



Non-linear N wave source Impedance model

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Summary

A systematic solution of the non-linear Helmholtz resonator equation driven by the N-wave source is derived asymptotically close to the resonance including the non-linear correction. The derived solution is used to obtain the impedance at various harmonics of the driven frequency. The amplitude regime is chosen such that when we stay away from the resonance condition, the non-linear terms are relatively small and neglected. Close to the resonance frequency, the non-linear terms can no longer be neglected and algebraic equations are obtained that describe the corresponding non-linear impedance for various harmonics of N wave.

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1. Introduction

The N wave sound also called as buzz-saw noise is very common in practise e.q. the current typical Dutch electronic dance music, EDM. Of particular interest and concern, is the buzz-saw sound produced by aircraft engine while take off when the blade tip mach number exceeds unity that evolves a shock wave pattern at the blade leading edge close to the duct wall. Since the geometry of each blade is not ideally same, the shock waves are not parallel and interact with each other in the upstream of the intake duct. This interaction which is (highly) non-linear in nature results in a sound field that is not only the first harmonic of the blade passing frequency but also contains several other harmonics [1]. The amplitude of each harmonic is inversely proportional to the it's frequency. Typically, there is a difference of about 6 dB between 1BPF and 2BPF and a difference of about 9.5 dB between 1BPF and 3BPF and Also, the shock interaction is much stronger in the upstream of the fan hence, the buzz-saw sound increases a bit when we move away from the fan and gains maturity at the shock interaction region and later starts to decrease.

Typically, we can describe N wave pressure field by the series

$$p'_{\rm ex} = F_0 \sum_{n=1}^{\infty} \frac{\sin(n\omega t)}{n} \tag{1}$$

where ω is the blade passing frequency, 1BPF and F_0 is the amplitude of excitation. The liners are usually constructed in such a way that the resonance frequency corresponds to 1BPF to absorb the dominant sound spectrum and the impedance is usually obtained after ignoring the non-linear effects. A time harmonic non linear model to predict the tone noise from the turbofan engine was constructed by [2]. In the frequency domain, the impedance information is vital to model the boundary condition hence a model which could describe the relationship between acoustic pressure and velocity is useful to maintain the understanding of the behavior of wall at the higher harmonics of N-wave.

The non-linear corrections for a Helmholtz resonator type impedance based on a systematic asymptotic solution of the pertaining equations was derived by [3] when the excitation source is harmonic in nature. The present work focus on the on same asymptotic analysis when the resonator is excited by N wave source as shown in Fig. 1. The excitation source is the summation of various harmonic sources but the problem is (weakly) non-linear hence, the final solution can not be represented as the linear combination of the individual solutions obtained by the single harmonic excitation. The asymptotic methods are used to solve the non-linear equation and the obtained solution is used to derive the impedance after the Fourier disintegration of the source and solution.

2. Mathematical formulation

The Helmholtz resonator considered is shown in Fig1. A simple and classic model, that includes non-linear separation effects for the air flow in and out the neck, is derived by [3]. If the volume of cavity V is large enough and the cavity neck is acoustically compact *i.e.* $k\ell \ll 1$, we can neglect compressibility in the neck and integrate the

⁽c) European Acoustics Association

line integral of the momentum equation along a streamline from a point inside to a point outside as

$$\rho_0 \int_{in}^{ex} \frac{\partial \boldsymbol{v}}{\partial t} \cdot \mathbf{ds} + \frac{1}{2} \rho_0 (u_{in}^{\prime 2} - u_{in}^{\prime 2}) + (p_{ex}^{\prime} - p_{in}^{\prime})$$
$$= \int_{in}^{ex} \mu \nabla^2 \boldsymbol{v}^{\prime} \cdot \mathbf{ds}. \quad (2)$$

Assuming that the streamline does not change in time (for example the center streamline) we have

$$\int_{in}^{ex} \frac{\partial \boldsymbol{v}}{\partial t} \cdot \mathbf{ds} = \frac{d}{dt} \int_{in}^{ex} \boldsymbol{v} \cdot \mathbf{ds}.$$
 (3)

The velocity line integral evidently scales on a typical length times a typical velocity. If friction effects are minor and the velocity is reasonably uniform, we can use the neck velocity u'_n with a corresponding length being the neck length ℓ , added by a small end correction δ [4] to take into account the inertia of the acoustic flow at both ends just outside the neck (inside and outside the resonator). Then we have:

$$\int_{\rm in}^{\rm ex} \boldsymbol{v} \cdot \mathbf{ds} = (\ell + 2\delta) u'_n. \tag{4}$$

