Acoustic properties of composite lightweight structures

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Summary
Lightweight structures find more and more applications in both vehicle and building engineering. To meet a growing demand, a variety of different types of double and sandwich panels have been developed during the last few decades. One of the problems to deal with is the assessment of the acoustic performances of such panels once they are already mounted in their final place. In this case, it can be of importance to find a way to characterise their dynamic and acoustic properties, such as bending stiffness, internal losses and sound transmission loss, through non destructive testing. In the following paper a method for a quick determination of the bending stiffness of a lightweight ribbed panel is presented. On the basis of the apparent bending stiffness and of the losses, it is possible to predict the transmission loss of the panel in a fairly simple way. The results obtained from the mobility tests have been compared to the measurements carried out in sound transmission rooms according to the ISO standard procedure. The model used for the post processing of the mobility data allows parameter studies of the sound transmission loss and of the sound radiation ratio for structures with different thicknesses of the laminates and the core once the main physical data are known.

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1. Introduction

The expression “sandwich panel” refers to a structure with a thick lightweight core with thin laminates bonded to each side of either a foam or honeycomb core. This type of plate combines low weight with high strength. However, for certain types of composite plates, the acoustic properties can be very poor. The absence of acoustic qualities can severely restrict the use of lightweight elements. It is therefore essential to optimize the acoustic properties of such structures through reasonable predictions. Some of the basic parameters of a sandwich structure can be determined by means of simple tests using a beam element of the structure, [1] and [2]. Using this method, some simple vibration measurements can determine a number of natural frequencies of the beam. Based on these results the apparent bending stiffness can easily be determined through a least mean square method applied to the general equation describing the dynamic characteristics of a composite material. The apparent bending stiffness of a composite structure at one of its natural frequencies is equal to the bending stiffness of a simple Euler beam having the same length, boundary conditions and weight as the sandwich structure at the same frequency. The procedure to determine the apparent bending stiffness of a sandwich structure is described in [1]. Obviously, this method presents some difficulties if applied to already mounted specimens, since it is not always possible to cut beams from a mounted structure.

In this paper, a method will be presented through which the material parameters can be determined from simple point mobility measurements on a plate element. In particular, this method has been applied to a kind of panel which cannot be strictly defined as sandwich, since it is made up of two external gypsum laminates and a thick ribbed wooden core. It will be shown that the technique allows to estimate the apparent bending stiffness also in this case, thus taking into account the real boundary conditions of the mounted panel. The technique can also be used for non isotropic panels.

The transmission loss results obtained from point mobility measurements will be compared to those found after the tests carried out in sound transmission rooms according to the existing ISO standards.
2. Bending stiffness from point mobility measurements

When an external harmonic force $F = F_0 \exp(i\omega t)$ is applied at a certain point of a dynamic system, it will vibrate with a certain velocity $v$. For this excitation point a mobility function $Y$ can be defined, as the complex ratio between the Fourier transform of velocity $v$ and the Fourier transform of the force $F$ measured at the same point:

$$Y(\omega) = \frac{\theta(\omega)}{F(\omega)}$$  (1)

The vibration behaviour of finite structures can be derived from that of infinite ones. In an infinite plate the bending waves excited by a point force can propagate indefinitely in the specimen. In a finite plate the same bending waves reach the boundaries of the plate, and are then reflected back. If a point force acts at a point of the plate, its velocity will depend upon the plate geometry and on the boundary conditions, thus the point mobility will change depending on the location and on frequency. However, a space and frequency average of the real part of the point mobility for a finite structure is in the mid and high frequency region equal to the real part of the point mobility of an infinite structure of the same material and thickness:

$$\text{Re}(\tilde{Y}(\omega)) = \text{Re}Y_{\infty}(\omega)$$  (2)

Consequently, the power input, injected in a finite panel by a force acting randomly in time and space, can be calculated as if the structure were infinite and excited by a point force with a power spectral density equal to the sum of the power spectral densities of all the sources acting on the finite structure.

