On the prediction of rail cross mobility and track decay rates using Finite Element Models

Benjamin Betgen
Vibratec, 28 Chemin du Petit Bois, 69130 Ecully, France.
Giacomo Squicciarini, David J. Thompson
ISVR, University of Southampton, Southampton SO17 1BJ, UK.

Summary

Rolling noise can in many cases be predicted with good confidence using analytical models such as TWINS. The most widely used rail model in TWINS consists of a Timoshenko beam on a continuous double elastic foundation. As the model cannot reproduce vertical-lateral cross mobility, this is normally accounted for by using an empirical cross-coupling term. Moreover, effects coming from the periodic support of the rail or deformation of the rail cross-section are not taken into account with a beam on continuous support. Measured decay rates are therefore often used when available in order to improve the reliability of TWINS predictions. The present paper shows how Finite Element Modelling can be used to predict vertical-lateral cross mobility as well as decay rates. This seems especially interesting for new track designs for which no measured decay rates are available. Computational results are compared with measurements performed on a test track and good agreement is found.

PACS no. 43.40.+s, 43.50.+y

1 Introduction

Within a speed range from approximately 50 to 300 km/h, rolling noise is the predominant railway noise source. It is excited by the combined roughness of the wheel and the rail which then causes both the wheel and track to vibrate and radiate sound. Frequency-domain models, notably TWINS [1], are widely used to quantify rolling noise and to study the effect of new designs. However, despite validation and widespread use, there remain aspects of the models and their use that deserve refinement.

Within the ACOUTRAIN project, several such areas have been studied [2]. Concerning the low frequency range, these include sleeper radiation and different aspects of ballast behaviour. At high frequencies, the effect of rail cross-section deformation and the effect of rail cross mobility have been investigated.

In this context, this paper describes the analyses done by means of measurements and with a Finite Element (FE) model of the track to understand and quantify the vertical-lateral coupling in the rail. The possibility of adopting this FE model to obtain an accurate estimation of the vibration decay along the track is also investigated.

1.1 Vertical-lateral coupling

Rail cross mobility is a quantity that identifies the amount of lateral rail vibration for an input in the vertical direction or vice versa. For a perfectly symmetric structure excited along its symmetry axis this is equal to zero. However the rail, once connected to the rest of the track, is no longer symmetric and moreover, in the practical condition of a train running over the track, the contact position is not always at the exact centre of the rail nor is the rail orientated exactly vertically. The rail head is curved, and the rail itself is inclined at an angle of between 1:20 (UK, France) and 1:40 (Germany) to the vertical.

The Timoshenko beam used in the *rodel* model of TWINS does not include any cross coupling directly. This is done 'artificially' by defining a cross mobility α_{vl} [1] which is determined from the geometrical average of vertical and lateral mobilities α_v and α_l as

$$\alpha_{vl} = 10^{(XdB/20)} \sqrt{\alpha_v \alpha_l} \quad (1)$$

The empirical factor *XdB* is generally set around -10; but no universally agreed value exists. Often, *XdB* is chosen as a function of sleeper type, i.e. monobloc or bi-bloc track, however, the influence of the sleeper will be shown to be quite small.

1.2 Decay rates

The decay of vibration along the track, or decay rate (DR), can be expressed in dB/m and quantified, as defined in standard EN 15461 [3], by

$$DR = 4.343 \Big/ \sum_{n=0}^{n_{\max}} \frac{|A(x_n)|^2}{|A(x_0)|^2} \Delta x_n \quad (2)$$

where $A(x_0)$ and $A(x_n)$ are respectively point and transfer accelerances, and Δx_n is the distance along the track associated with each measurement position x_n. Decay rates are separately measured in vertical and lateral directions.

The decay rate is the most important descriptor of track dynamics with respect to rolling noise. Low decay rates lead to a greater length of radiating rail per wheel-rail contact and thus to high rolling noise emission from the track. High decay rates result in lower noise and can be obtained for example by the use of stiff pads between rail and sleepers. However, soft pads are often used for non-acoustic reasons, e.g. to minimise sleeper damage or ground borne vibrations.

