Outdoor ground impedance models

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Summary
Time domain calculations of outdoor sound propagation are increasingly popular but require ground impedance models to be physically admissible. A single-parameter (effective flow resistivity) semi-empirical equivalent fluid power law model (the Delany and Bazley model), and its modification by Miki are used widely to represent frequency-dependent soil impedance. A three-parameter version of the Miki model introduces additional parameters of porosity and tortuosity. A second modification has adjusted coefficients in the Miki model in an attempt to satisfy requirements for physical admissibility in the time domain. However, it is shown that, as is the case with the Delany and Bazley model on which they are based, the various forms of the Miki model lead to non-physical predictions of the real part of complex effective density and the surface impedance of a hard-backed layer at low frequency. Several rigid-frame porous material models including the Zwikker and Kosten phenomenological model, a variable porosity model, the Hamet and Berengier model, the Wilson model and a slit pore microstructural model are physically admissible and, using classical expressions for the field due to a point source over an impedance plane, enable better fits to short-range measurements of level difference spectra between vertically-separated microphones over many outdoor ground surfaces than obtained by using the single-parameter impedance models. Models that require porosity and tortuosity as well as flow resistivity can be simplified to two-parameter forms by assuming an inverse power law relationship between tortuosity and porosity. Although several models enable acceptably accurate predictions for high flow resistivity grassland surfaces, the two parameter variable porosity impedance model enables the best fits to the short range data. Single-parameter semi-empirical models result in significantly poorer predictions of short-range propagation over relatively low flow resistivity ground surfaces including railway ballast, gravel and forest floors than obtained using other impedance models. Estimates are made of the potential discrepancies in predictions of outdoor sound propagation at longer ranges that could result from use of single parameter models.

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1. Introduction

Often ground effects associated with the interference between sound travelling directly to a receiver and sound arriving at the receiver after being reflected at the ground are important in predicting the propagation of outdoor sound. According to ISO 9613-2 [1], any ground surface of low porosity may be considered acoustically-hard and any grass-, tree-, or potentially vegetation- covered ground is to be considered acoustically-soft. Although this might be an adequate representation in some circumstances, it is an oversimplification of the considerable range of properties and resulting effects. For example, different types of 'grassland' can yield significantly different ground effects. This has been recognized in recent prediction schemes [2,3] and by the development of standard methods for deducing ground impedance from short-range propagation measurements[4,5]. For typical noise predictions such as for noise from surface transport, the choice of impedance models can be confined to those representing the acoustical properties of air-filled porous materials with rigid frames. It is common to deduce parameter values for impedance models by fitting short range level difference spectra using 'template' methods [4,5]. Subsequently these models and parameter values can be employed in prediction schemes. As a result of its relative simplicity, a one parameter semi-empirical model [6] has been used widely for outdoor sound prediction. However there are many other impedance models for the acoustical properties of rigid-porous materials. Some of these models for arbitrary microstructures require values or estimates of many parameters, such as pore shape factors, characteristic lengths and pore size distributions, which are typically unknown for outdoor ground surfaces. Other models [7,8], require only three parameters but these can be shown to relate to low frequency/high flow resistivity approximations of models that assume identical tortuous pores [9]. By using a relationship between tortuosity and porosity for spheres, the required number of adjustable parameters can be reduced to two viz. flow resistivity and porosity.

