Sound pressure fields in two coupled rooms - Comparison of a finite element approach and an analytic solution

Maximilian Neusser

Hannes Konder

Thomas Bednar

Research Center of Building Physics and Sound Protection, TU Vienna, Austria.

Summary

This paper presents a comparison of a finite element method (FEM) Fluid Structure Interaction Model and an analytic solution of the sound pressure field in two neighboring, coupled and rectangular rooms with different source locations at the low frequency sound spectrum. The FEM Model described in the paper combines the pressure acoustics of a linear elastic fluid in the air volumes and of structural mechanics to connect the acoustics pressure variations in the fluid domain with the structural deformation in a the partition wall. A parametric study and comparison between both models is shown which has different source positions, room dimensions, wall properties and the mesh density of the FEM model as parameters. It is shown that the solutions of the FEM Model fit in the dependence of the mesh density to the described analytic solution of the physical problem.

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1. Introduction

Sound isolation is an important building component property. Especially the sound insolation at the low frequency sound spectrum is gaining importance due to the growing use of light weight constructions and the intensified distribution of HIFI audio systems with low frequency output. Referring to the fact that the research in building acoustic is focusing in calculation methods for the acoustic properties of building elements below 100Hz, these calculation methods have to deal with non-diffuse sound pressure fields and a low eigenmode density in the sending and receiving rooms at the investigated frequency ranges. A sound field driven by a low frequency sound source in small rooms is characterized by a location depended pressure field, due to the low eigenmodes density in the low frequency range. Some of the applied building acoustic simulation methods, for example statistical energy analysis, have problems dealing with those low frequencies caused by the low eigenmode density. For FEM calculations the only problem with none diffuse sound fields is the higher calculation effort, because the whole air domain around the investigated building part has to be taken into account to simulate the interaction of the air and the structural domain. The FEM is a general method for solving differential equations over complex domains, so discretizing large domains cause in FEM-simulations high numbers of mesh elements. So those simulations are mainly limited by the available computational power. This paper gives an overview about the necessary mesh density in FEM by comparing the results of a simulated sound transmission measurement by FEM and an analytical physical equivalent model of two through the velocity field of a homogenous partition wall coupled sound pressure fields. The pressure field of the air volume in the receiving room is driven by the displacement velocity of the separating wall of the two rooms. Both, the analytical model and the FEM-simulation calculate the described phenomena stepwise. In a first step the sound pressure field in the sending room caused by a point source is calculated. Then the velocity field of the partition exited by the pressure field along the wall is determined. The last step calculates the sound pressure field in the receiving room produced by the radiated energy of the partition wall. Source frequencies from 80-200Hz are investigated.

2. Methodology

1.1. Analytic Model

As described in the introduction the calculation of the sound pressure field in the receiving room is separated into three steps. The following section of the paper should give an overview about the used solution method. The full solving path and assumption list for the investigated case is published in [1].

1.1.1. Sound pressure field in the sending room

The inhomogeneous differential equation (1) describing the sound pressure p in the air domain is referred as Helmholtequation. We shall characterize the monopole source q as mass flux having the unit $kg \cdot m^3 \cdot s^{-1}$

$$\Delta p + \frac{\omega^2}{K} \cdot p \cdot p = -i \cdot \omega \cdot p \cdot q \qquad (1)$$

Solving this type of equations analytically for simple room shapes and simple boundary conditions is possible and many solution can be found in literature. The analytic discussion of equation (1) shows that the solution of the equation can be written as summation of the product of the eigenmodes ψ_N and their amplitudes $\tilde{p}_N$ if $\tilde{p}_N$ is fitted to the inhomogeneous source condition.

$$p = \sum_N p_N \cdot \psi_N \qquad (2)$$

Subscript N identifies the N-th combination of the three running counters l, m and n of the natural vibrations or modes N of the rectangular room with dimensions Lx, Ly and Lz.

$$\psi_N = \cos\frac{l \cdot \pi \cdot x}{L_x} \cdot \cos\frac{m \cdot \pi \cdot y}{L_y} \cdot \cos\frac{n \cdot \pi \cdot z}{L_z} \cdot \qquad (3)$$

