A quasi-potential flow formulation for the prediction of the effect of the circulation on the acoustic shielding from a lifting body by means of a finite element method

Simone Mancini, R. Jeremy Astley, Gwenael Gabard and Samuel Sinayoko
Institute of Sound and Vibration Research, University of Southampton, SO17 1BJ, Southampton, UK

Michel Tournour
Siemens Industry Software NV, Interleuvenlaan 68, 3001, Leuven, Belgium

Summary

This paper presents a new simplified approach for the prediction of the acoustic shielding from lifting bodies by means of a finite element method. It extends a method already validated for aerodynamic applications to Aeroacoustics on the basis of the small perturbation expansion. A numerical model to represent the effect of the circulation developed by a slender body on the noise radiation in an unbounded domain is provided. A quasi-potential flow formulation is adopted by introducing a simplified shear layer model: a frozen wake with finite thickness and extent. The effect of the sound diffraction in the wake region is accounted for by applying the continuity of pressure across the wake line in the aeroacoustic field. An incompressible steady mean flow is considered and the Kutta condition is applied to overcome the singularity at the trailing edge. The circulation in the mean flow is predicted by reducing the potential solution to a single-valued problem. The non- uniform base flow is superimposed on the wave propagation and the linear aeroacoustic problem is solved by means of the full acoustic potential equation. The finite element method is applied both to the solution of the base flow and to the acoustic radiation. The acoustic field scattered by a 2D airfoil from a line source in presence of a non-uniform base flow is predicted as a numerical example of the proposed model. The circulation modifies the extent of the acoustic shielding by altering the wave propagation around the slender body, the wave diffraction at the trailing edge and the refraction in the shear layer.

PACS no. 43.28.+h

1. Introduction

Sound radiation and scattering from moving bodies is numerically challenging owing to the physical phenomena involved. For wave scattering from complex geometries, the impact of a base flow might be relevant. In particular with slender bodies, the development of lift alters the wave propagation and the wake shed causes wave refraction; diffraction occurs at the trailing edge. Predicting these effects in large-scale domains implies high computational costs. All these problems are major concerns in the investigation of acoustic installation effects for aircrafts.

The Finite Element Method (FEM) is an effective approach to solve for scattering with non-uniform flows [1]. Although the method can be applied to different physical models, such as, Linearised Euler Equations (LEE) and Linearised Navier-Stokes Equations (LNS), the computational cost for applications to unbounded-large-scale domains can be prohibitive at the current state of development of the computer science. On the other hand, a potential formulation is effective in reducing the problem to one unknown; a loss in terms of accuracy of the physical model is the downside: refraction of acoustic waves by rotational flows is not represented by this formulation.

In computational Aeroacoustics, the mean flow is generally superimposed on the wave propagation by considering small acoustic perturbations. For an isentropic potential formulation, the development of lift cannot be described. For this reason, the use of rotational mean flows in combination with an acoustic potential formulation is a common practice, even though some hypotheses are dropped; it is considered acceptable, however, under the assumption of small acoustic perturbations.

This paper presents a simplified approach to predict the Aeroacoustics of slender bodies by using FEM. A quasi-potential mean flow is provided on the basis of the work by Baskharone [2] and the shear layer model, introduced by Eversman [3], is used to model wave refraction. The structure of the paper is as follows: Section 2 presents the physical model. Section 3 describes the numerical method used to solve wave propagation by considering a quasi-potential flow formulation. Finally, the numerical example of the scattering by a NACA 0012 airfoil from a monopole source with a non-uniform mean flow is presented by accounting for the development of lift.

2. Governing equations and boundary conditions

We consider potential isentropic flows. An extension to a quasi-potential formulation is obtained; a simplified wake model is described for both the mean flow and the noise propagation. The mean flow is superimposed on the wave propagation under the assumption of small acoustic perturbations. In the following sections we indicate with the subscript "$_0$" the mean flow quantities in order to distinguish them from the associated acoustic perturbations.

2.1. Base flow

2.1.1. Potential Flow

The mean flow is assumed homentropic and irrotational. This hypothesis applies for all the flow particles that do not come in contact with the body or the wake. An incompressible base flow is considered; it is a reasonable assumption if the model is restricted to slender bodies and low Mach numbers, namely, $M_\infty \leq 0.3$. Under these hypotheses, the evolution of the base flow can be described by means of the mass conservation and it is expressed as a function of the velocity potential Φ_0:

$$\nabla^2 \Phi_0 = 0 \tag{1}$$

In order to complete the formulation, the impermeability condition on the body and the uniform velocity at infinity are prescribed according to the specific problem:

$$\nabla \Phi_0 \cdot \mathbf{n}_b = 0 \quad \text{on} \quad \mathbf{x} \in B \tag{2}$$

and

$$\nabla \Phi_0 = \mathbf{u}_{0,\infty} \quad \text{for} \quad \mathbf{x} \to \infty \tag{3}$$

where $\mathbf{n}_b$ is the normal vector to the body and $\mathbf{u}_{0,\infty}$ is the uniform flow velocity.

