Uncertainty estimation in multi-port measurements

Stefan Sack and Mats Åbom
Department of Aeronautical and Vehicle engineering, Royal Institute of Technology, Teknikringen 8, SE-100 44 Stockholm, Sweden

Hervé Denayer, Wim De Roeck and Wim Desmet
Department of Mechanical Engineering, KU Leuven
Celestijnenlaan 300, 3001 Leuven, Belgium

Summary

The generation and scattering behaviour of fluid machines in connected duct or pipe systems is of great interest to minimize disturbing and harmful sound emission, for instance of air condition systems. Within the framework of the European project 'IdealVent' the acoustic behaviour of air conditioner systems in aircraft is investigated in detail in order to develop strategies to abate sound emission and hence augmenting safety and comfort within the airplane and at the aircraft ramp. One approach to handle such systems is to apply a linear multi-port model that includes direction-depending transmission and reflection coefficients for the propagating wave modes. Those parameters may be ascertained either numerically or experimentally. Once this characteristic data are determined for all elements of interest, the sound scattering and emission behaviour of every considerable combination of those elements can be calculated. In order to determine the system scattering, a number of external sound fields dominating the existing sound field are applied. This operation consists of matrix inversions and wave-number assumptions, which amplify uncertainties induced by the measurement procedure. The paper in hand shows a method to estimate the uncertainties for a multi-port of the order 6 by applying a condition number approach and a Monte-Carlo simulation.

PACS no. xx.xx.Nn, xx.xx.Nn

1. Introduction

The cabin comfort of civil aircraft is essentially affected by the noise emitted from the ducts and pipes in the air-conditioning systems. A high level of air-conditioner noise causes stress symptoms and tiredness for both, the passenger and the crew and has detrimental effects on their safety. However, the Environmental Control System (ECS) is important to adjust temperature and air quality and is hence indispensable in modern aircraft.

The mechanisms of noise scattering and generation in the ECS are complex and usually involve high Helmholz-numbers, mainly due to the large radius of the duct-elements, which triggers acoustic modes of higher order even within the perceptible frequency range.

Within the framework of the European project 'Idealvent' we investigate the generation and scattering of (higher order) acoustic modes in an ECS in detail. We determine the scattering behaviour and the source strength of common elements in ECS, e.g. ducts, fans, orifices, valves, and bends. Therefore, we apply a linear, time invariant multi-port model and ascertain its parameter either numerically or experimentally.

The measurement approach has been presented in previous work [8, 10]. However, the quality of the measurement results is strongly influenced by the measurement setup, namely by the sensor and source placing. The literature shows investigations on the two-microphone method [1], but established approaches are difficult to apply into the test-rig design for higher order multi-ports. The paper in hand shows therefore a method to first estimate the quality of a test-rig setup utilizing the condition number, and secondly to compute the uncertainty propagation through the scattering calculation using a Monte-Carlo simulation. Schultz et al. did similar investigations, but again only on the two microphone method. They showed, however, that Monte-Carlo simulations are better-suited for uncertainty investigations on multi-port measurements than linear multivariate uncertainty analysis, mainly due to non-linear uncertainty propagation within the modal decomposition [11]. We finally compare the theoretical results with measurements taken on an empty duct, where we know the analytical solution [4].

2. Theory

2.1. Multi-port model

We consider an acoustic element connected to a straight duct (figure 1). We might describe it as a linear time-invariant multi-port

$$\mathbf{p}_{+}(\omega) = \underline{\underline{\mathbf{S}}}^{\mathbf{s}}\mathbf{p}_{-}(\omega) + \mathbf{p}^{\mathbf{s}}_{+}(\omega) \quad , \tag{1}$$

where $\underline{\underline{\mathbf{S}}}^{\mathbf{s}}$ denotes the scattering matrix and $\mathbf{p}_{-}$, $\mathbf{p}_{+}$ are vectors containing the pressure amplitudes of the propagating modes in the +/- direction, and ω is the signal frequency [8]. For the investigations in this paper, we are only interested in scattering part of the multi-port. Let an external, uncorrelated sound source dominate the acoustic field. The source vector $\mathbf{p}^{\mathbf{s}}_{+}(\omega)$ then vanishes in equation 1 and the scattering matrix can be computed [7]. In [8] an approach to decompose the sound field from measurement data is given. As $\mathbf{p}_{+}$ and $\mathbf{p}_{-}$ include the spectra of all acoustic modes of the N-port in both propagating directions, the spatial sampling of the sound field must be performed by measuring the acoustic pressure for at least 4 N spatial positions. To ascertain the scattering matrix, a set of 2 N independent sound fields has to be determined. To determine these propagating wave amplitudes, a wave decomposition is performed using an array of pressure sensors. For a sound field measured at different spatial positions, a set of equations can be written in matrix notation