Measured decay rates are widely used for the computation of rolling noise with models such as TWINS. Although TWINS also permits the estimation of decay rates from track dynamic properties, the most commonly used model of a Timoshenko beam on a continuous foundation does not account for effects from the periodic support or rail cross-section deformation. In most situations adopting measured decay rates instead of analytically computed ones results in more accurate noise predictions. As an alternative to measurements this paper explores the possibility of using an accurate FE representation of the track to predict decay rates. Comparisons with decay rates measured on a test track have been made for validation purposes.

2 Measurements

To understand the implications that the cross mobility has on rolling noise, a series of mobility measurements has been performed at the University of Southampton test track. The cross mobility has been measured for different lateral shifts of the vertical excitation position with respect to the centre of the rail head. Figure 1 shows the points selected on the rail. Tests have been performed with an impact hammer and accelerometers. The driving point mobility has been measured at points *i* (*i*=1,…,5) and transfer mobilities between each point *i* and 6.

The test track is a ballasted track of length 32 m with monobloc concrete sleepers and UIC 60 rails. The rail pads installed during the tests have a vertical dynamic stiffness of around 120 MN/m. Decay rates for the vertical and lateral directions have also been measured on the test track according to [3].

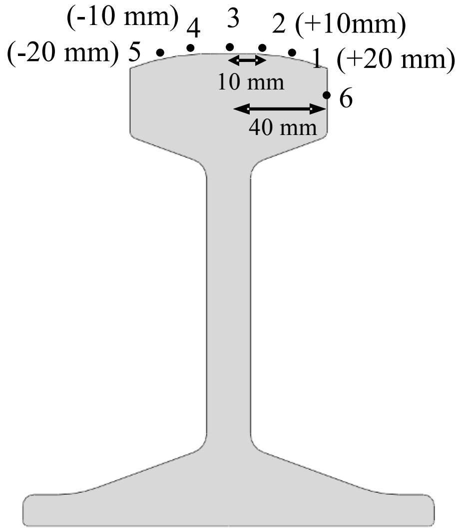

Figure 1. Cross-section of rail with measurement grid for assessing the effect of lateral offset of contact point on vertical-lateral coupling.

3 Track Finite Element Models

A monobloc track of total length 34.8 m has been modelled in MSC Nastran, including 6 m of "anechoic terminations" on either end (i.e. the length of useful rail is 22.8 m). These terminations are obtained by gradually increasing damping of the rail from 2% to 100% and prevent any reflections while the length of modelled track remains reasonable. Both rail and sleepers are modelled with solid elements, while rail pads and ballast are introduced as springs (15 springs per rail pad, arranged as 5x3).

This model represents approximately 1.8 m degrees of freedom. The resolution of each frequency step (direct frequency response of type 'SOL108') took about 10 minutes using 12 cores in parallel and 96 Gb RAM memory.

A bi-bloc sleeper track has been built as well, where only one rail is modelled and sleepers are represented as concentrated masses of 120 kg. This perfectly symmetric track has been used to assess the influence of sleeper geometry on the rail cross mobility.

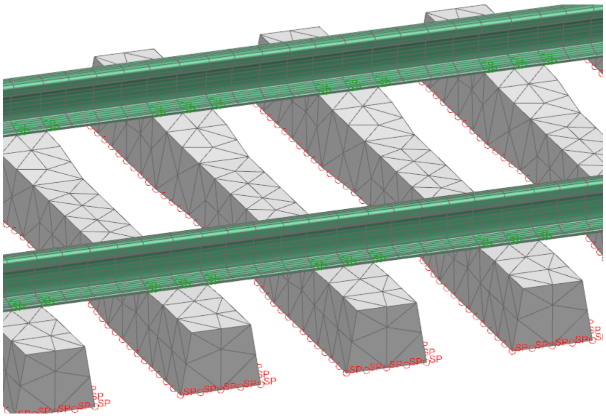

Figure 2. FE model of a monobloc sleeper track.