For the stress term line integral we observe that, apart from u'_n itself, it will depend on flow profile, Reynolds number, wall heat exchange, turbulence, separation from sharp edges, and maybe more. Following Melling [5], we will take these effects together in a resistance factor R, which will a priori be assumed to be relatively small, in order to have resonance and a small decay per period.

$$\int_{\rm in}^{\rm ex} \mu \nabla^2 \boldsymbol{v}' \cdot \mathbf{d}\boldsymbol{s} = R u'_n.$$
⁽⁵⁾

Due to separation from the outer exit, we have with outflow $u'_{in} \simeq 0$ with $u'_{ex} = u'_{in}$ jetting out, while similarly during inflow, $u'_{ex} \simeq 0$ with $u'_{in} = u'_n$ jetting into the cavity. The pressure in the jets, however, has to remain equal to the surrounding pressure (p'_{ex} and p'_{in} respectively) because the boundary of the jet cannot support a pressure difference. Therefore, we have altogether

$$\rho_0(\ell+2\delta)\frac{\mathrm{d}}{\mathrm{d}t}u'_n + \frac{1}{2}\rho_0 u'_n |u'_n| + Ru'_n = p'_{\mathrm{in}} - p'_{\mathrm{ex}}(6)$$

The second equation between p'_n and u'_n is obtained by applying the integral mass conservation law on the volume V of the cavity. The change of mass must be equal to the flux through the cavity neck, which is in linearised form for the density perturbation ρ'_{in} :

$$V\frac{\mathrm{d}\rho_{\mathrm{in}}'}{\mathrm{d}t} = -\rho u_n' S_n \approx -\rho_0 u_n' S_n \tag{7}$$

Assuming an adiabatic compression of the fluid in the cavity, we have $p'_{in} = c_0^2 \rho'_{in}$. Elimination of ρ'_{in} and u'_n from



Figure 1. Helmholtz Resonator

(6) by using (7) and redefining $(\ell + 2\delta) := \ell$ yields the non-linear Helmholtz resonator equation

$$\frac{\ell V}{c_0^2 S_n} \frac{\mathrm{d}^2 p'_{\mathrm{in}}}{\mathrm{d}t^2} + \frac{V^2}{2\rho_0 c_0^4 S_n^2} \frac{\mathrm{d} p'_{\mathrm{in}}}{\mathrm{d}t} \left| \frac{\mathrm{d} p'_{\mathrm{in}}}{\mathrm{d}t} \right| + \frac{RV}{\rho_0 c_0^2 S_n} \frac{\mathrm{d} p'_{\mathrm{in}}}{\mathrm{d}t} + p'_{\mathrm{in}} = p'_{\mathrm{ex}}$$
(8)

where V is the volume of the cavity, c_0 speed of sound, ℓ the cavity neck length including the correction term, R is the resistance term and S_b , S_n represents the base and neck area respectively. Next, we non-dimensional the equation in order to understand the relative effect of the various terms. By dividing the non-linear damping term by the acceleration term we find the pressure level $2\rho_0 c_0^2 \ell S_n/V$ at which the non-linear damping would be just as large as the other terms. So for a pressure that is a small fraction ε of this level we have a problem with only little non-linear damping. In addition we assume that the linear damping is small and of the same order of magnitude as the nonlinear damping. Also, the driving amplitude $p'_{\rm ex}$ will be an order smaller than p'_{in} . In order to make all this explicit we introduce a small parameter (via the external forcing amplitude), and make dimensionless

$$\begin{split} \tau &= \omega_0 t, \qquad \qquad R = \varepsilon \rho_0 c_0 (\ell S_n / V)^{1/2} r, \\ p'_{\rm in} &= 2\varepsilon \rho_0 c_0^2 (\ell S_n / V) y, \quad p'_{\rm ex} = 2\varepsilon^2 \rho_0 c_0^2 (\ell S_n / V) F, \end{split}$$

where $0 < \varepsilon \ll 1$. Suppose we excite the Helmholtz resonator harmonically by external forcing $p'_{\text{ex}} = C \sum_{n=1}^{\infty} \frac{\sin(n\omega t)}{n}$ with resonance frequency ω . In the scaled variables τ and F this becomes $F = F_0 \sum_{n=1}^{\infty} \frac{\sin(n\omega t)}{n}$ with $\omega = \Omega \omega_0$.