$$\mathbf{p}(\omega) = \underline{\underline{\mathbf{M}}}(\omega) \begin{bmatrix} \mathbf{p}_{+}(\omega) \\ \mathbf{p}_{-}(\omega) \end{bmatrix} \quad , \tag{2}$$

where $\mathbf{p}(\omega)$ is a row vector containing the spectra of all spatial measurement points and, for a set of n vector positions, $\mathbf{M}$ is the $[\mathrm{n} \times 2\,\mathrm{N}]$ projected modal matrix containing eigenfunctions and exponential propagation factors for the modes. Solving equation 2 for a set of at least 2 N independent sound fields for each side of the multi-port, we get the needed input for equation 1. In the next step we solve equation 1 by means of the Moore-Penrose pseudoinverse

$$\underline{\underline{\mathbf{S}}}^{s} = \underline{\underline{\mathbf{p}}}_{+}\underline{\underline{\mathbf{p}}}_{-}^{-1} \quad , \tag{3}$$

where $\underline{\underline{\mathbf{p}}}_{+}$ and $\underline{\underline{\mathbf{p}}}_{-}$ contain the decomposed acoustic modes as column-vectors.

2.2. The condition number of the sensor- and source matrix

Due to singularities within the matrix inversion of the projected modal matrix $\underline{\underline{\mathbf{M}}}$ and the incident matrix $\underline{\underline{\mathbf{p}}}_{-}$, we easily arrive at disturbed measurements caused by axial mode coupling (weak coupling) or azimuthal mode coupling (strong coupling) [10]. A good indicator for such disturbances is the condition number of the projected modal matrix which we calculate as a function of frequency and then use its values as a fist estimator of stability and for the uncertainty propagation through the modal decomposition. This approach is advantageous as those calculations are commonly fast and can hence be included directly into the test-rig design and optimization of the sensor array. The matrix of incident waves can be treated in the same way. But the sound fields have to be computed, e.g. with a Green's function approach [3, 6], where we assume an array of loudspeakers, modelled with point sources

$$\mathbf{p}(\mathbf{z}, \boldsymbol{\phi} \mid \mathrm{z}^{\mathrm{s}}, \phi^{\mathrm{s}}) = -\mathrm{j}\omega\rho\mathbf{g}^{\mathrm{s}}(\mathbf{z}, \boldsymbol{\phi} \mid \mathrm{z}^{\mathrm{s}}, \phi^{\mathrm{s}}) \quad , \tag{4}$$

with $(\mathbf{z}, \boldsymbol{\phi})$ denoting the sensor positions, $(\mathrm{z}^{\mathrm{s}}, \phi^{\mathrm{s}})$ are the coordinate of the source. We have to decompose these sound fields in order to calculate the condition number of the incident waves $\underline{\underline{\mathbf{p}}}_{-}$. Hence, we cannot come to a conclusion only for the source matrix, as we always arrive at a mixed source-sensor solution and the result will depend both on the sensor (microphone) and source (loudspeaker) array used.

2.3. Uncertainty analysis

There are commonly two ways to carry out uncertainty analysis on measurement data or analytical models. A Taylor Series Method (TSM), which includes only first order terms from a Taylor series expansion but gives an analytic solution which can be assessed quickly or a Monte Carlo Method (MCM) that uses a statistical approach to also describe higher-order terms; the latter might be used to describe uncertainty propagation in highly sophisticated systems but occasionally requires extensive computation [12]. However, Schultz et al. showed for the two microphone method, that MCM treat uncertainty found in common engineering applications with higher precision than TSM [11]. As the equations describing the two microphone method can be solved analytically, a TSM was reasonable for this study. However, for the ahigher number of microphones or over-determined system matrices that are solved in means of the pseudo-inverse, the mathematical equations for the general N-port case become too complex to be treated with TSM. For the paper in hand we hence applied an MCM into the wave decomposition and scattering calculation in order to investigate their uncertainty propagation. A main focus is on the uncertainties in the pressure values acquired with the microphone sensors. We assume uncorrelated uncertainties for amplitude and phase, following [9].