2. Physical admissibility

The semi-empirical model for the acoustical properties of fibrous sound absorbing materials developed by Delany and Bazley, based on a single parameter (effective flow resistivity) regression fits to impedance tube data for a wide selection of fibrous materials [6]. While this model has the advantage of using only a single parameter (effective flow resistivity) to predict the acoustical properties of ground surfaces, it has the known disadvantages that (a) it predicts non-physical, i.e. negative, values for the real part of the surface impedance of a layer when extrapolated to low frequencies, and (b), for a porous material with known flow resistivity, it overestimates of the imaginary part of the propagation constant and porous material [10]. To better reproduce Delany and Bazley's original measured data at low frequency and avoid non physical predictions for the surface impedance of a layer of fibrous material both Miki [11,12] and Komatsu [13] modified Delany and Bazley's equations. Miki [11] changed the regression coefficients but retained the structure of the equations. Komatsu [13], while keeping the basic parameters the same, changed the fundamental structure also. Dragna and Blanc-Benon [14] have proposed modifications to the Miki model [11] to make it physically admissible for use in time domain calculations of outdoor propagation. Henceforth their result is termed the 'modified Miki' model. The Delany and Bazley [6] and Miki [10] expressions for characteristic impedance ($Z = R + iX$) and propagation constant ($k = \beta + i\alpha$) (assuming $\exp(+i\omega t)$ time dependence) have the form:

$$R = \rho_0 c_0 \left\{1 + a\left(\frac{f}{\sigma}\right)^b\right\}, \quad (1)$$

$$X = -\rho_0 c_0 \left\{c\left(\frac{f}{\sigma}\right)^d\right\} \quad (2)$$

$$\alpha = \frac{\omega}{c_0} p \left(\frac{f}{\sigma}\right)^q \quad (3)$$

$$\beta = \frac{\omega}{c_0}\left\{1 + r\left(\frac{f}{\sigma}\right)^s\right\} \quad (4)$$

where $\omega = 2\pi f$ represents angular frequency (rad/s), ρ_0 and c_0 are the density and adiabatic sound speed in air, σ is the flow resistivity (Pa s m^{-2}) and a, b, c, d, p, q, r and s are constant coefficients deduced from fitting a large body of impedance tube data for fibrous materials.

After comparing his original model with low frequency/high flow resistivity forms of the identical capillary pore model for rigid porous media [10], Miki proposed a three-parameter form [12] in which the right-hand sides of equations (1) and (2) are multiplied by $\sqrt{T/\Omega}$ and those of equations (3) and (4) are multiplied by $\sqrt{T}$, where T is tortuosity and Ω is porosity but the coefficients in Eqs. (1) to (4) are the same as in Miki's original model [11]. Miki [12] suggested that the three-parameter version of his model could be used to represent outdoor ground impedance.

The values of the coefficients a, b, c, d, p, q, r and s for the Delany and Bazley [6], three-parameter Miki [12] and modified Miki [6] models are listed in Table 1. The impedance of a hard-backed porous layer of thickness L can be deduced from equations (1) - (4) and Table 1 using:

$$Z(L) = Z\coth(ikL) \quad (5).$$

Table 1 Coefficient values in the Delany and Bazley, three-parameter Miki and modified Miki models

Model/ coefficient	*a*	*b*	*c*	*d*
Delany and Bazley	0.0497	−0.754	0.0758	−0.732
Three-parameter Miki	0.070	−0.632	0.107	−0.632
Modified Miki	0.251	−0.632	0.384	−0.632
	p	***q***	***r***	***s***
Delany and Bazley	0.169	−0.595	0.0858	−0.700
Three-parameter Miki	0.160	−0.618	0.0109	−0.618
Modified Miki	0.351	−0.632	0.539	−0.632

Physical admissibility has traditionally been tested in respect of the real part of surface impedance. However impedance models of the form of equations (1) to (4) represent equivalent fluid models, so effective (complex) density and (complex) sound speed are more fundamental quantities than surface impedance. The complex effective density ratio for an equivalent fluid impedance model can be calculated from

$$\rho/\rho_0 = (Z/\rho_0 c_0)(k/(\omega/c_0)) \quad (6)$$

Kirby [15] has investigated low frequency predictions of the Miki and Komatsu models, i.e. predictions for small values of f/σ, and has found that they lead to physically-inadmissible results, predicting negative values for the real part of effective complex density. Other impedance models including that due Wilson [16] have been shown to be physically admissible [14,15]. The Wilson model give more or less identical predictions to those of a model for a rigid porous medium that assumes an idealised microstructure of parallel tortuous slits [9].