So we have a weakly non-linear forced oscillator as in (9) where the initial conditions are not important as we are interested in the stationary state.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} + \varepsilon \frac{\mathrm{d}y}{\mathrm{d}\tau} \left| \frac{\mathrm{d}y}{\mathrm{d}\tau} \right| + \varepsilon r \frac{\mathrm{d}y}{\mathrm{d}\tau} + y = \varepsilon F_0 \sum_{n=1}^{\infty} \frac{\sin(n\Omega\tau)}{n}$$
(9)

Near resonance when $1 - \Omega^2 = O(\varepsilon)$, the pressure amplitude rises to levels of O(1), and the assumption that the non-linear damping is negligible is not correct. This also corresponds with the most achieved damping. As the physics of the problem essentially change when $\Omega^2 = 1 + O(\varepsilon)$, we assume that $\Omega = 1 + \varepsilon \Delta$. To obtain a uniform approximation later [6, sec 15.3.2], we remove the ε -dependence from the driving force, so we make again a

slight shift in the time coordinate. In addition, we translate the origin by an amount $\theta(\varepsilon)$, such that the location of the sign change of y' is fixed and independent of ε . So we introduce $\tilde{\tau} = \Omega \tau - \theta(\varepsilon)$ to obtain to the leading orders

$$(1+2\varepsilon\Delta)\frac{\mathrm{d}^2 y}{\mathrm{d}\tilde{\tau}^2} + \varepsilon\frac{\mathrm{d}y}{\mathrm{d}\tilde{\tau}} \left|\frac{\mathrm{d}y}{\mathrm{d}\tilde{\tau}}\right| + \varepsilon r\frac{\mathrm{d}y}{\mathrm{d}\tilde{\tau}} + y \tag{10}$$

$$=\varepsilon F_0 \sum_{n=1}^{\infty} \frac{\sin n(\tilde{\tau}+\theta)}{n}.$$
 (11)

where θ is to be chosen such that $y'(\tilde{\tau}) = 0$ at $\tilde{\tau} = (2N+1)\pi/2$. When we substitute the assumed Poincaré expansions $y(\tilde{\tau};\varepsilon) = y_0(\tilde{\tau}) + \varepsilon y_1(\tilde{\tau}) + \varepsilon^2 y_2(\tilde{\tau}) + \dots$ and $\theta(\varepsilon) = \theta_0 + \varepsilon \theta_1 + \dots$, and collect like powers of ε , we find for y_0

$$\frac{\mathrm{d}^2 y_0}{\mathrm{d}\tilde{\tau}^2} + y_0 = 0, \qquad y_0'(\frac{(N+1)\pi}{2}) = 0 \tag{12}$$

with general solution $y_0(\tilde{\tau}) = A_0 \sin(\tilde{\tau})$. The next order y_1 is

$$\frac{\mathrm{d}^2 y_1}{\mathrm{d}\tilde{\tau}^2} + y_1 = F_0 \sum_{n=1}^{\infty} \frac{\sin n(\tilde{\tau} + \theta_0)}{n} - 2\Delta \frac{\mathrm{d}^2 y_0}{\mathrm{d}\tilde{\tau}^2} - \frac{\mathrm{d}y_0}{\mathrm{d}\tilde{\tau}} \left| \frac{\mathrm{d}y_0}{\mathrm{d}\tilde{\tau}} \right| - r \frac{\mathrm{d}y_0}{\mathrm{d}\tilde{\tau}} = F_0 \sum_{n=2}^{\infty} \frac{\sin n(\tilde{\tau} + \theta_0)}{n} + 2\Delta A_0 \sin(\tilde{\tau}) - A_0 |A_0| \cos(\tilde{\tau}) |\cos(\tilde{\tau})| - r A_0 \cos(\tilde{\tau})$$

with $\cos(\tilde{\tau}) |\cos(\tilde{\tau})| = \sum_{n=0}^{\infty} \frac{-1^{(n+1)} \cos(2n+1)\tilde{\tau}}{\pi(n^2 - \frac{1}{4})(n+\frac{3}{2})}$. When we suppress the cos- and sin-terms, including the first term of the Fourier expansion of $\cos(\tilde{\tau}) |\cos(\tilde{\tau})|$, we get the equations

$$F_0 \cos \theta_0 = -2\Delta A_0, \quad F_0 \sin \theta_0 = \left(\frac{8}{3\pi}|A_0| + r\right) A_0$$

with solution

$$\left[\left(\frac{8}{3\pi} |A_0| + r\right)^2 + (2\Delta)^2 \right] A_0^2 = F_0^2$$
(13)
$$\tan(\theta_0) = -\frac{\frac{8}{3\pi} |A_0| + r}{2\Delta}.$$
(14)