For MCM based on calculations, one could use a normal distribution and a fixed standard deviation for all frequencies and microphones. However, this approach

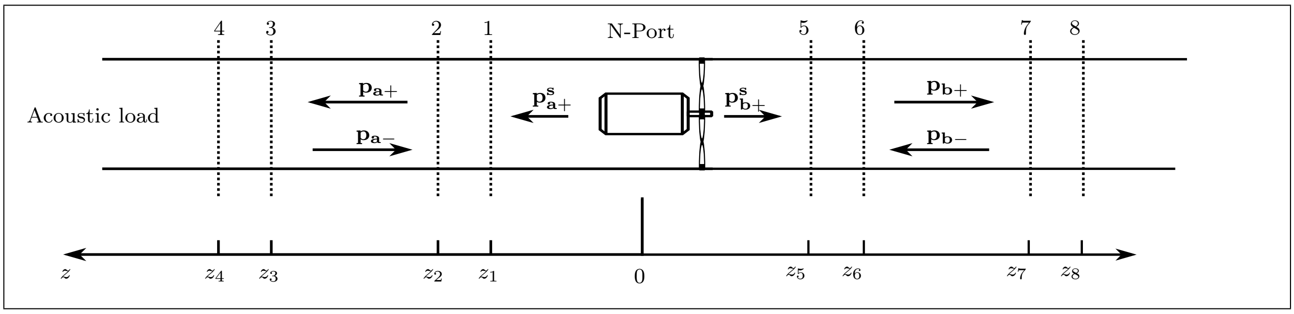

Figure 1. Sketch of an acoustic multi-port. The source vector $\mathbf{p}^s_+$ contains the acoustic modes evoked by the source. The sound waves created by the acoustic load are reflected and transmitted at the multi-port. The indexes $_+$ and $_-$ denote the direction of the propagation. The dotted lines indicate the microphone sections.

is not valuable for the evaluation of real measurement data, since the uncertainties in the microphone signals vary due to their position, e.g. close to pressure nodes. We calculate the standard deviation in the sensor pressure by means of the coherence function γ_{lm} between the microphones l and loudspeakers m, based on [9]

$$\sigma_m = \frac{(1-\gamma_{lm}^2)^{1/2}}{|\gamma_{lm}|\sqrt{2n}} \tag{5}$$

where n is the number of averages used to calculate the transfer function.
The uncertainties in temperature and mean flow velocity can be ascertained in various ways. One could use general information provided by the sensor manufacturer or evaluate time-samples of the sensor values under measurement conditions. For the paper in hand, we use a mixed method, were we compare uncertainties obtained from time samples with the full wave decomposition described in [2], where we fit the wave number for all plane wave frequencies to estimate the variation in the Mach-number values.

3. Measurements

3.1. Measurement setup

The test-rig used in this paper was designed to meet the requirements which were mainly imposed by the properties of the test objects, namely the blade passing frequency of an axial fan (2800 Hz). The exact frequency range is defined in the 'IdealVent' project description to be from 600 Hz to 3500 Hz, which contains a total of maximal six propagating modes. Table I shows the properties of the test rig. According to equation 1, we require a total number of 12 sensors and 6 sound sources to perform the full multi-port characterization. However, it was shown that an over-determination in sources and sensors may significantly improve the measurement results [5, 10]. For the paper in hand we realize a source over-determination of 6 and a sensor over-determination of 6, which results in a total number of 6 sources and 9 sensors up- and downstream. As sensors, we flush-mount 6 BK 4938-A11 and 12 PCB 378C10 high pressure microphones to the channel walls.

Table I. Properties of the test rig, the medium is air.

