Excitation of a single cut-on mode by means of a planar mode synthesizer composed of several point sources – theory and experiment

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Summary
The paper presents a method proposed by the authors to generate a single user-selected waveguide cut-on mode by means of mode synthesizer consisting of a set of point sources, each of them modelled by thin pipe outlet and driven by a separate loudspeaker. Analysis of sound propagation in duct-like facilities frequently calls for considering the presence of higher cut-on modes together with the phenomena of their reflection/coupling occurring on outlets, impedance variation, etc. And so, with increasing reduced frequency (Helmholtz number), description of the field becomes more and more complex. To avoid this complexity, the simplest approach seems to be to generate a single cut-on mode by means of a properly constructed source. Based on theoretical analyses carried out by means of the in-duct Green's function the authors designed mode synthesizer composed of $N \leq 13$ point sources which was constructed and integrated into the measurement duct. The measurements were carried out for some reduced frequencies allowing for propagation of at least four cut-on modes. Combination of rotary motion of the mode synthesiser and microphone positioning along the duct diameter allowed to measure automatically the acoustic pressure in points uniformly distributed on the duct cross section.

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1. Introduction

The complex nature of acoustic phenomena occurring in waveguides due to their geometry [1–8], different properties of ducts surfaces [1, 2, 6, 9–20], variety and diversity of acoustic sources operating in them [19, 21–25] acting sometimes also as sources of a flow of medium [9–12, 18, 26–28, 40, 43], and various active or passive elements inserted inside [27–30] result in complexity of mathematical description of the acoustic field propagated along such ducts and radiated from their outlets. Another difficulty consists in comparing experimental results with theoretical predictions. Additionally, description of the field becomes more and more complex with increasing values of the reduced frequency (Helmholtz number) [12–14], as the number of the cut–on modes increases approximately with the square of the parameter. As regards the geometry, most of available studies consider straight cylindrical waveguides of constant radius [4, 8, 15–19,], but also elliptical ducts [2, 13] or those with varying cross section [1, 5, 7] are analysed. As far as the sources operating in duct-like systems are concerned, the most frequently analysed cases include rotors, fans, compression chambers, but also turbofan jet engines [14, 21–24], all of which additionally enforce the flow of medium, to mention only heating, ventilation, and air-conditioning (HVAC) systems. Until now, analytical solutions are known only for sources far less complicated than the ones mentioned above. Thus, some new techniques based on reconstruction of sources operating inside the duct from measurement data [31–34] or reconstruction of modal content of the considered sound field [35–37] have been developed. But even in the waveguide type most probe to mathematical modelling, i.e. in a straight duct with constant circular cross-section and rigid walls, description becomes more complicated when some passive/active elements such as mufflers or silencers are incorporated inside the duct [29, 31] or the duct ends are provided with unflanged outlet [4, 8, 15, 19, 26, 38–40]. As is can be seen from analytical solutions, any change in conditions of propagation results not only in reflection of each of the propagating modes but also in its transformation/coupling into all cut-on modes of the same circumferential order [38, 39]. Therefore, propagation of n cut-on modes of a given circumferential order *m*, namely (*m,* 1), (*m,* 2), ..., (*m, n*) is uniquely determined by $2n \times 2n$ elements of the scattering matrix comprising four $n \times n$ sub-arrays (reflection and transmission matrices for waves travelling in both directions). When such a wave heading towards an unflanged duct outlet is considered, the diffraction phenomena should be accounted for, which requires knowledge of $n \times n$ elements of the reflection/transformation matrix, calculated in this case as a result of solving the wave equation with appropriate boundary condition by means of the Wiener-Hopf method [4, 39, 40]. To derive the scattering matrix, the system must be excited at least as many times as the number of propagating modes and field measurements have to be carried out over four duct cross sections, of which two are located upstream and the other two downstream the inserted element [9, 37]. To derive the reflection matrix at the open end of a duct it is sufficient to take measurements on two duct cross sections.

As it can be seen from the above, the analysis of a multimode sound wave propagating in a duct-like system is rather complicated and it might have been convenient to be able to generate a single selected cut-on mode. This would allow to simplify and improve accuracy of predictions concerning, for example, acoustic properties of mufflers, silencers, or duct orifices of different shapes which would be undoubtedly of interest to designers and constructors of such devices.

In experiments carried out by laboratories dealing with duct acoustics and making an attempt at producing the acoustic field with prescribed modal content, acoustic drivers/microphones were mounted flush with the duct wall [35–37] and their number varied from 10 [35] up to 133 [42].

In our work, we have focused on solutions involving much lower financial outlays and decided to construct a duct set-up in which different field configurations would be obtained by means of a planar source synthesiser, i.e. a matrix containing a variable number of “point” sources. This paper reports theoretical background of the research undertaken to synthesise a sound field with prescribed modal content and technological solutions employed in constructing such mode synthesiser allowing to collect automatically the experimental data taken on the whole cross-section of the duct model.