 2Δ

The y_1 equation then

$$\begin{aligned} \frac{\mathrm{d}^2 y_1}{\mathrm{d}\tilde{\tau}^2} + y_1 &= \sum_{n=1}^{\infty} \left[F_0 \frac{\sin(n+1)(\tilde{\tau}+\theta_0)}{(n+1)} \right. \\ &+ A_0 |A_0| \frac{(-1)^n \cos(2n+1)\tilde{\tau}}{\pi (n^2 - \frac{1}{4})(n+\frac{3}{2})} \right] \end{aligned}$$

has the general solution

$$y_1(t) = A_1 \cos \tilde{\tau} + B_1 \sin \tilde{\tau} - \sum_{n=1}^{\infty} \left[\frac{F_0 \sin(n+1)(\tilde{\tau} + \theta_0)}{n(n+1)(n+2)} + \frac{A_0 |A_0|}{4\pi} \frac{(-1)^n \cos(2n+1)\tilde{\tau}}{n(n+1)(n^2 - \frac{1}{4})(n+\frac{3}{2})} \right].$$

with the condition that $y'_1(\frac{(2N+1)\pi}{2}) = 0$, The constant A_1 is determined. Once A_1 is determined, we move to the next order asymptotic solution from (11), collecting the like coefficients of ϵ^2 , we have

$$y_{2}'' + y_{2} = \left[\Delta^{2}A_{0}\sin\tilde{\tau} + 2\Delta A_{1}\cos\tilde{\tau} + 2\Delta B_{1}\sin\tilde{\tau} + ..\right] \\ + \left[2\Delta A_{0}^{2}\cos^{2}\tilde{\tau} - 2A_{0}A_{1}\sin\tilde{\tau}\cos\tilde{\tau} + 2A_{0}B_{1}\cos^{2}\tilde{\tau} - 2A_{0}F_{0}\sum_{n=1}^{\infty}\frac{\cos\tilde{\tau}\cos(n+1)(\tilde{\tau}+\theta_{0})}{n(2n+1)} - \frac{2A_{0}^{3}}{4\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n}(2n+1)\sin(2n+1)\tilde{\tau}\cos\tilde{\tau}}{n(n+1)(n^{2}-\frac{1}{4})(n+\frac{3}{2})}\right] * \\ + \left[rA_{1}\sin\tilde{\tau} - rB_{1}\cos\tilde{\tau} - r\Delta A_{0}\cos\tilde{\tau} + ...\right] \\ + \theta_{1}F_{0}(\cos\tilde{\tau}\cos\theta_{0} - \sin\tilde{\tau}\sin\theta_{0}).$$

Suppressing the coefficients of the sine and cosine terms as done previously, we can obtain the value of B_1 and θ_1 from the linear equations

$$(16\frac{A_0}{3\pi} - r)B_1 + (F_0\cos\theta_0)\theta_1 = -2\Delta A_1 - \frac{16\Delta A_0^2}{3\pi} + r\Delta A_0 -\frac{2A_0F_0}{\pi} \sum_{n=1}^{\infty} \frac{6\cos(\frac{n\pi}{2}) + 2\cos(\frac{3n\pi}{2})}{n(n^2 - 1)(n + 3)}\cos(n + 1)\theta_0 (2\Delta)B_1 - (F_0\sin\theta_0)\theta_1 = -\Delta^2 A_0 + \frac{8A_0A_1}{3\pi} - \frac{A_0^3}{27\pi^2}(80 - 9\pi^2) - rA_1 + \frac{2A_0F_0}{\pi} \sum_{n=1}^{\infty} \frac{3\cos(\frac{n\pi}{2}) + \cos(\frac{3n\pi}{2})}{n(n + 1)(n^2 - 1)(n + 3)}\sin(n + 1)\theta_0.$$

The series we see are truncated for a finite summation. The solution $y = y_0 + \varepsilon y_1 + O(\varepsilon^2)$ ascertains in principle (for small ε) a better approximation of y than the leading order approximation y_0 , which is important to obtain the approximation of the impedance for higher harmonics. Consider first the leading order approximation. Equation (14) for A_0 has 2 real symmetric solutions (of which we normally need to consider only the positive one), but solving $A_0 = A_0(\Delta)$ is not straightforward. Therefore, it is useful to consider the inverse, $\Delta = \Delta(A_0)$, given by

$$4\Delta^2 = \frac{F_0^2}{A_0^2} - \left(\frac{8}{3\pi}|A_0| + r\right)^2 \tag{15}$$

Since $\Delta^2 \ge 0$ we see immediately that solutions exists only for a finite interval in A_0 , while $\Delta \to \infty$ only when $A_0 \to 0$. In particular, we have

$$A_0 \simeq \frac{F_0}{2|\Delta|} \qquad \theta_0 \simeq -\frac{r}{2\Delta} + n\pi,$$
 (16)

which is in exact agreement with the asymptotic behavior for $\Omega = 1 + \varepsilon \Delta$, Δ large, corresponding to the linear solution for harmonic source. In fact, by tracing the solution