Frequency range	600 Hz - 3500 Hz
Cut-on frequency	2363 Hz
Diameter	84 mm
Center-line velocity	32 ms
Temperature	18 C

In order to solve equation 1, two matrix inversions have to be performed. Those matrices however, which are defined by the sensor and source positions, become easily singular for certain frequencies. Therefore, the microphone and loudspeaker sections are optimized according to the procedures described in [10]. Figure 2 shows the condition number of the projected modal matrix (solid line) and the source matrix (dashed line). A peak only appears at the cut-on frequency of the (1,0)-mode. The mean condition number for the source and the sensor matrix is close to one for both, the source and the sensor matrix. However, due to the decreasing sensor and source over-determination for (1,0)-mode, the condition number increases, which indicates an augmenting sensitivity for measurement uncertainties. The condition number of the source array is slightly higher than the condition number of the sensor array after the cut-on.

3.2. Measurement procedure

To induce the external sound fields, we excite the loudspeaker sources with stepped-sine signals, whereas we determined the necessary sample time for each frequency point utilizing the coherence between loudspeakers and microphones in previous test drives, see equation 5. We measure the temperature with a thermocouple module and determine the center-line velocity with a Schiltknecht MiniAir20 turbine flow meter.

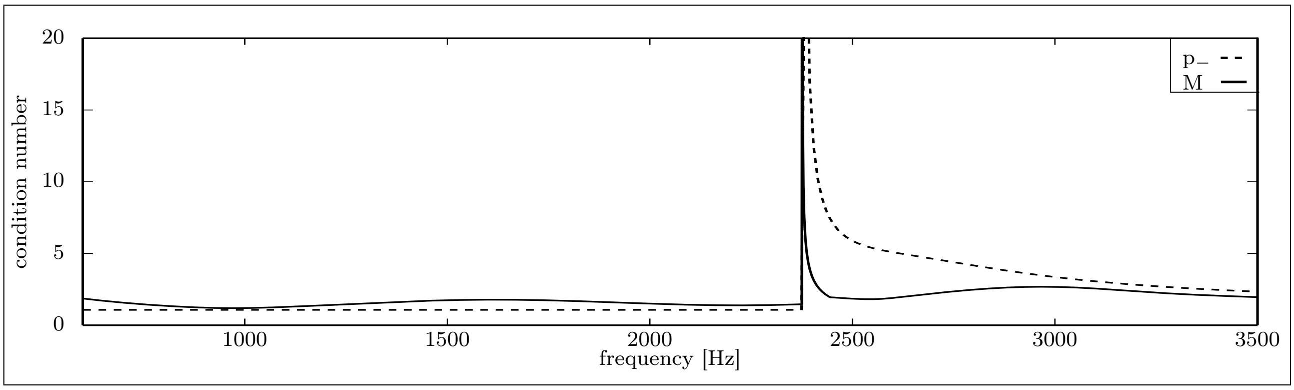

Figure 2. Condition number of the projected modal matrix and the source matrix over frequency.

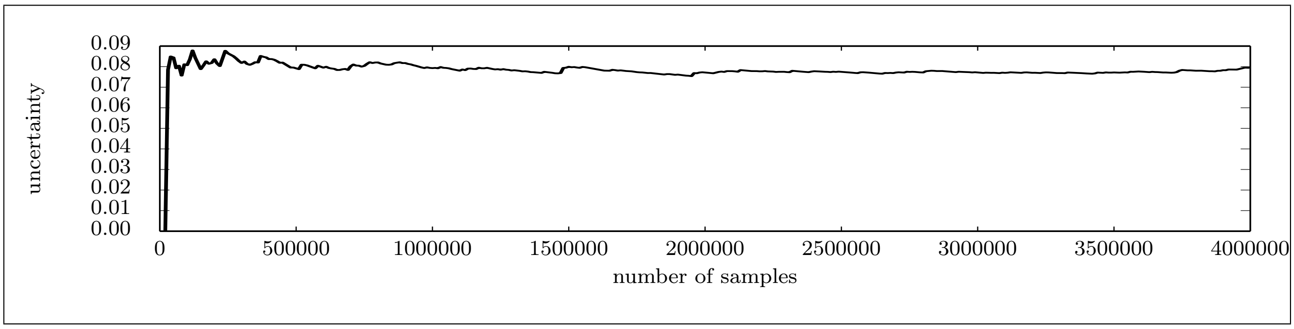

Figure 3. Convergence study of the MCM at 3000 Hz. The deviation in the results of the MCM is strongly influenced by the number of calculated samples. The method approaches convergence at around 2 million calculations.