2.1.2. Extension to a quasi-potential formulation

The solution of equation 1 is multivalued for a lifting body in a simply connected domain. A circulatory flow still satisfies the boundary conditions in equations 2-3; the circulation represents the difference between the physical solution and the multivalued one. The development of lift is generally predicted by considering viscous flows. In order to account for the same effect in an inviscid flow, the model is reduced to a single-valued problem by considering a simply connected domain; this is done by introducing an infinitely thin shear layer. If Γ_0 is the circulation developed by a lifting body, we assume that $\Gamma_0 \neq 0$ on the wake line.

The Kelvin's theorem is still applicable in the whole domain except for the points on the body and the wake. The impermeability condition on the body is still valid. The solution on the wake line, however, is not potential anymore. Therefore, the continuity of pressure and velocity across the shear layer [4] [5] is imposed:

$$\Delta p_0 = 0 \quad \text{on} \quad \mathbf{x} \in W \tag{4}$$

with $\Delta p_0 = p_0^+ - p_0^-$ the difference in the mean flow pressure above and below the wake line;

$$\mathbf{u}_{0,N} = \mathbf{u}_{0,w_N} \quad \text{on} \quad \mathbf{x} \in W \tag{5}$$

where $\mathbf{u}_{0,N}$ and $\mathbf{u}_{0,w_N}$ represent respectively the normal velocity of the flow near the shear layer and the normal velocity of the particles on the wake surface; the total velocity of the wake surface is

$$\mathbf{u}_{0,w} = \frac{1}{2}(\mathbf{u}_0^+ + \mathbf{u}_0^-) \tag{6}$$

where $\mathbf{u}_0^+$ and $\mathbf{u}_0^-$ are the mean flow velocities immediately above and below the wake line. The normal velocity across the wake line is continuous whereas the tangential component is discontinuous. These conditions are obtained by considering the mass conservation and the momentum equation. From a physical point of view, we ensure that the material points cannot cross the wake line.

The Kutta condition is enforced at the trailing edge. The continuity of the normal velocity to the camber line is the condition to be satisfied for a finite angle trailing edge. In case of a cusp angle, the velocity vector has to be continuous at the trailing edge.

2.2. Wave propagation with non-uniform mean flows

The wave equation in presence of a non-uniform base flow is introduced. The acoustic perturbations are considered of small amplitude if compared to the background flow. By considering ϕ the acoustic velocity potential, for a steady mean flow one has [6]:

$$\frac{\partial}{\partial t}\left(\frac{\rho_0}{c_0^2}\frac{D_0\phi}{Dt}\right) - \nabla\cdot\left(\rho_0\nabla\phi - \frac{\rho_0}{c_0^2}\frac{D_0\phi}{Dt}\mathbf{u}_0\right) = 0 \tag{7}$$

where $D_0()/Dt = \partial()/\partial t + \mathbf{u}_0 \cdot \nabla()$ is the material derivative, with $\mathbf{u}_0$ the mean flow velocity.

Equation 7 is applicable in the whole aeroacoustic domain except for the wake line. At the shear layer, the continuity of the acoustic velocity normal to the wake line and pressure are formulated as a function of the acoustic velocity potential:

$$\Delta \frac{\partial \phi}{\partial n} = 0 \quad \text{on} \quad \mathbf{x} \in W \tag{8}$$

and

$$\frac{D_w \Delta \phi}{Dt} = 0 \quad \text{on} \quad \mathbf{x} \in W \tag{9}$$

where $D_w()/Dt = \partial()/\partial t + \mathbf{u}_{0,w} \cdot \nabla()$ is the material derivative along the shear layer. For clarity, equation 9 is obtained from the linearised Bernoulli equation.

3. Numerical method

We consider a harmonic problem; equation 7 is written in the frequency domain. A weak variational formulation is introduced both for the mean flow and the full potential acoustic equation. The development of circulation in the mean flow is achieved by using a nodeless variable approach. For wave propagation, the condition on the wake line is introduced by applying a penalty function method.