Expressions for the complex density ρ, the complex compressibility C, the propagation constant and the characteristic impedance in a rigid-framed medium containing identical tortuous slit-like pores may be written in terms of a dimensionless parameter λ:

$$\rho(\lambda) = \rho_0/G(\lambda), \quad (7a)$$

$$C(\lambda) = (\gamma P_0)^{-1}[\gamma - (\gamma - 1)G(\lambda\sqrt{(N_{DD})})] \quad (7b)$$

$$G(\lambda) = 1 - \tanh(\lambda\sqrt{(-i)})/(\lambda\sqrt{(-i)}),$$

$$\lambda = \sqrt{\left(\frac{3\rho_0\omega T}{\Omega\sigma}\right)} \quad (7c)$$

$$k = \omega[T\rho(\lambda)C(\lambda)]^{0.5}, \quad (7d)$$

$$Z = (\rho_0 c_0)^{-1}[(T/\Omega^2)\rho(\lambda)/C(\lambda)]^{0.5} \quad (7e)$$

Recently the slit pore model has been shown to be physically admissible [16]. Both the three-parameter form of the Miki model and the slit pore impedance model given by equations (7) can be modified to require only two parameters (effective flow resistivity and porosity), by using the relationship $T = 1/\sqrt{\Omega}$, which applies to a packing of spheres [17]. However, the description 'three-parameter' is retained to label the relevant version of the Miki model.

A low frequency/high flow resistivity approximation for the surface impedance of a rigid-porous medium in which the porosity decreases exponentially with depth (the 'variable porosity model') at a rate α /m is [17]

$$Z = (1 + i)/\sqrt{(\pi\gamma\rho_0)}\sqrt{(\sigma/f)} + (ic_0\,\alpha)/8\pi f \quad (8)$$

This model has been shown to satisfy conditions for physical admissibility [14]. Equation (8) has the same form as a low frequency/high flow resistivity approximation of the impedance of a thin non-hard-backed layer of thickness $4/\alpha$ [17].

Fig. 1(a) shows predictions of the real part of complex effective density divided by the density of air and Fig. 1(b) shows predictions of the real part of the relative (to air) surface impedance of a layer of thickness 0.1 m as a function of f/σ using parameter values consistent with those of a low flow resistivity ground such as snow or a forest floor (flow resistivity 10 kPa s m^{-2}, porosity 0.7) [16]. For a flow resistivity of 10 kPa s m^{-2}, the Delany and Bazley, three-parameter Miki and modified Miki models predict that the real part of normalised effective density becomes negative below 100 Hz whereas the slit pore model does not. The physically-inadmissible predictions of negative real part of complex density ratio by the modified Miki model result despite the fact that with the same parameter values it predicts that the real part of surface impedance of a hard backed layer is positive.

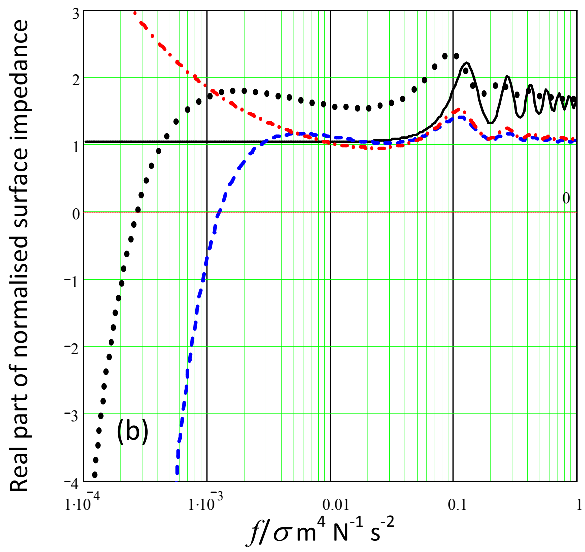

Figure 1 Predictions as a function of frequency divided by flow resistivity of real parts of (a) normalised complex density for a 'snow-like layer' ground (flow resistivity 10 kPa s m^{-2}, porosity 0.7, (hard-backed) layer thickness 0.1 m) according to the slit pore model (solid line), Delany and Bazley model (broken line), three-parameter Miki model (dotted line) and modified Miki model [6] (dash-dot line).