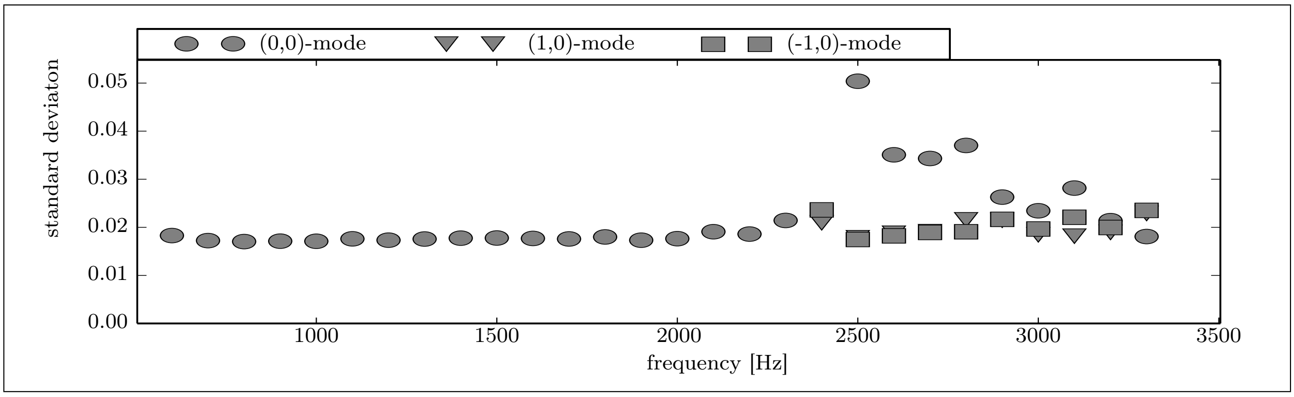

Figure 4. Simulated uncertainties (Green's-function approach and MCM.)

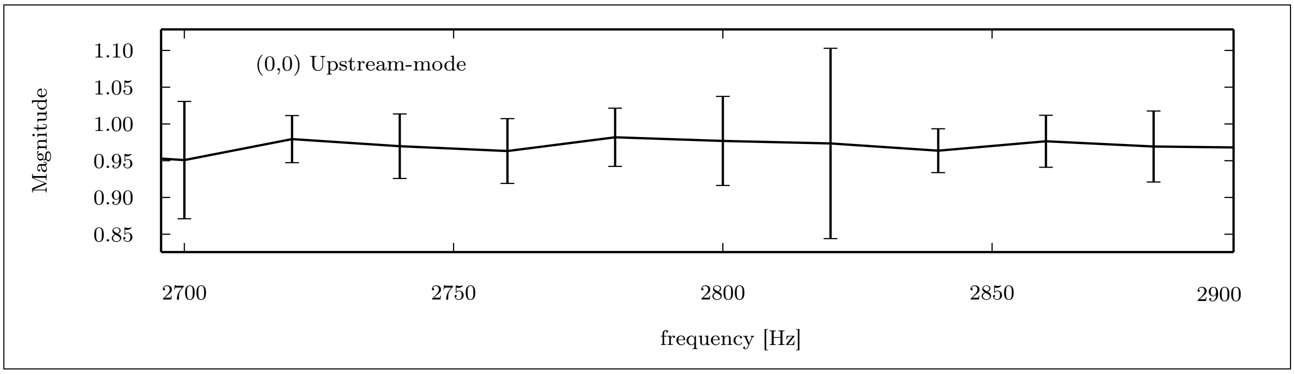

Figure 5. Transmission of the (0,0)-mode with error bars. The uncertainties show a smooth curve for most frequencies, but there are jumps at 2700 Hz and 2820 Hz.