2. Theoretical background

The theoretical background of the reported research consists in the Green’s function formalism [43–45] applied to a number of point sources operating inside the duct [44] and the mode decomposition method [35–37].

Assuming the harmonic time dependence of the considered sound field in the form $e^{i\omega t}$ the in duct acoustic field can be expressed as an infinite number of modes being particular solutions of the Helmholtz equation [45] and determined by two indices representing their circumferential (*m*) and radial (n) orders, out of which only the cut-on modes will be taken into account in the forthcoming considerations [45]. Thus, in cylindrical co-ordinates system, the acoustic pressure inside the duct can be represented by a sum

$$p(\rho, \varphi, z) = \sum_{m,n} P_{mn} \psi_{mn}(\rho, \varphi)\, e^{-i\gamma_{mn} z}, \quad (1)$$

where P_{mn} is the (*m,n*) pressure mode amplitude, γ_{mn} is the axial wave number and ψ_{mn} is the mode shape function

$$\psi_{mn}(\rho, \varphi) = \Lambda_{mn}^{-1} e^{im\varphi} J_m(\beta_{mn}\rho). \quad (2)$$

In the latter formula Λ_{mn} is the modal normalisation constant introduced to satisfy the orthogonality condition on the duct cross section $\langle \psi_{mn} | \psi_{m'n'} \rangle = \delta_{mm'}\delta_{nn'}$ [41], $J_m(x)$ is the Bessel function of the first kind, and β_{mn} is the radial wave number fulfilling the hard wall duct boundary condition $J'_m(\beta_{mn}a) = 0$. In what follows it will be assumed $\beta_{mn}a = \mu_{mn}$. The radial wave number satisfies the relation $\beta_{mn}^2 + \gamma_{mn}^2 = k^2$, where k is the wave number in a free-field. For propagating modes the axial wave number must be real and so for these modes $\mu_{mn} < ka$, where *ka* is the Helmholtz number.

The in-duct Green’s function [45], representing the field of an acoustic monopole of a unit amplitude operating inside the duct and positioned at (ϱ', φ', z') takes the form

$$G(\varrho,\varphi,z,\varrho',\varphi',z') = \sum_{m,n} i\,\frac{\psi_{mn}(\varrho,\varphi)\psi^{*}_{m'n'}(\varrho',\varphi')}{(-2\gamma_{mn})}\,e^{-i(z-z')\gamma_{mn}}, \quad (3)$$

where the asterisk means the complex conjugate.
For a number Q of point sources with complex amplitudes A_q, the acoustic pressure can be expressed as a linear superposition of the consecutive Green's function representing each of the monopoles multiplied by amplitude of an corresponding mode. On the grounds of orthogonality properties of the mode shape functions, the resultant pressure attributed to mode (m,n) coming from Q point sources operating inside the duct is

$$P_{mn}^{(Q)} = \sum_{q=1}^{Q} A_q P_{mn}\left(\vec{r'}_g\right) \quad (4)$$

The above formula can be used to determine analytically the complex amplitudes (modules and phases) for each of the point sources required to obtain the field of prescribed modal contents, in particular composed of only one user-selected mode [44] Assuming that the Helmholtz number ka is such that the number of the cut-on modes is possible, at least theoretically, to determine the amplitudes of consecutive point sources A_q, in such a way that they will excite a wave in which the only non-zero mode amplitude will be P_{mn}, namely this of the selected mode. More details on the method leading to determination of amplitudes with individual sources of the in-duct matrix must be driven can be found in [44].

3. Planar mode synthesiser

The constructed planar mode synthesiser integrated into the duct model is depicted in Figs. 1 and 2. The experimental duct set-up is composed of two one-meter- and two half-meter-long segments made of 6.3-mm thick steel pipe with the inner radius 0.103 m and adapted to carry the microphone positioning device, allowing the microphone to be moved along the duct diameter. Measurements inside the waveguide were taken by means of B&K 4939 ¼″ measuring microphone with resolution of 5 mm in the duct radius.

Figure 2. A picture of the modular measuring set-up with planar mode synthesiser mounted at one end.