This assertion is valid if the modal density within a band is independent of boundary conditions, which is true for the medium and high frequency bands. This means the exclusion of the first few modes corresponding to the low frequency ranges. In order to extend this assertion to the low frequency range, a certain number of modes have to be included within each frequency band. It can be shown that the number of modes within a band should be at least 5 in order to have a fair accuracy. For obtaining a space average $\text{Re}(\tilde{Y})$ of the mobility which is representative of the dynamic behaviour of the entire panel, the mobility must be measured over a sufficiently large number of points, randomly distributed over the surface of the panel.

Under these conditions the mobility of a finite structure can be predicted through the mathematical formulation of the mobility for an analogous infinite structure [2]. Following this analysis, the frequency average of $\text{Re}(\tilde{Y})$ can be written as [2]:

$$\text{Re}(\tilde{Y}) = \frac{1}{8} \frac{1}{D_p \mu}$$  (3)

where, $D_p$ and $\mu$ are the bending stiffness per unit width and the mass per unit area of the panel, respectively.

The bending stiffness per unit width of the panel at the central frequency of each band is therefore obtained as:

$$D_p = \frac{1}{64 \mu^2 [\text{Re}(\tilde{Y})]^2}$$  (4)

For modes $(m,n)$ having $m=0$ or $n=0$, and then for the first modes, it can be shown that [2]:

$$\text{Re}(\tilde{Y}) = \frac{2}{8} \frac{1}{D_p \mu}$$  (5)

Therefore, for the first natural frequencies corresponding to such modes, the measured mobility should be divided by a factor 2.

An important aspect for lightweight and homogeneous structures is the influence of the mass of the transducer used during the mobility measurements. As discussed in [2], the dynamic response of the structure, and then its modal behaviour, may be modified by the presence of the added mass of the transducer. The measured point mobility should then be corrected according to the following correlation:

$$Y_{\text{measured}} = \frac{Y}{1 + \gamma \Delta \mu / \omega}$$  (6)

In the low frequency region the effect of the added mass on the motion of the structure is negligible. For increasing frequencies the denominator tends to increase while the amplitude of the point mobility can decrease significantly.

This problem occurs when the mobility is measured by using an impact hammer and an accelerometer, while an impedance head is less sensitive.

3. Sound Transmission Loss

The forced response of a structure can, when predicted in third octave bands, be estimated fairly accurately by using the apparent bending stiffness of the structure [3].

The sound transmission loss $R$ for single leaf panels is discussed in [4], where it is evaluated as a function of the bending stiffness of the plate and of a number of other parameters.
The expressions presented in [4] can, after some modifications, also be used for sandwich plates. It is convenient to introduce the critical frequency \( f_c \), for which the wavenumber in air is equal to the wavenumber for flexural waves on the plate. The frequency \( f_c \) for which \( k_{plate} = k_{air} \) is given by

\[
f_c = \left( \frac{c}{2\pi} \right) \sqrt{\frac{k_{plate}^2}{k_{air}}} = \left( \frac{c^2}{2\pi} \right) \sqrt{\frac{\mu}{\rho_p}}
\]

(7)

where \( c \) is the speed of sound in air. For a thin single-leaf panel, \( f_c \) is a constant. For a sandwich plate the bending stiffness in equation 7 should be written as \( D_p(f_c) \). The critical frequency for a sandwich structure must be calculated by numerical methods.

The sound transmission loss \( R \) in decibels for a plate is \(-10\log(\tau_d)\), where \( \tau_d \) is the sound transmission coefficient for diffuse incidence, defined as

\[
\tau_d = 2 \int_0^{\pi/2} \tau(\varphi) \cdot \cos \varphi \sin \varphi \, d\varphi
\]

(8)

The transmission coefficient \( \tau(\varphi) \) at the angle of incidence \( \varphi \) is given by

\[
\tau(\varphi) = \frac{\left\{ 1 + \frac{\mu \omega}{2pc} \cos \varphi \cdot \left( \frac{f_c}{f_c'} \right)^2 \left( \sin \varphi \right)^4 \eta \right\}^2}{\left\{ 1 + \frac{\mu \omega}{2pc} \cos \varphi \cdot \left( \frac{f_c}{f_c'} \right)^2 \left( \sin \varphi \right)^4 \right\}^2 - 1}
\]