Figure 2 compares the predictions of the models used for Fig.1 as a function of f/σ using parameter values representative of a grass-covered ground (flow resistivity 200 kPa s m^{-2}, porosity 0.4, layer thickness 0.03 m) [12]. For 'grass-covered ground' parameter values, the Delany and Bazley and two Miki-based models lead to negative values of real normalised complex density below 2 kHz. The Delany and Bazley layer model predicts negative values of the real part of normalised surface impedance below 10 Hz. It is likely that, for 'grassland' parameters, the three-parameter Miki model will also predict a negative real part of impedance at an even lower frequency. Although, again, the modified Miki (layer) model does not predict a negative real part of surface impedance, it predicts values of the real part of the normalised surface impedance much larger than those predicted by the slit pore model below 1 kHz.

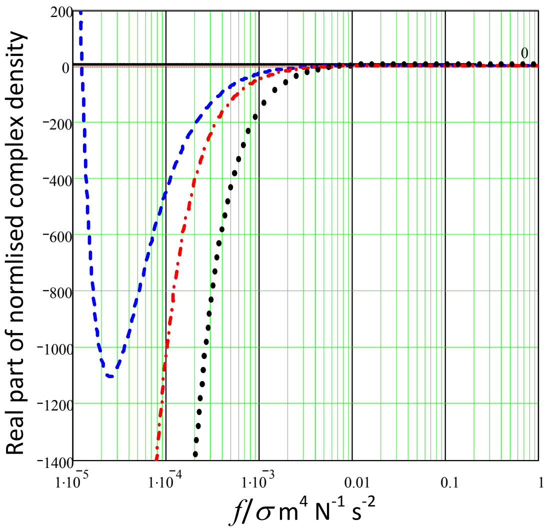

Figure 2 Predictions of real part of normalised complex density as a function of frequency divided by flow resistivity for a 'grass-covered' ground (flow resistivity 200 kPa s m^{-2}, porosity 0.4, (hard-backed) layer thickness 0.03 m) according to slit pore (solid lines) Delany and Bazley (broken lines), three-parameter Miki (dotted lines) and modified Miki (dash-dot lines) models.

3. Short range data and predictions

NT ACOU 104 *Ground surfaces: Determination of the Acoustic Impedance* [4] describes the fitting of predictions based on the Delany and Bazley impedance model to third-octave data for the difference in levels recorded between vertically separated microphones at a short range from a point source. The method uses a single geometry (source height 0.5 m, receiver heights at 0.5 m and 0.2 m, separation 1.75 m). The fits are used to place a given ground surface in one of twelve impedance classes based on values of effective flow resistivity.

Figures 3 and 4 show data obtained in connection with NT ACOU 104 [4] and best-fit predictions (through equations (9)) using the Delany and Bazley, modified Miki and variable porosity impedance models for two grass covered ground surfaces. Use of the Delany and Bazley or Miki impedance models enables reasonably good fits to these data. However, as has been reported elsewhere [9] for these and many other grassland data, the two-parameter variable porosity impedance model yields better fits.