With equation (2) and (3) and the assumptions made in [1] the pressure field in the sending room can by characterized by equation (4)

$$p_N = -\frac{i \cdot \omega \cdot \rho \cdot K}{\Lambda_N \cdot (\omega^2 \cdot \rho - \omega_N^2 \cdot \rho_N)} \cdot q \cdot \psi_N(x_0, y_0, z_0) \qquad (4)$$

1.1.2. Velocity field in the partition wall

The sound field equation of a vibrating wall referred to [1] can be written as follows:

$$B \cdot \left(\frac{\partial^4}{\partial y^2} + 2 \cdot \frac{\partial^4}{\partial y^2 \cdot \partial z^2} + \frac{\partial^4}{\partial z^2}\right) \cdot v + m'' \cdot \frac{\partial^2 v}{\partial t^2} = \frac{\partial \Delta p}{\partial t} \qquad (5)$$

The solution of the inhomogeneous differential equation corresponds to the solution of the homogeneous differential equation (5), if only the coefficient of the homogeneous solution, so the amplitudes of the velocity distribution, are adapted to the inhomogeneity. The inhomogeneity represents the temporal change of the pressure differential that excites at the plate boundary. Because the sound pressure in the sending room is substantially greater than that in receiving room, follwing approximation can be made.

$$\Delta p = p_S - p_E \approx p_S \qquad (6)$$

In equation (6) p_E symbolizes the sound pressure in the receiving room behind the partition wall. With equation (6) the inhomogeneity in (5) results only from the temporal change of the sound pressure in the sending room. The sound field around a clamped over a sending room indirectly excited rectangle wall is thus determined. The equation with which the velocity field can be calculated theoretically, is summarized as follows:

$$v_W = \sum_{N_W} \frac{16 i \cdot \omega \cdot m_W \cdot n_W}{m'' \cdot \pi^2 \cdot (\omega_{N_W}^2 - \omega)} \cdot \sum_{N_S} \frac{p_{N_S}}{(m_s^2 - m_W^2) \cdot (n_s^2 - n_W^2)} \cdot \varphi_{N_i} \qquad (7)$$

With the described set of formulas it is possible to determine the sound field in an by a point source excited rectangle space and its possible to calculate the transfer of acoustic energy from this source room in one of the room enclosing, clamped rectangular wall.

1.1.3. Sound pressure field in the receiving room

While the calculation of the spatial modes of the receiving room runs by the same method as the calculation of the modes of the sending room, and is therefore considered as known, it's only necessary to determine the NE-th amplitude of $\tilde{p}_{NE}$ in equation (4) to characterize the sound pressure field in the receiving room. This, to the inhomogeneity adapted, amplitudes can be calculated the following equation.

$$p_{N_E} = -\frac{i \cdot \omega \cdot \rho_E \cdot K}{\Lambda_{N_E} \cdot (\omega^2 \cdot \rho_E - \omega_{N_E})} \cdot \sum_{N_W} \frac{4 \cdot L_y \cdot L_x \cdot m_W \cdot n_W}{\pi^2 \cdot (m_W^2 - m_E^2) \cdot (n_W^2 - n_E^2)} \cdot v_{N_W} \qquad (8)$$

3. FEM – Model

The used FEM software environment is COMSOL Multiphysics in the version 5.0. COMSOL offers a pressure acoustic module and a structural mechanics module which were used in the presented parametric simulations.

The physics interface of the pressure acoustics module in COMSOL can be used for linear acoustics described by a scalar pressure variable. It includes domain conditions to model losses in a homogenized way, so-called fluid models for porous materials, as well as losses in narrow regions. Domain features also include background incident acoustic fields, as well as domain monopole and dipole sources. The plane wave