(9)

where \( pc \) is the wave impedance, equal to 415 kg/(m²s) in air and \( f_c \) is the critical frequency satisfying equation 7. For a homogeneous single leaf panel the bending stiffness is constant. However, for a sandwich structure the bending stiffness varies with frequency. The sound transmission loss of a sandwich structure can be derived by replacing \( f_c \) in equation 9 with a frequency dependent parameter \( f_c' \) where

\[
f_c' = \left( \frac{c^2}{2\pi} \right) \sqrt{\frac{\mu}{\rho_{apparent}(f)}}
\]

(10)

The sound transmission loss for the plates described hereafter was predicted according to equation 8 and equation 9 and compared to measurements.

4. Specimens under test

The investigated specimen is made up of three parts: two identical external laminates of gypsum fibreboard, 12.5 mm, and a wooden ribbed core. The ribs are 25 mm x 50 mm studs, thus the overall thickness of the panel is 75 mm.

The dimensions of the panel are 1.7 m x 1.1 m. Figure 1 shows a layout of the ribs position, together with the spacing between the ribs. The gypsum fibreboards and the ribs were kept together by screws and a thin layer of glue. The panel was tested in three configurations:

- Double panel filled with mineral wool (density of the mineral wool: 40 kg/m³);
- Double panel without mineral wool.

The mass per unit area of the two specimens are 33.15 kg/m² and 30.95 kg/m², respectively.

Figure 1. Layout of the ribs and main dimensions of the panel.

4.1. Test methods and materials

The panel was mounted in the opening dividing the reverberant rooms. The position of the panel mounted in the wall is shown in Figure 2.

Figure 2. Position of the opening in the dividing wall.

The short lateral side of the panel was sealed by placing mineral wool in the gap between the
wooden frame and the wall. To avoid sound leakage, silicon glue was used to seal the border, on both sides.

On one fibreboard panel, 20 point mobility measurement positions were spread on the external surface: 10 positions on the ribs and 10 positions in between them.

A PCB 288D01 impedance head was attached to a nylon stinger and then to a Bruel & Kjaer 4809 exciter fed with a white noise. The force and acceleration signals coming from the transducer were acquired by an OROS 36 multi-channel system able to compute directly the real and imaginary parts of the mobility function. The frequency span of the acquisition was selected from 0 to 6.4 kHz, 1 Hz resolution.

5. Point mobility measurements

The point mobility measurements were performed directly in situ, with the panel mounted in the opening between the two rooms. Figure 3 shows how the measurement positions were distributed across the panel surface.

![Figure 3. Measurement positions for the point mobility.](image)

The post processing of the data was carried out by exporting the text data from the OROS NVGate software. At this point it was possible to compute the average mobility for the 20 measurement positions, and to apply a correction for taking into account the weight of the transducer. Finally, the mobility value was computed starting from the corrected and averaged mobility synthesizing the values into extended 1/3 octave bands in order to have at least 5 modes inside the frequency span defined by each band. Once the average mobility is known, it is possible to compute the related bending stiffness and to use this value for determining the apparent bending stiffness through the least mean square method applied to a set of points \( f_n \), \( D_n \), and to the equation

\[
\frac{A}{f} D_x^{3/2} - \frac{B}{f} D_x^{1/2} + D_x - C = 0
\]

which describes the general behavior of the apparent bending stiffness \( D_x \) for a panel made up of two laminates separated by a core [1].

Since the modal density in the low frequency range is low, there is some lack of points for computing the bending stiffness. For this reason a fictitious bending stiffness point \( D_0 \) has been introduced in order to “guide” the curve in the very low frequency region. The static bending stiffness \( D_0 \) can be computed once some geometrical and material parameters are known through the equation:

\[
D_0 = \frac{E_t h_c h_l}{2}
\]

Where \( E_t \) is the Young’s modulus for one laminate, \( h_c \) is the core thickness and \( h_l \) is the thickness of one laminate.

5.1. Determination of the loss factor

Before starting with the computation of the sound transmission loss of the panels, it is necessary to determine the internal losses. The determination of the losses was made through the evaluation of the structural reverberation time.