Table II. Results from the wave-number fitting. v in [m/s] and t in [K]

	t measured	**t fitted**	
600 Hz,	295.15	292,65	
1000 Hz	295.15	292.85	
	v measured	**v fitted**	$\mathbf{v_m/v_c}$
600 Hz	33.1	24.2	0.77
1000 Hz	33.1	25.5	0.73

3.3. Results

During the testrig design, we calculated the condition number of the source matrix and the sensor matrix (figure 2). We furthermore investigated the theoretical distribution of the uncertainties in the scattered modes. We applied a MCM into equation 3. The sound-fields were calculated with virtual sources in equation 4 and decomposed solving equation 2. The data from the MCM for pressure uncertainties of 5 % is shown in figure 4. The results showed stable uncertainties until the cut-on frequency of the (1,0) mode. Beyond the cut-on, the (0,0)-mode was more sensible for uncertainties than the (1,0)-mode.
We measured the scattering matrix of the empty duct to demonstrate the uncertainty analysis with the MCM on measurement data. The transfer functions between the microphones and the loudspeakers and their coherences were ascertained. Using equation 5, we estimated the uncertainties in the pressure values. The data showed increasing uncertainties for higher frequencies. We used the full wave decomposition to compute the temperature and mean flow velocity, which showed a good agreement with the measured values (table II) and low standard deviations for different frequencies. For the MCM investigation, we used a temperature of (21.3±1.5) °C and a mean flow velocity of (25±1) m/s.
In a first step, we performed a convergence study on the measured data at 3000 Hz for the MCM. We found from figure 3, that the MCM converges after 2 million samples. The results of the scattering calculation and the MCM can be seen in figure 6. We obtained very small uncertainties for the (0,0)-mode up to the cut-on of the (1,0)-mode (less than 1 % in the 95 % confidence interval). The uncertainties increased close to the cut-on and beyond. To investigate the smoothness of the obtained values, we increased the resolution of the calculations between 2700 Hz and 2900 Hz. We found a smooth curve for those frequencies (figure 5), even though we obtained uncertainty jumps at 2700 Hz, 2820 Hz and 2900 Hz. In general, the uncertainties increased with the frequency. We furthermore found agreement with the calculated data, where the (0,0) mode was more sensible to uncertainties than the (1,0)-mode (by a factor of 1,5 - 2).

4. CONCLUSIONS

In the present paper we applied a Monte Carlo Method into the scattering calculation of an acoustic multi-port. We showed, that a theoretical model with virtual sources can be used to qualitatively predict uncertainty propagation through a certain measurement setup. The actual uncertainty of a measurement has to be calculated as a part of the post processing. We used coherence functions to calculate the pressure perturbations, which were then used as an input for the MCM. The investigation resulted in smooth uncertainty curves for most frequencies.

5. Acknowledgement

The project has been funded by the European Research and Technological Development Project 'IdealVent', part of the 7th Framework Programme. We should like to acknowledge Christophe Schram, who is coordinating the project. The research of Hervé Denayer is funded by a fellowship of the Agency for Innovation by Science and Technology in Flanders (IWT).

References

[1] M. Åbom and H. Boden. Error analysis of two-microphone measurements in ducts with flow. *The Journal of the Acoustical Society of America*, 83:2429, 1988.

[2] S. Allam, M. Åbom, and H. Bodén. Over-determiantion in acoustic two-port data. *ICSV*, 2006.

[3] I. Davis. *Experimental investigations in turbomachinery noise-source identification.* PhD thesis, Trinity College, Dublin, 2013.

[4] M. E. Goldstein. *Aeroacoustics.* 1976.

[5] A. Holmberg, M. Åbom, and H. Bodén. Accurate experimental two-port analysis of flow generated sound. *Journal of Sound and Vibration*, 330(26):6336–6354, Dec. 2011.

[6] G. J. Bennett, C. J. O'Reilly, and H. Liu. Modelling multi-modal sound transmission from point sources in ducts with flow using a wave-based method. *ICSV*, (July):5–9, 2009.

[7] J. Lavrentjev and M. Åbom. A Measurement Method for Determining the Source Data of Acoustic Two-Port Sources, 1995.

[8] J. Lavrentjev and M. Åbom. Characterization of Fluid Machines as acoustic multi-port Sources. *Journal of Sound and Vibration*, 197:1–16, 1996.

[9] J. S. Bendat and A. G. Piersol. *Random Data.* Wiley, 4. edition, 2010.

[10] S. Sack, M. Åbom, C. Schramm, and K. Kucukcoskun. Generation and scattering of acoustic modes in ducts with flow. *AIAA*, pages 1–11, 2014.

[11] T. Schultz, M. Sheplak, and L. N. Cattafesta. Uncertainty analysis of the two-microphone method. *Journal of Sound and Vibration*, 304(1-2):91–109, July 2007.