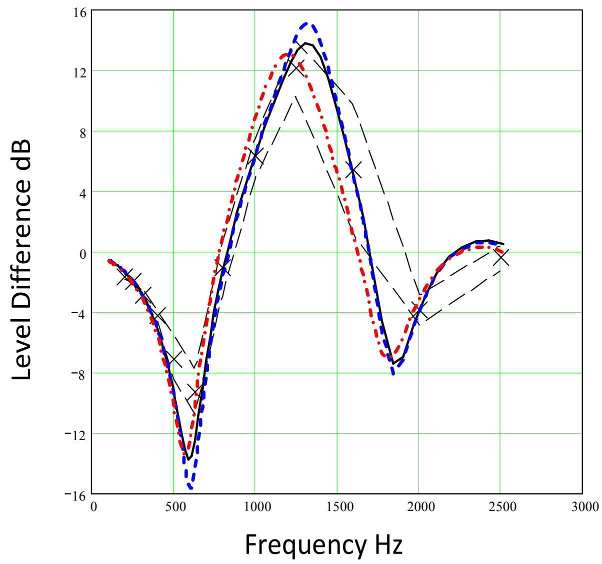

Figure 3 Third octave band level difference data (×) [thin black broken lines indicate 90% confidence limits (±1.65S.D.); NORDTEST geometry)] and best fit predictions for lawn (site #30) [14] using the variable porosity model (solid black line, effective flow resistivity 366.5 kPa s m^{-2}, porosity rate –79.5/m), the Delany and Bazley model (broken blue line, effective flow resistivity 746 kPa s m^{-2}) and the modified Miki model (red dash-dot line, effective flow resistivity 565 kPa s m^{-2}).

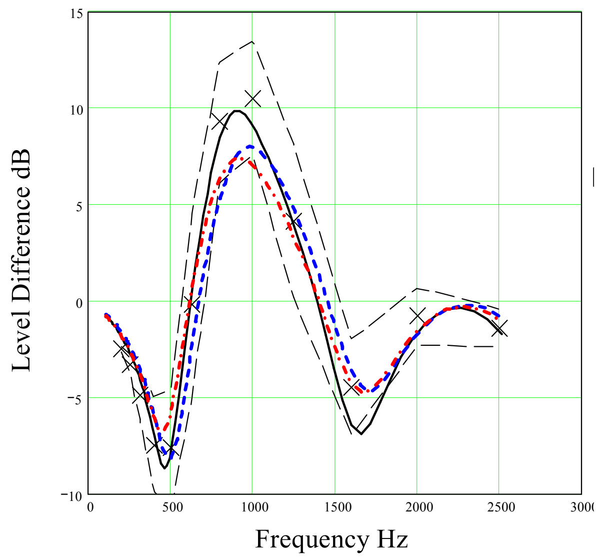

Figure 4 Third octave band level difference data (×) [thin black broken lines indicate 90% confidence limits (±1.65S.D.); NORDTEST geometry)] and best fit predictions for a long grass site (site #20) using the variable porosity model (solid black line, effective flow resistivity 20 kPa s m^{-2} and porosity rate 50/m); the Delany and Bazley layer model (broken blue line, effective flow resistivity 110 kPa s m^{-2} and effective layer depth 0.019 m); and the modified Miki layer model (red dash-dot line, effective flow resistivity 100 kPa s m^{-2}, effective layer depth 0.025 m)

Use of the Delany and Bazley and the modified Miki impedance models in equations (9) yields poor fits to short range data obtained over relatively low flow resistivity surfaces (for example forest floors and gravel in a pit). Figure 5 shows an example of the fitting to data for a pine forest floor.