For both monobloc and bibloc models the rail is excited at 1.2 m from one of the anechoic terminations, i.e. the length of rail available for the determination of DRs is 21.6 m. According to [3] this is sufficient for an assessment of DRs down to 0.2 dB/m.

4 Results

4.1 Vertical-lateral coupling

In the Finite Element model the rail pad stiffness has been chosen to be 180 MN/m in vertical and 36 MN/m in lateral direction as this gives best agreement with the measured mobility. Rail pad loss factors are of 20 % in all directions.

Vertical-lateral coupling has been tested using the same excitation points as for measurements. As there is a rail inclination of 1:20, excitation positions on either side from the rail centre do not lead to identical results. A positive direction denotes a shift towards the centre of the track; a higher vertical-lateral coupling can be expected than for a shift outwards.

Direct and cross mobilities are shown in Figures 3 to 5 for lateral offsets of 0 mm, +10 mm and +20 mm. The estimation of direct vertical mobility and cross-coupling as used by TWINS is indicated as well. In this case, for each lateral shift the blue dotted line shows the results obtained with the cross coupling factor giving the best fit with measurements. These figures are presented with a 1/6th octave band frequency resolution. Measured mobilities have been averaged over the frequency band while FE calculations have been performed at the centre frequency of each sixth octave band only.

Figure 3 shows the results obtained at the nominal contact point. As expected the coupling is very low in this case and above 300 Hz the measured cross mobility is about 20 dB below the direct measurement. FE results show a similar trend although the cross mobility is underestimated by about 5 dB. The analytical model with the same pad parameters as in the FEM fits measurements well and FEM in the direct case, except around 200 Hz. The best fitting cross-coupling factor adopted in TWINS to estimate the cross mobility is equal to -15 dB.

Increasing the lateral shift (Figures 4 and 5) increases the cross mobility but does not change the direct one much. The FE model shows a similar behaviour but it is capable of following the measurements only in an average sense without capturing all the details. A TWINS cross-coupling factor (*XdB*) of -12 dB for the +10 mm position and -7 dB for the +20 mm position gives the best fit with measurements. These values have been selected among calculations spanning *XdB* between -1 and -20 dB, with a 1 dB step. The best fit has been judged by a graphical comparison between measurements and calculations. Also this model cannot capture the detailed fluctuations of the measurements, which is to be expected since it is based on an empirical combination of beam models that do not account for any cross-section deformation or torsion.

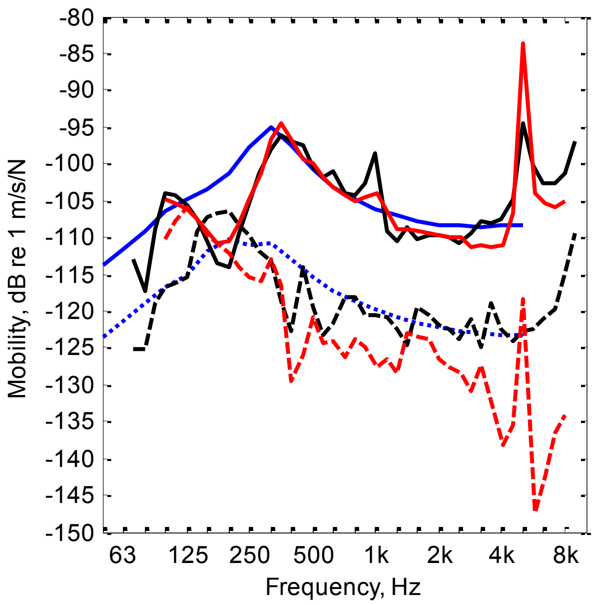

Figure 3. Track mobilities for lateral shift of 0 mm. TWINS best fit is found adopting coupling factor equal to -15 dB. ——: Measured direct vertical; – – –: Measured cross (vertical to lateral); ——: FEM direct; – – – :FEM cross; ——: TWINS direct; ······: TWINS cross (*XdB*=-15 dB).