Figure 2. Solution of amplitude A_0 and phase θ_0 as a function of Δ , while $r = F_0 = 1$.

parametrically as a function of Δ , we can see that if we start with $\theta_0 = 0$ for $\Delta \to -\infty$, we end with $\theta_0 = \pi$ for $\Delta \to \infty$. In this way, we have obtained the expression for A_0 and θ_0 shown in Fig. 2. It is also proved [3] that the A_0 solution is stable and does not grow with time.

This way, we know the asymptotic solution $y = y_0 + \varepsilon y_1$ correct till $O(\varepsilon)$. Next we formulate the impedance calculation.

3. Impedance Calculation

The impedance Z formulation starts with the calculation of the neck velocity, u'_n from p'_{in} . We have

$$\frac{V}{c_0^2}\frac{\mathrm{d}p'_{\mathrm{in}}}{\mathrm{d}t} = \mathrm{i}\omega\frac{V}{c_0^2}p'_{\mathrm{in}} = -\rho u'_n S_n.$$

The external normal velocity u'_{ex} is defined as the average over the whole area and therefore includes the porosity factor S_n/S_b *i.e.* $u'_{ex} = \frac{S_n}{S_b}u'_n$.

Impedance is a linear concept and if we want to generalize for non-linear problem, the most natural way would be the ratio of component of pressure and velocity over a particular frequency.

Hence, We define the impedance as the negative of the ratio of the Fourier transform of the external pressure p'_{ex}



Figure 3. Impedance at observed frequency $\eta = n\omega$ and N wave excitation $\Omega = 1$. $\frac{S_n}{S_n} = 0.5$, r = 0.2, $\frac{\omega_0}{2\pi} = 1447Hz$, L = 0.035m and $\ell = 0.002$ m.

to the Fourier transform of the external velocity u'_{ex} . The Fourier transform is taken over the frequency of interest, which happen to be the integral multiple of the resonance frequency ω_0 in our case *i.e.*

$$Z(\eta) = -\frac{\frac{1}{2\pi} \int\limits_{-\infty}^{\infty} p'_{\rm ex} e^{-i\eta t} dt}{\frac{1}{2\pi} \int\limits_{-\infty}^{\infty} u'_{\rm ex} e^{-i\eta t} dt} \qquad (\eta = n\omega_0).$$
(17)

Defined in this way, the impedance gives an understanding of the behavior of the wall for different harmonics of the N-wave. In other words, it is the response of the wall (acoustic velocity) to a particular pressure component of the source, characterized by its frequency. Of Most importance are the first 3 harmonics of N-wave because the later harmonics are practically cut off.

4. Results

The above analysis is performed for a typical geometry of the cavity liner with $\frac{S_n}{S_b} = 0.5$, r = 0.2, $\frac{\omega_0}{2\pi} = 1447Hz$, L = 0.035m and $\ell = 0.002$ m. The acoustic pressure and velocity are Fourier transformed to obtain the impedance using (17).

Shown in 3 is the impedance in the dimensional form for different harmonics of the resonance frequency calculated for different driving amplitudes in SPL dB. We notice that at $\omega = \omega_0$, the impedance has the same value as when the Helmholtz resonator is driven by the harmonic source (and hence should be compared with [7]) and increases for higher harmonics. The resistance $\operatorname{Re}(Z)$ term is strongly dependent on the driving amplitude and grows much higher for higher amplitudes. The reactance $\operatorname{Im}(Z)$ term on the contrary, is practically independent of the driving amplitude. Essentially, the wall behaves like a hard wall for very high harmonics of the natural frequency ω_0 . The above analysis could be useful to optimize the liner and choose a suitable geometry to find an agreement to kill several dominant harmonics.

5. CONCLUSIONS

A systematic approximation of the hydrodynamically nonlinear Helmholtz resonator equation driven by N-wave is obtained, including the resulting impedance if the resonator is applied in an acoustic liner. The resistance $\operatorname{Re}(Z)$ term is found to be strongly dependent on the driving amplitude and grows much higher for higher amplitudes. The reactance $\operatorname{Im}(Z)$ term on the contrary, is practically independent of the driving amplitude. The wall behaves like a hard wall for very high harmonics of the natural frequency ω_0 . A much deeper insight in the problem could be obtained by comparing the results with numerical computation [8], [9]. A comparison with the LES calculations is planned ahead.

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