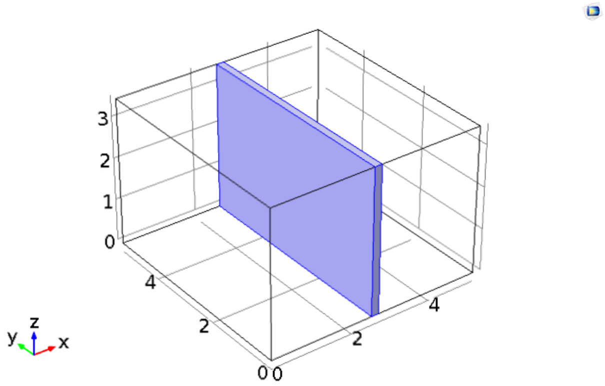


Figure 1. 3D Illustration of the FEM-Model in COMSOL

attenuation behavior of the acoustic waves may be entered as a user-defined quantity, or defined to be bulk viscous and thermal losses. The physics interface solves the Helmholtz equation in the frequency domain for given frequencies, or as an eigenfrequency or modal analysis study [7]. Acoustic-structure interaction refers to a multiphysics phenomenon where the acoustic pressure causes a fluid load on the solid domain, and the structural acceleration acts on the fluid domain as a normal acceleration across the fluid-solid boundary. A dedicated multiphysics coupling condition is defined for the fluid-solid boundary and sets up the fluid loads on the solid domain and the effect of the structural accelerations on the fluid. Caused by the stepwise calculation method in the analytical solution the FEM-Model simulates the described physical problem in an equivalent decoupled stepwise way to avoid differences in the results, caused by different physical assumptions.

In FEM-simulations the user always have to reach a compromise between solution convergence or accuracy and computational effort. When using a FEM-technique a reasonable number of elements per wavelength are of the order three to four [5]. In the case of the investigated frequencies from 80-200Hz this would require a maximum element size of approximately 1-0.40m.

4. Parametric study

In the parametric study the following parameters of the rooms, wall and source position are varied, with each possible combination of parameters examined.

- Density partition wall ρ=800, 2300 kg/m³
- Poisson's ratio partition wall μ=0.4, 0.33
- Thickness partition wall d=0.1, 0.24 m
- E-Module partition wall E=10^{10}, 30^{10} Pa
- Length of the test chambers Lx=2, 5 m
- Width of the test chambers Ly=5, 6 m
- Source position x_0=0.5, 1; 0.5, 2 m
- Source position y_0=0.5, 2.5; 0.5, 3 m
- Source position z_0=0.5, 1.5 m
- Soruce frequency f_0=80, 125, 200Hz

Goal of the parametric study is to produce different sound pressure fields in the source and receiving room induced by different source positions, wall properties and room dimensions, to have a spread over the possible combinations referred to real situations.

Table I. Mesh properties in COMSOL

	Coarse	Normal	Fine	Extra Fine
Maximum element size in m	1,01	0,53	0,42	0,18
Minimum element size in m	0,21	0,10	0,05	0,01
Maximum element growth rate	1,70	1,50	1,45	1,35
Curvature factor	0,80	0,60	0,50	0,30
Resolution of narrow regions	0,30	0,50	0,60	0,82

All mesh parameters shown in Table 1 are set automatically by the software COSMOL referred to the type of physics selected and the mesh type given by the user from extra fine to coarse. Although all shown parameters of the finite element discretization can be edited by the user if necessary.