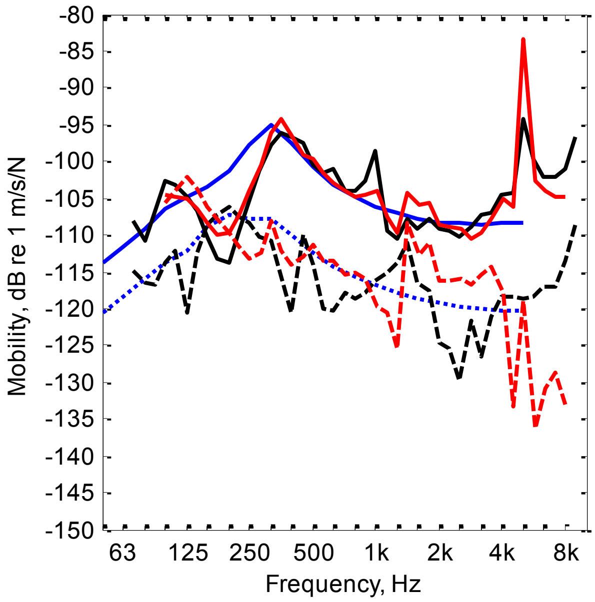

Figure 4. Track mobilities for lateral shift of +10 mm. TWINS best fit is found adopting coupling factor equal to -12 dB. ——: Measured direct vertical; – – –:Measured cross (vertical to lateral); ——:FEM direct; – – –:FEM cross; ——: TWINS direct; ⋯⋯: TWINS cross (*XdB*=-12 dB).

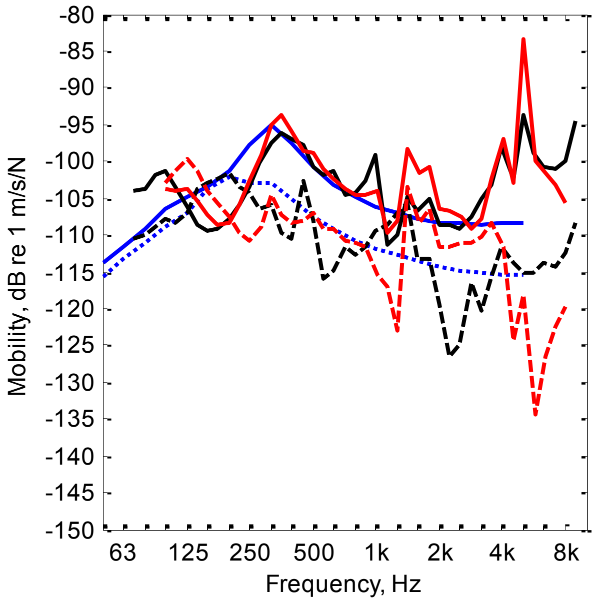

Figure 5. Track mobilities for lateral shift of +20 mm. TWINS best fit is found adopting coupling factor equal to -7 dB. ——: Measured direct vertical; – – –: Measured cross (vertical to lateral); ——: FEM direct; – – –: FEM cross; ——: TWINS direct; ⋯⋯: TWINS cross (*XdB*=-7 dB).

Measurements and simulations for a lateral shift of -10 mm show results very similar to the +10 mm position. However, a further increase of the outwards offset (to -20 mm) leads to a negligible change in the vertical-lateral coupling, which is due to rail inclination. These results for outwards offsets are not plotted here but can be found in [2]. Table 1 shows the TWINS cross-coupling factors for all positions leading to the best fit with measurements.

Table 1. Lateral offset versus TWINS cross coupling factor.