A small accelerometer was placed in each one of the points used for the measurement of the mobility, and then the panel was hit by means of an impact hammer very close to the transducer. The resulting impulse response was then post processed determining the decay for each 1/3 octave band of interest. Then it was possible to compute the losses through the formula:

\[
\eta_0 = \frac{6}{2\pi f_0 T_R \log_{10} e} \approx \frac{2.2}{f_0 T_R}
\]

where \( f_0 \) is the central frequency in hertz of the 1/3 octave band of interest and \( T_R \) is the measured reverberation time in seconds for each frequency band. The decay for the different frequency bands was obtained by post processing the impulse response signals by a very short exponential averaging and a multi spectrum technique. Then the resulting decays, computed for the frequency
bands of interest, were further postprocessed in order to obtain the losses as a function of frequency (Figure 4).

Figure 4. Computed losses for the two panels tested (black – two laminates and wool inside, grey – two laminates and no wool inside).

6. Transmission loss computed from mobility measurements

Once the losses have been determined and the apparent bending stiffness derived from the mobility measurements, it is possible to compute the sound transmission loss of a panel. The computation is carried out according to the theory described in the previous sections.

Starting from the dimensions of the panel, its mass for unit area and the mobility measurements, the bending stiffness is computed in the frequency span 1 Hz – 5 kHz. Once the bending stiffness is computed, the critical frequency \( f_c \) can be easily determined and the radiation losses calculated. The losses determined through the use of the reverberation time method are then added to the previously determined radiation losses.

When these data are available, it is possible to compute also the sound transmission loss of the panel taking into account the limitation on the angle of incidence due to its finite dimensions according to the work of Davy [5] and the following formula:

\[
\theta_{lim} = \cos^{-1}\left(\frac{\lambda}{2\pi j L_x L_y}\right) \tag{14}
\]

where \( \lambda \) is the wavelength in air, \( L_x \) and \( L_y \) the main dimensions of the panel.

Finally, the sound transmission loss and the single rating value are computed. In Figure 5 the computed sound transmission loss for the panels tested is shown. The dashed curves correspond to the computed sound transmission loss.

7. Measurements in sound transmission rooms

After the mobility measurements were performed on the panels, the specimens were tested in sound transmission rooms according to the international standards in order to determine their sound transmission loss. Two source positions were used so to have a good average of the sound fields and ten sound pressure level measurements were performed for both the source and the receiving rooms. The difference between the average sound pressure level of the source room \( (L_{SR}) \) and the average sound pressure level of the receiving room \( (L_{RR}) \) was weighted for the size of the partition and the sound absorption area of the receiving room to compute the transmission room according to the formula:

\[
TL = L_{SR} - L_{RR} + 10 \log_{10}\left(\frac{s}{A_{RR}}\right) \tag{15}
\]

Figure 5 shows the comparison between the sound transmission loss measured using the procedure given by the standard and the transmission loss resulting from the mobility measurements. The two cases considered are:

- \( \Delta \): double panel with studs frame no mineral wool inside (solid: sound transmission rooms, dotted: simulation);
- \( \circ \): double panel with studs frame mineral wool inside (solid: sound transmission rooms, dotted: simulation);

Figure 5. Measured (solid) vs predicted (dashed) sound transmission loss for the two kinds of panels tested.

7.1. Baffle effect

The measured sound transmission losses of a panel mounted in transmission room with and without a frame or baffle are different. In particular this is the case in the low frequency region as
demonstrated in [2] and [6]. Using a correction for the baffle effect the resulting predicted sound transmission loss for the double gypsum panel with mineral wool is shown in Figure 6, together with the experimental data obtained in the sound transmission rooms for the same panel. The prediction of the transmission loss follow the procedures given in [2]. The theoretical model described in [2] does not include any limiting angle for incident waves discussed in Section 6.

The technique described presents a very simple procedure for estimating the sound transmission loss of a double wall structure. The input data required for the prediction are dimensions and weight of panel, frequency and space average of point mobility measured on panel. In addition the losses of the panel should be determined.

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References