[12] H. W. Coleman and G. W. Steele. *Experimentation, Validation, and Uncertainty Analysis for Engineers.* Wiley, 3 edition, 2009.

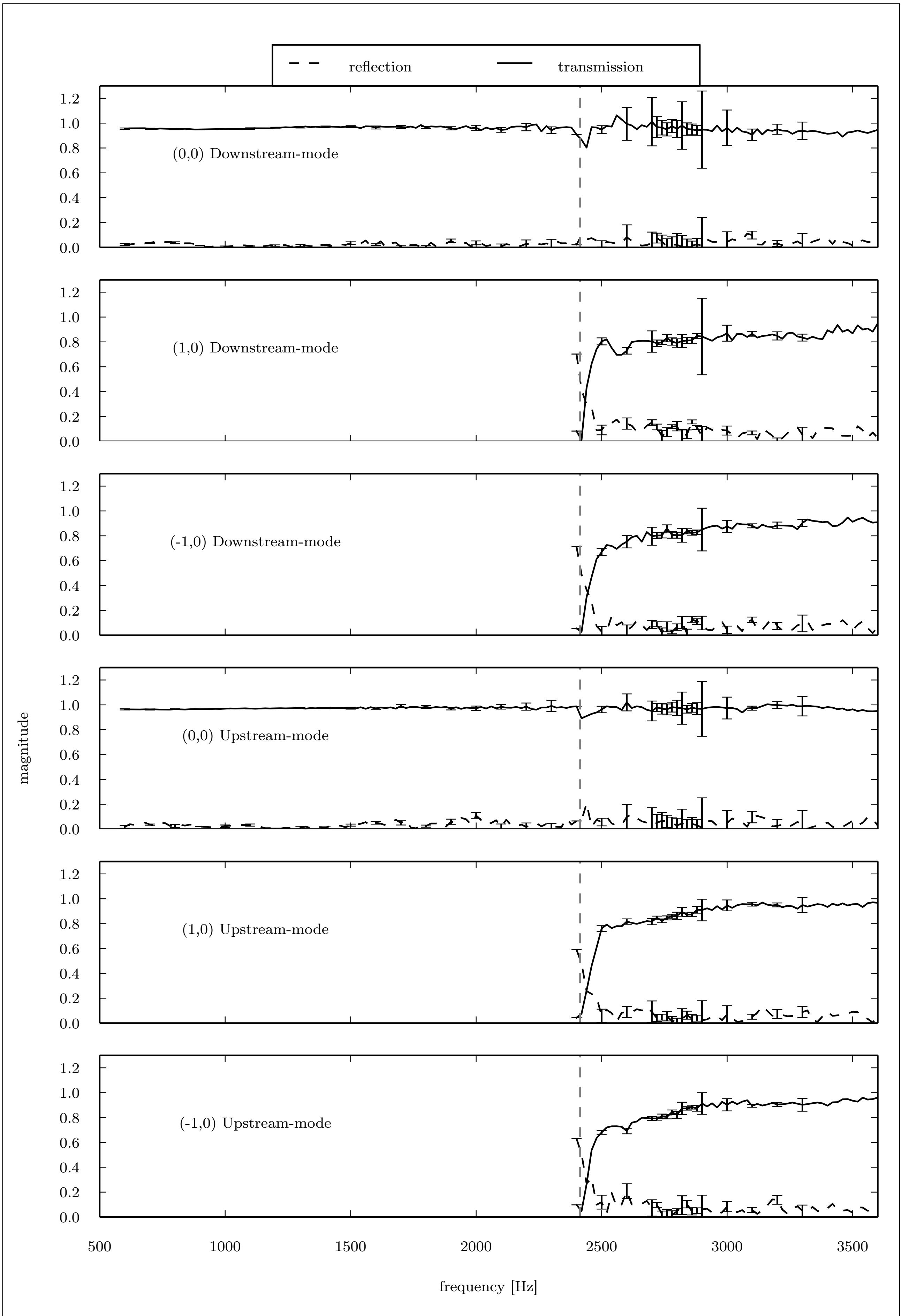

Figure 6. Transmission and reflection of all modes with error bars (higher resolution between 2700 Hz and 2900 Hz).