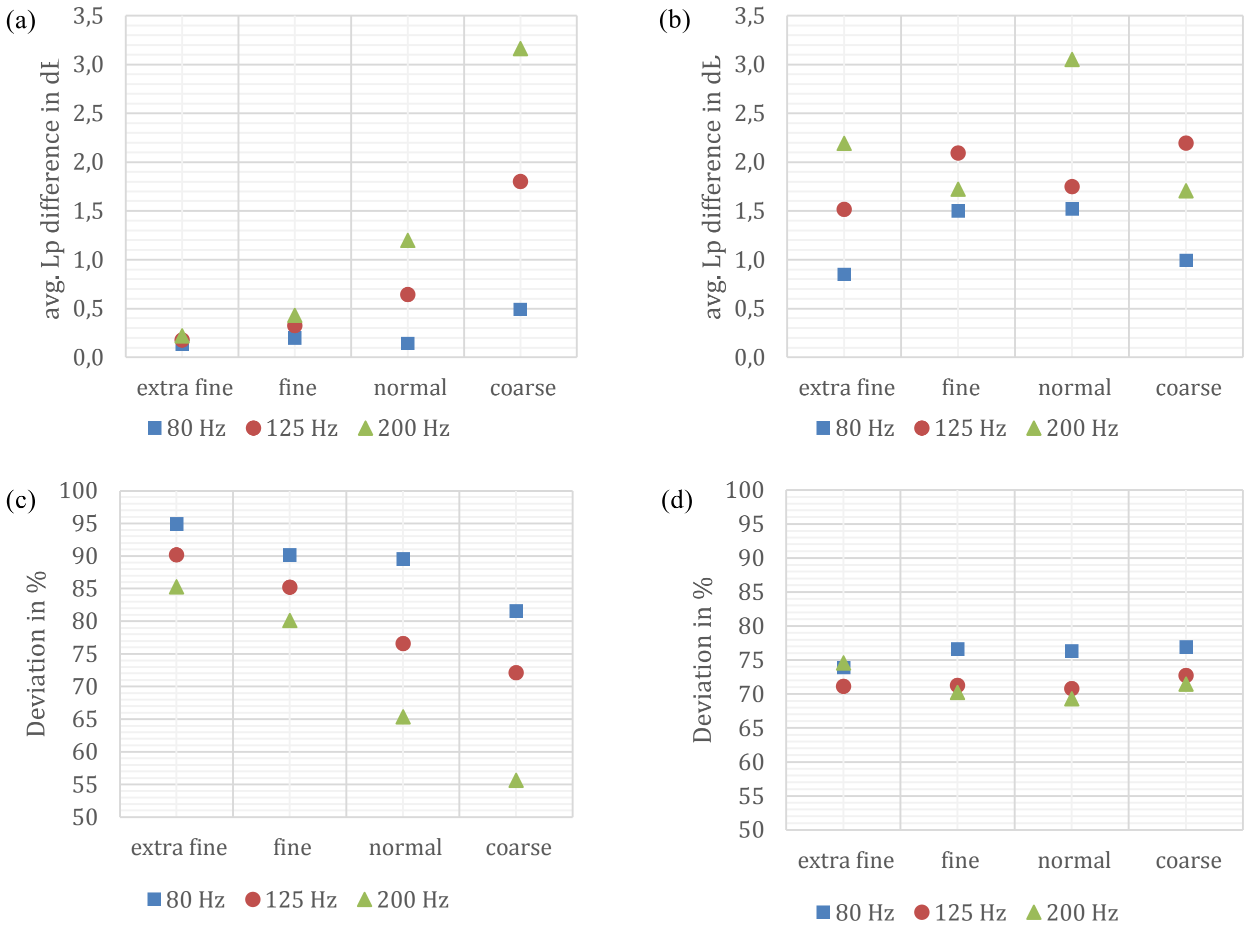


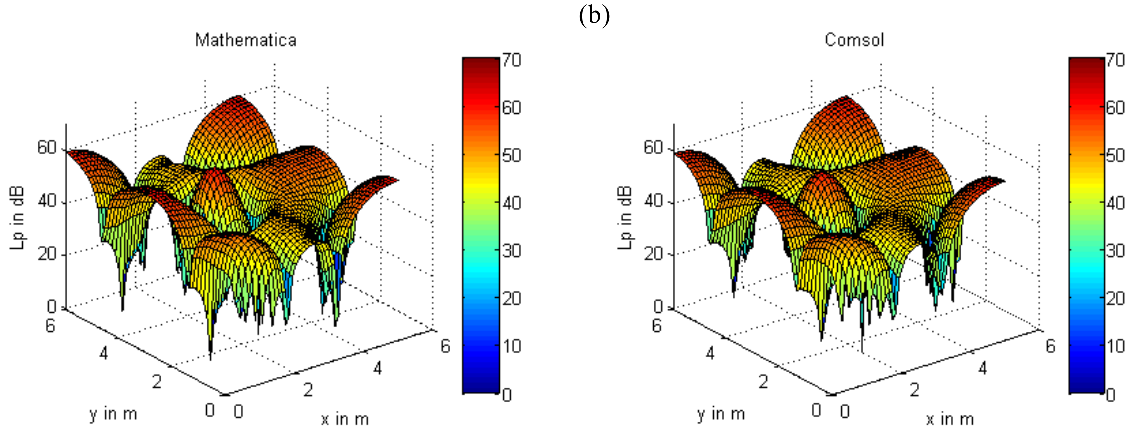
Figure 2. Illustration of the energy weighted average sound pressure level difference between the analytic solution and the FEM – simulations with different mesh densities in the sending room (a) and in the receiving room (b) Illustration of the average percentage of points in the FEM solution with less than 1 dB difference from the exact analytic solution in the sending room of all variations (c) and receiving room (d)