Lateral offset	***XdB***
+20 mm	-7 dB
+10 mm	-12 dB
0 mm	-15 dB
-10 mm	-12 dB
-20 mm	-11 dB

Figure 6 shows a comparison between simulations with monobloc and bi-bloc sleepers (180 MN/m pads). Differences are limited to the region below 300 Hz where the rail is strongly coupled to the sleepers. The small difference in resonance frequency near 400 Hz is due to the different sleeper mass. With a large lateral offset, both models give similar results even at low frequencies, which indicates that the lateral shift then dominates the asymmetry rather than the sleeper type.

A similar picture (not shown) is obtained with stiffer pads (360 MN/m) for both types of sleeper; the differences between the two tracks then remain up to approximately 500 Hz.

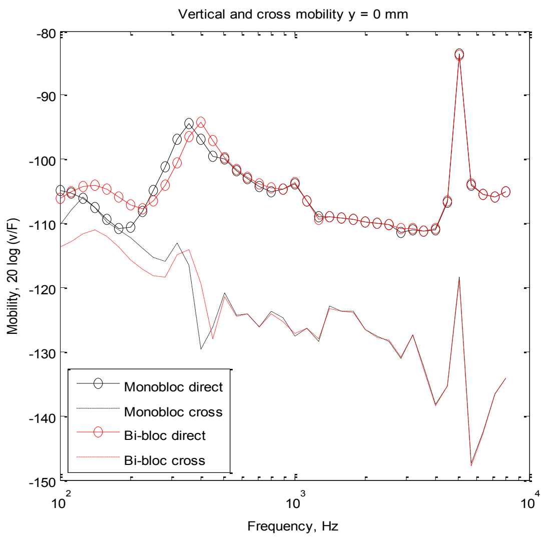

Figure 6. Vertical and vert.-lat. track mobilities for a lateral shift of 0 mm with monobloc and bi-bloc sleepers and soft pads (180 MN/m)

4.1.1 Implications for rolling noise

To give an indication of the potential effect of the cross mobility on rolling noise a set of calculations has been run using a TWINS-like software implemented in Matlab. The chosen model consists of a regional train at 80 km/h on a track with soft pads (100 MN/m), a situation where the track component of noise dominates over the wheel. It has been found that the effect of decreasing the cross-coupling factor from -7 dB to -15 dB (the same range found in measurements between +20 mm and 0 mm lateral shift) potentially leads to a decrease in overall noise of about 2 dB(A).

4.2 Track decay rates

The post-processing of decay rates from the FE model has been performed analogously to measurements, i.e. by following the procedure of EN 15461 [3].
Figure 7 and Figure 8 show the predicted vertical and lateral decay rates in comparison with measurements. The adopted rail pad stiffness is the same as in above figures, i.e. 180 MN/m vertical stiffness.
The vertical DR is well predicted, including the effect of rail cross-sectional deformation around 5 kHz. In the lateral direction, this cross-sectional deformation occurs already around 3 kHz and its effect is under-estimated by the model.

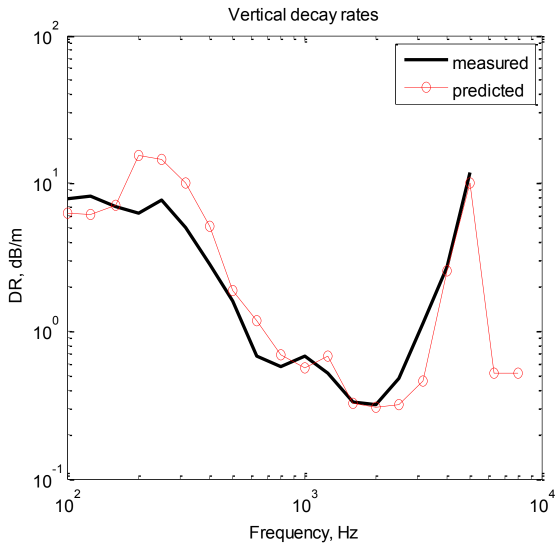

Figure 7. Vertical decay rates for monobloc sleeper track, measured (test track) and computed (with rail pads of 180 MN/m stiffness)

