calculated scattering coefficient values were determined. Also, the measurement uncertainty calculated using Monte Carlo simulation was compared with the error propagation method recommended by ISO standard. Tests has shown, that presumption of lack of correlation between input quantities causes uncertainty values calculated according to ISO 17494-1 to be underestimated about 10%.

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### References