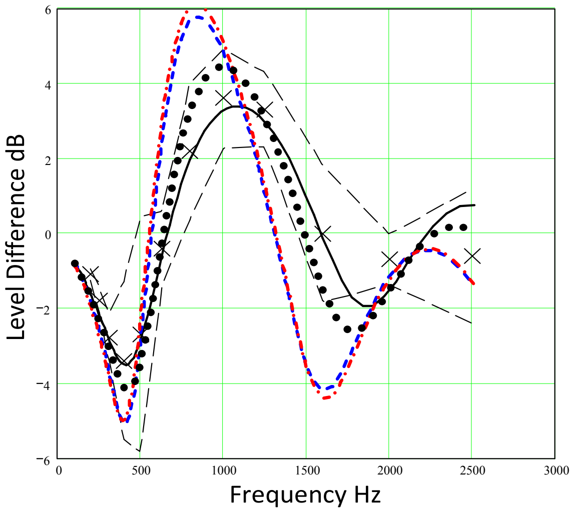

Figure 5 Third octave band level difference data (×) for a pine forest floor (site #5) [thin black broken lines indicate 90% confidence limits (±1.65S.D).; NORDTEST geometry)] and best fit predictions using the two-parameter slit pore model (solid black line, effective flow resistivity 22.75 kPa s m^{-2} and porosity 0.43), the Delany and Bazley model (broken blue line, effective flow resistivity 48 kPa s m^{-2}), the modified Miki model (red dash-dot line, effective flow resistivity 61 kPa s m^{-2}) and the three-parameter Miki model (black dotted line, effective flow resistivity 15 kPa s m^{-2}, porosity 0.559).

In a similar manner to the procedure described in NT ACOU 104 [4], the fitting errors (E) are calculated from:

$$E = \sum_f |LD_M(f) - LD_C(f)| \quad (9a)$$

$$LD_C = EA(1) - EA(2) \quad (9b)$$

$$EA(1) = 20\lg\left[\left|1 + (QR_2/R_1)e^{k(R_2-R_1)}\right|\right] \quad (9c)$$

$$EA(2) = 20\lg\left[\left|1 + (QR_4/R_3)e^{k(R_4-R_3)}\right|\right] \quad (9d)$$

In Eqs. (9a) and (9b), LD_M are the measured level difference magnitudes, LD_C are the predicted level difference magnitudes between microphones at distances R_1 and R_3 from the source; R_2 and R_4 are the corresponding reflected ray path lengths.

In Equations 9(c) and 9(d), EA(1) and EA(2) are the predicted excess attenuation magnitudes and Q is the spherical wave reflection coefficient which depends on the surface impedance and the source-receiver geometry according to equations (9e - 9h)

$$Q = R_p + (1 - R_p)F(w) \quad (9e)$$

$$R_p = \frac{\cos\theta - \beta}{\cos\theta + \beta} \quad (9f)$$

$$F(w) = 1 + i\sqrt{\pi}w\exp(-w^2)\mathrm{erfc}(-iw) \quad (9g)$$

$$w = \sqrt{ikR_2}(\cos\theta + \beta),\ w = \sqrt{ikR_4}(\cos\theta + \beta) \quad (9h)$$

where θ is the angle of incidence and β is the surface admittance (1/Z).

The fitting errors calculated from Eqs. (9) corresponding to the predictions shown in Figs. 3 to 5 are listed in Table 2.

Table 2 Fitting errors calculated from Eqs. (9) for the predictions in Figs. 3 to 5.

Ground	*Model*	*E*
Lawn	Variable porosity	***10.2***
	Delany & Bazley	13.9
	Modified Miki	15.2
Long grass	Variable porosity	***9.7***
	Delany Bazley layer	16.9
	Modified Miki layer	15.0
Pine forest floor	Slit pore	***4.9***
	Delany and Bazley	19.8
	Modified Miki	21.5
	Three-parameter	8.6

Although, for the forest floor example, the 3-parameter form of the Miki model enables significantly better fits than obtained with either the Delany and Bazley or modified Miki models, it is clear that the (semi-infinite) slit pore model enables an even better fit. According to NT ACOU 104 [4], a site is not classifiable if the fitting error exceeds 15 dB. Using this criterion, the forest floor and grass sites would be marginally classifiable if any of the Miki models are used in fitting whereas clearly classifiable when using the slit pore and variable porosity models respectively for fitting the short range data.