3.1. Mean flow

The weighted residual formulation associated to equation 1 is presented. A double integration by parts of the integral projection of equation 1 over a test function W is performed:

$$\int_V \nabla W \cdot \nabla \Phi_0 dV = \int_{\partial V} W(\nabla \Phi_0 \cdot \mathbf{n}) dS \tag{10}$$

with $\mathbf{n}$ the outgoing normal unit vector to the boundary surface. By assuming $\Phi(\mathbf{x}) \cong \sum_{i=1}^{m} N_i(\mathbf{x})\Phi_i$ and adopting the same polynomial expansion for $W(\mathbf{x})$, the linear system associated to the finite element method is determined on the basis of the discretised domain.

In this domain, the wake is represented as a line of discontinuity for the velocity potential; a straight fixed wake model is proposed. In the finite element model, the shear layer is represented by a line of bridging elements located on the same side with respect to the wake line; a bridging element has at least a node on the wake line. Figure 1 depicts the finite element discretisation for the shear layer.

The circulation is introduced as a nodeless Degree of Freedom (DoF) for each element on the splitting boundary; it vanishes for the nodes that are not on the line of discontinuity:

$$\Phi_w(\mathbf{x}) \cong \sum_{i=1}^{m} N_i(\mathbf{x})\Phi_i + N_g(\mathbf{x})\Gamma \tag{11}$$

where $N_g(\mathbf{x}) = \sum_{j=1}^{r} N_r(\mathbf{x})$ is the sum of all the shape functions which are non zero on the wake line, namely, the shape functions associated to the nodes on the splitting boundary; Φ_w is the velocity potential on the wake line. This approach generates a single-valued problem; the circulation represents the potential jump across the shear layer. In the finite element formulation, equation 10 accounts for the additional DoF on the wake line; the linear discrete system of equations solves for the circulation such that the development of lift is predicted.

The Kutta condition is enforced at the trailing edge by an iterative procedure: the extent of the wake is tuned in order to minimise the jump of the normal velocity at the trailing edge. The domain in which the mean flow is non-uniform is related to this condition. This process ensures the development of the right amount of circulation.

3.2. Aero-Acoustics

The weak variational formulation for the full potential equation is considered. Equation 7 is reformulated in the frequency domain; the transformed equation is projected on a test function Υ and some of the terms integrated by parts:

$$\begin{aligned} \int_V \rho_0 \nabla \Upsilon^* \cdot \nabla \tilde{\phi} dV - \int_V \frac{\rho_0}{c_0^2} (\mathbf{u}_0 \cdot \nabla \Upsilon^*)(\mathbf{u}_0 \cdot \nabla \tilde{\phi}) dV \\ + i\omega \int_V \frac{\rho_0}{c_0^2} [\tilde{\phi}(\mathbf{u}_0 \cdot \nabla \Upsilon^*) - \Upsilon^*(\mathbf{u}_0 \cdot \nabla \tilde{\phi})] dV \\ - \omega^2 \int_V \frac{\rho_0}{c_0^2} \Upsilon^* \tilde{\phi} dV \\ = \int_{\partial V} \frac{\rho_0}{c_0^2} [c_0^2 \Upsilon^* \nabla \tilde{\phi} - \mathbf{u}_0 \Upsilon^* (\mathbf{u}_0 \cdot \nabla \tilde{\phi}) - i\omega \mathbf{u}_0 \Upsilon^* \tilde{\phi}] \cdot \mathbf{n} dS \end{aligned} \tag{12}$$

with $\mathbf{n}$ the outgoing normal unit vector to the boundary surface; $\tilde{\phi}$ is the acoustic velocity potential in the frequency domain and the superscript "*" indicates the complex conjugate.

In the acoustic field, the Kutta condition is imposed as a constraint at the trailing edge: the continuity of the acoustic velocity normal to the camber line is enforced. Wake refraction is accounted by following Eversman [3]. A fixed wake with a straight geometry is considered; since we assume an incompressible mean flow, only a zero acoustic pressure jump is imposed on the wake line. A penalty integral is introduced over the jump of pressure for the elements which belong to the wake line,

$$\begin{aligned} \mathcal{L}(\mathbf{x}, \omega) = \int_{\partial V} \frac{\rho_0}{c_0^2} [c_0^2 \Upsilon^* \nabla \tilde{\phi} - \mathbf{u}_0 \Upsilon^* (\mathbf{u}_0 \cdot \nabla \tilde{\phi}) \\ - i\omega \mathbf{u}_0 \Upsilon^* \tilde{\phi}] \cdot \mathbf{n} dS + \mu \int_{V_W} \Upsilon^* \Delta p dV \end{aligned} \tag{13}$$

where $\mathcal{L}(\mathbf{x}, \omega)$ is the l.h.s. of equation 12 and μ is the penalty factor; we set $\mu = 10^5$.