5. Results

With the described analytic model the sound pressure level field in the sending room and the receiving room is calculated and compared to the FEM-simulation resulting in a 0,05m XY - grid in 1m height. For Figure 2(a) and 2(b) ten random points are drawn out of the grid of the FEM and analytic solution and the difference of the two energy weighted average sound pressure level of those ten points is calculated for the different mesh densities in the FEM software environment. This was done for all variations of parameters described in chapter 3 and an average for all variants was generated and illustrated for the different mesh densities and source frequencies. These results depend on the ability of the FEM model to reproduce the analytic solutions but also strongly depend on the homogeneity of the sound pressure level fields.

To get a second reference about the reproducibility of the sound pressure fields by the FEM model a second indicator is needed. For one variant of room geometry, source position and wall type all sound pressure levels in the 0,05m grid calculated by the analytic model are compared to the sound pressure field simulated by the FEM model. Again an average of all deviations of all variants was formed. The Illustration 2(c) and (d) shows the average percentage of points which got a smaller deviation than 1dB between the two models for the different source frequencies and mesh densities. With this indicator an overall judgment about the fit between the two solutions is possible. Figure 3 shows an excerpt of the results in the parametric study and points out the comparison between the sound pressure field calculated by the analytic solution of equation (1) and the result of the FEM-simulation.

(a) (b)



(c)

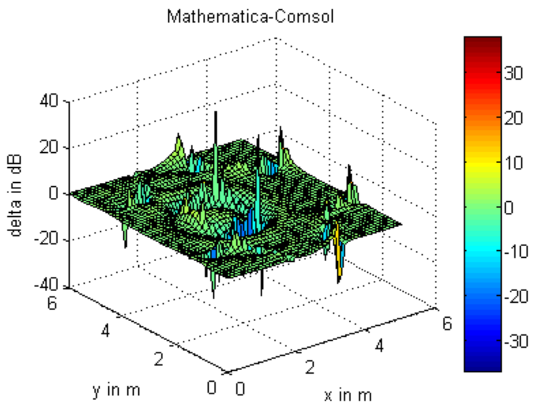


Figure 3. Example of comparison between the sound pressure level field in dB of the analytic solution (a) and the FEM simulation with extra fine mesh in COMSOL (b) of the variant Lx=5m, Ly=6m, ρ=800kg/m³, μ=0.4, x0=1m, y0=2.5m; z0=1.5m in the sending room at 125Hz source frequency; (c) shows the difference between the two sound pressure fields in dB

6. Conclusions

In general Figure 2 shows that the FEM-Simulation performs very well in pretty much all investigated frequencies, reproducing the sound pressure field in the sending room, when the mesh has a maximum element size smaller then 0.42m. Looking at the comparison of the results in the calculations of the sound pressure field in the receiving room in Figure 2 (b) and (c) there is obviously a bigger difference between both results, caused by the interpretation of the physics of structural deformation of the wall between the two models. The deviation in the average sound pressure level and in the amount of points with 1dB difference shows that the analytic solution and the FEM-simulation deliver non matching outcomes in the receiving room, although the difference in the average sound pressure level is for all variants smaller than 3 dB. The difference area in Figure 3 (c) shows that the FEM-Simulation is able to reproduce good the sound pressure field calculated by the analytic solution in the sending room excited by a point source. The Illustration shows that also complicated mode structures can be simulated with FEM with acceptable errors if the discretization and so the mesh density is fitted to physical problem. The big differences occur only in the mode vales were the gradient of the pressure is very high. In this narrow regions it is not possible to completely analyze the appeared differences because the errors of the analytic solution caused by grid or other approximations.

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