4. Short and longer range predictions

A recent study of the effects of ground roughness on impedance and spatial variation of impedance [18] has used the (original) Miki (layer) model for fitting short range data obtained over a 'natural' grass covered field. The average fitted Miki model parameters for 'natural' grass are flow resistivity 212 kPa s m^{-2} and layer thickness 0.0154 m. Figure 6 compares predictions of level difference spectra for a source height 1 m, receiver heights 1 m and 0 m at a horizontal distance of 4 m from the source. The predictions use the modified Miki (layer) model with these parameter values, the Delany and Bazley (layer) model (flow resistivity and layer thickness m), the three parameter Miki model (flow resistivity porosity, tortuosity and layer thickness m) and the slit pore layer model (flow resistivity , porosity and layer thickness).

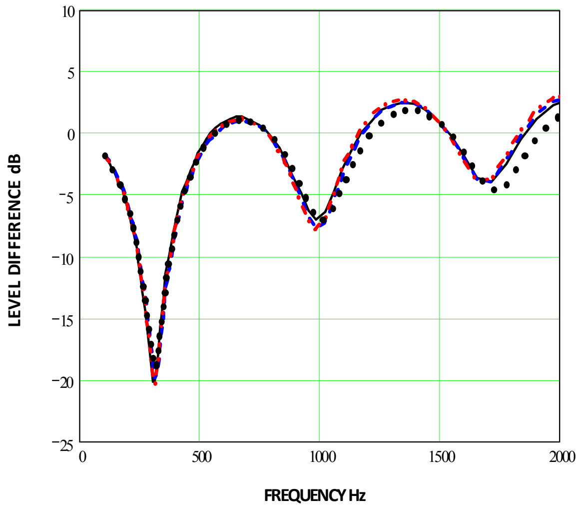

Figure 6 Predictions of Level Difference spectra (allowing for moderate turbulence) for source height 1 m and receivers at heights of 1 m and 0 m at a horizontal distance of 4 m from the source. Predictions use the slit pore layer model (black continuous line), Delany and Bazley layer model (broken blue line) modified Miki model (red dash-dot line) and three parameter Miki layer model (blacked dotted line).

These predictions are almost identical. However, since they imply rather difference surface impedance spectra, they give rise to different excess attenuation spectra predictions at a longer range. Figure 7 compares predictions (including moderate (Gaussian) turbulence, mean squared refractive index 10^{-4} and outer scale of turbulence 1 m [16]) for a 'tyre noise geometry' (source height 0.01 m, receiver height 1.5 m and range 100 m). Near 400 Hz there is more than 5 dB difference between the predicted excess attenuation spectra.

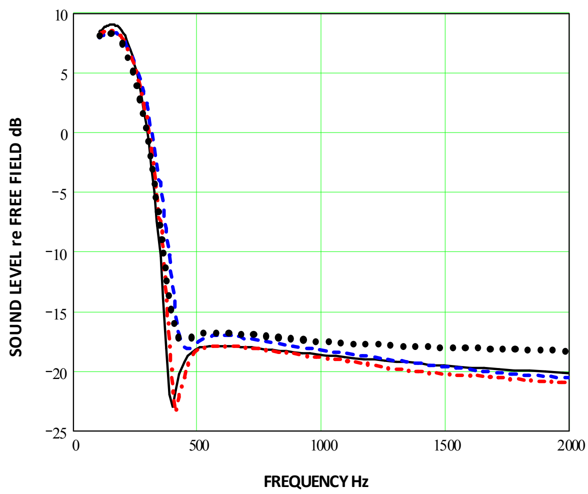

Figure 7 Predicted Excess Attenuation spectra for source height 0.01 m, receiver height 1.5 m and range 100 m. Key to curves as for Fig.6.

5. Conclusions

Single parameter semi-empirical impedance models [6, 11-14] (a) are physically inadmissible, (b) do not enable as good fits to short range data over low flow resistivity surfaces as physically-admissible models and (c) result in predictions at longer ranges that may be significantly different from those obtained with two parameter models.

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