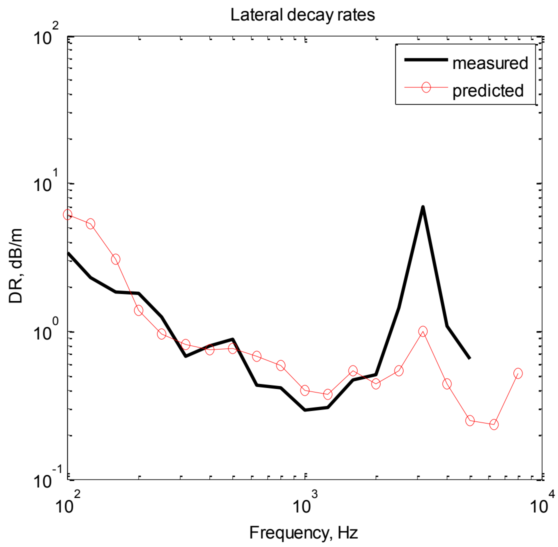

Figure 8. Lateral decay rates for monobloc sleeper track, measured (test track) and computed (with rail pads of 180 MN/m stiffness)

The main sources of error in the prediction of DRs are believed to be:

- The discrete distribution of 15 springs to represent each rail pad. This may lead to an over-estimation of stiffness when the pads are deformed around the longitudinal axis. This is the case for lateral excitation where the pad mainly works in the vertical direction (besides shearing in the lateral direction) but in opposite directions at both extremities.
- Neglect of fastening system (which is possibly stiff in the lateral direction).

5 Conclusions

The cross mobility (i.e. vertical-lateral coupling) of the rail is known to have a non-negligible effect on rolling noise. Results from measurements and predictions have shown that a lateral shift of the vertical excitation position on the rail significantly impacts the cross mobility. The TWINS coupling term *XdB* resulting in the best fit with measurements increased by almost 10 dB between central excitation of the rail and a 20 mm offset towards the centre of the track.

The FE model predicts a similar vertical-lateral coupling behaviour to the measurements but it only follows the measurements in an average sense without capturing all the details. FE simulations have also shown that sleeper geometry has a low impact on the cross mobility, at least for the pad stiffness considered, limited to the low frequency region. A value of -10 dB for the factor *XdB* as commonly used in TWINS can be confirmed to be realistic. A higher coupling may occur, but is likely to be a consequence of worn rails than of any asymmetry introduced by the sleepers.

Decay rates have been determined numerically from the FE simulations by post-processing computation results in analogy with EN 15461 [2]; the minimum length of rail recommended for measurements therefore has to be respected for the model as well. In addition, an anechoic termination is needed to ensure the absence of reflections at the track termination

Decay rates measured on a test track have been correctly predicted by the FE model, in particular within the frequency region between 500 Hz and 2 kHz where the low decay rates of this track result in the dominant track noise emission.

Further work will consist in the validation of the model for different types of tracks, e.g. slab tracks. Also, the benefit of a more detailed modelling of the rail fastening system is worth investigating.

Acknowledgement

The work presented in this paper has been performed within the ACOUTRAIN project. The support from the European Commission, grant agreement no. 284877 is gratefully acknowledged.

References

[1] D.J. Thompson, B. Hemsworth, N. Vincent, “Experimental validation of the TWINS prediction program for rolling noise, part 1: description of the model and method”. *Journal of Sound and Vibration* 193(1), 123-135, 1996.

[2] G. Squicciarini et al.: Acoutrain deliverable 2.7 – Improved model components, ACOUTRAIN project, 2014.

[3] Standard EN 15461:2008+A1:2010:E. Railway applications. Noise emission. Characterization of the dynamic properties of track selections for pass by noise measurements, 2010.