The advantage of constructing the duct of four movable segments is that they can be arranged in a different way with some additional devices such as mufflers easily incorporated into the duct according to current needs. Both or only one end of the duct can be equipped with 1-m long anechoic termination to simulate the mathematical model of, respectively, infinite or semi-infinite duct. One of the anechoic terminations is also adapted for mounting the planar mode synthesiser. The mode synthesiser itself has a shape of a circle made of steel with holes allowing to mount firmly and precisely up to 13 acoustic monopole models.
The model of a point source was realised as an outlet of a thin steel tube with outer and inner radius of 6 mm and 5 mm, respectively, driven by a loudspeaker. To verify the effect of the tube outlet termination type, a number of different solutions were tested [46] by measuring directivity patterns produced by them in the AGH anechoic chamber. The directivity characteristics were measured in two planes, horizontal and vertical,

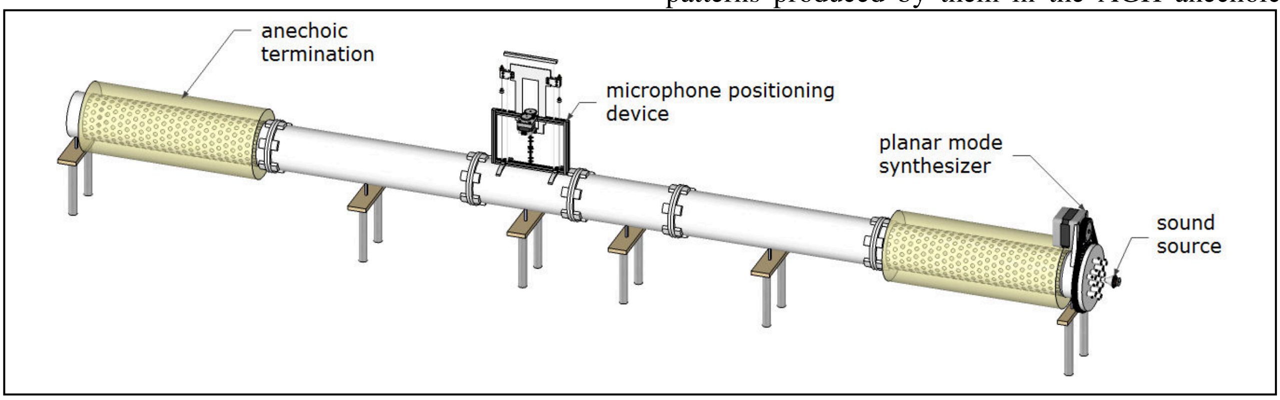

Figure 1. A sketch of the modular measuring set-up with planar mode synthesiser mounted at one end

for three frequencies selected from lower ($f = 435$ Hz), middle ($f = 4760$ Hz), and upper ($f = 9995$ Hz) range of the generated signal. Some results of the test are presented in Fig. 3.

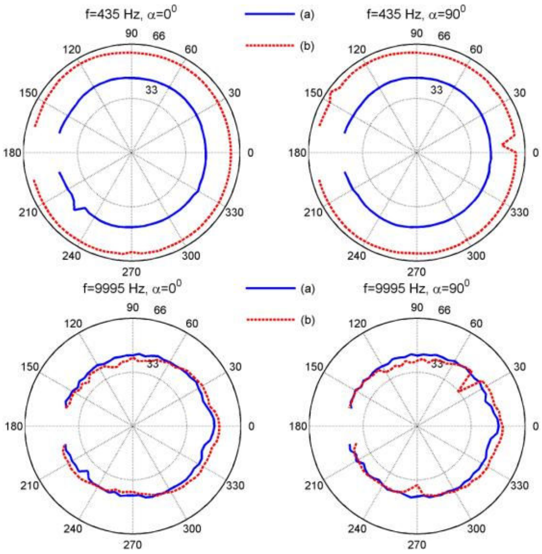

Figure 3. Free-space directivity characteristics for a thin tube tipped with (a) simple circular outlet and (b) the outlet modified by attaching a perforated sphere with radius of 20 mm and 57 holes for two frequencies limiting the examined range.

Rotation of the mode synthesiser is realised by means of a step motor controlled by a program written in MATLAB computing environment. The microphone position along the duct diameter is controlled by another proprietary program. Combination of these two motions results in the possibility to scan the duct cross-section with resolution of 15 degrees in azimuthal angle and 5 mm in the diameter.

4. Results of the experiment

Before starting experiments with the above-described single mode generator, some preliminary tests were carried out aimed at validation of the measuring component of the set-up, i.e. checking whether it will be possible to achieve accuracy sufficient to verify the applied mathematical model.

The measurements of the sound pressure distribution on the duct cross section confirmed accordance of the applied mathematical model of the infinite/semi-infinite cylindrical duct with properties of the constructed set-up. This conclusion is based on the following results: (i) for symmetrical excitation, the sound pressure distribution measured on the duct cross section was also symmetrical, thus signifying propagation of radial modes (Fig. 4a), while for asymmetric excitation, occurrence of circumferential modes was evident (Fig. 4b); (ii) the frequencies at which the consecutive modes became the cut-on ones and propagated along the duct agreed well with those predicted theoretically.

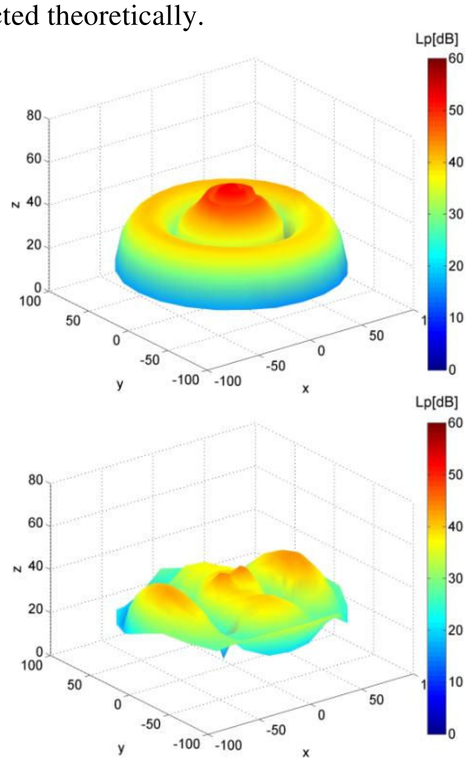

Figure 4. Distribution of the sound pressure level on the duct cross section in case of (a) symmetrical and (b) asymmetrical excitation ($ka = 7.2$; $f = 3820$ Hz).

For the Helmholtz number $ka = 4$ it is possible to generate each of four cut-on modes, namely the plane wave (0, 0) plus modes (1, 1), (2, 1), and (0, 1) with the use of six monopoles located in the mode synthesiser, each driven by means of a signal with appropriate amplitude and phase (Fig. 5).

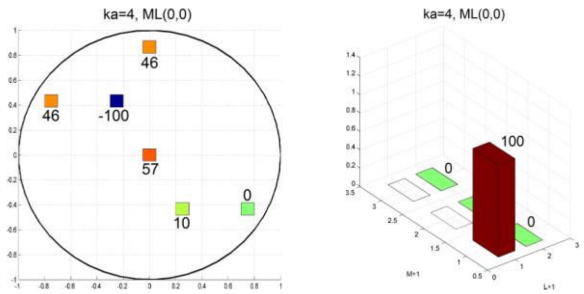

Figure 5. Arrangement of six monopoles capable to generate a single cut-on mode at $ka = 4$

The following table lists amplitudes and phases calculated by means of the in-duct Green's function which, according to the theory, are expected to assure that only one cut-on mode will be generated.

Table 1. Amplitude and phases of the signals exciting the consecutive monopoles

	mode (0,0)		mode (1,1)		mode (2,1)		mode (0,1)	
Source	Amp	Phase	Amp	Phase	Amp	Phase	Amp	Phase
1	46	0	0	0	5	0	46	π
2	46	0	100	π	51	0	46	π
3	100	π	80	0	100	π	100	0
4	57	0	0	0	87	0	3	0
5	10	0	80	π	89	π	10	π
6	0	0	100	0	46	0	0	0

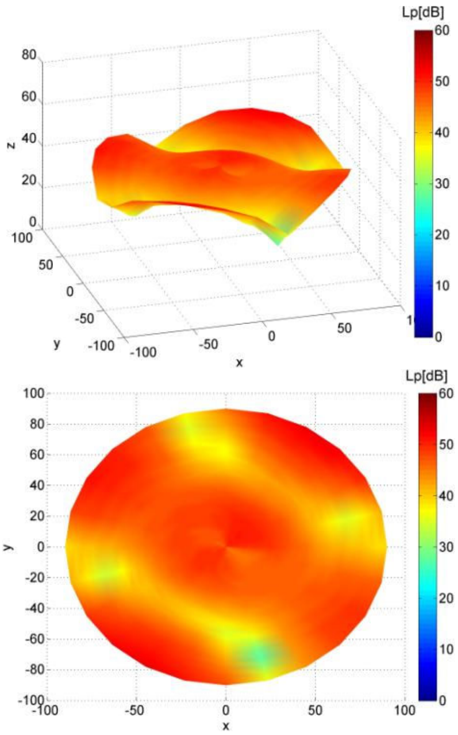

Figure 6**.** Distribution of the sound pressure level in the field generated by six monopoles driven to produce the mode (2, 1) at $ka = 4$.

5. Conclusions

In the stage described in this paper, experiments aimed at generating a single cut-on mode by means of the proposed and constructed planar mode synthesiser were focused on obtaining the (2, 1) mode at the Helmholtz number $ka = 4$. by means of six point sources distributed on the duct cross section as presented in Fig. 5 and driven by loudspeakers with signals characterised with relative amplitudes and phases listed in Table 1. Although the results are promising, they need to be improved and sources of errors resulting in the variation of the sound pressure level on the duct cross section have to be found. Some more experimental results will be presented during the Conference.

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