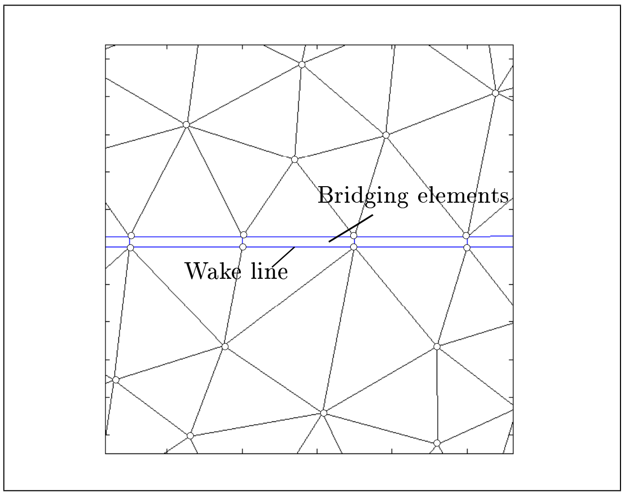

Figure 1. Bridging elements on the shear layer.

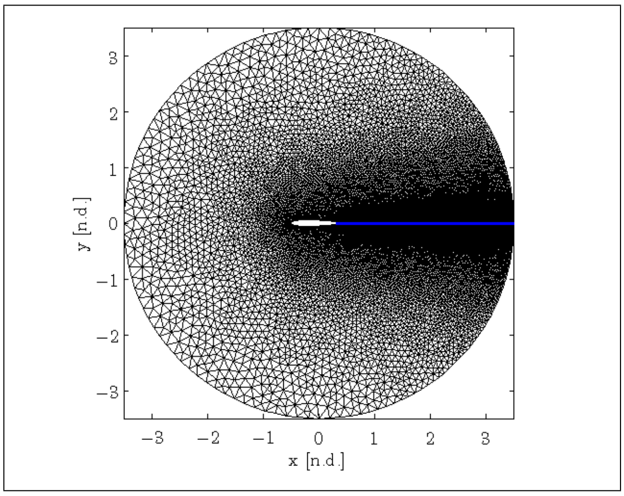

Figure 2. Aerodynamic mesh. Domain extent associated to $\alpha = 6°$.

In the finite element model, the wake is represented as for the mean flow. The condition on the pressure jump constraints the nodes on the bridging elements with the same x-axis coordinate. It is worth noticing that the acoustic pressure can be expressed as a function of the acoustic velocity potential:

$$p = -\rho_0(i\omega\tilde{\phi} + \mathbf{u}_0 \cdot \nabla\tilde{\phi}) \tag{14}$$

Therefore, the problem is still dependent on one variable. The constraint system of equations is obtained by introducing a polynomial expansion of ϕ and Υ in equation 13 by accounting for equation 14.

4. Discussion of the results

The method described in Section 3 is applied to study the wave propagation around a NACA 0012 from a monopole source with a non-uniform flow. The 2D problem is solved in the frequency domain by means of FEM. The unbounded domain is represented by using a Perfectly Matched Layer (PML) [7]. The problem is investigated for $M_\infty \leq 0.3$ by limiting the angle of attack to $\alpha \leq 10°$; an incompressible quasi-potential mean flow is considered. We want to assess the effect of lift on the shielding effect provided by the airfoil. We assume $\rho_0 = 1.22\ kg/m^3$ and $c_0 = 340\ m/s$. The characteristic length of the problem is the unit chord length L of the airfoil.

4.1. Assessment of the base flow

This section provides a validation of the mean flow field around the airfoil. The model presented is assessed against the numerical prediction from Eleni et al. [8] made by using a viscous formulation; the comparison is reported for $\alpha \leq 10°$.

Except for the wake line, an unstructured triangular grid is used; linear shape functions are defined. The reference mesh is refined up to $5 \cdot 10^{-3}L$ for the elements in proximity of the wake line in order to minimise the thickness of the shear layer and represent accurately the trailing edge of the airfoil; it is coarsened up to $0.4L$ in the outer elements. On the wake line, quadrilateral linear elements are arranged; the wake line extends from the trailing edge to the end of the domain.

Figure 2 shows an example of the mesh grid for the prediction of the mean flow. The extension of the domain is determined by the length of the wake; it is tuned in order to satisfy the Kutta condition. At the external boundary of the domain, the velocity is uniform; in the aeroacoustic problem, a uniform flow field is assumed for all the points external to this domain.

Figure 3 shows the comparison of the lift coefficient predicted by the model proposed and the work by Eleni et al. [8]. A good agreement is shown for $\alpha \leq 10°$. A larger difference is noted at low angles of attack: the overestimation is attributed to the iterative procedure used to determine the extension of the domain; a tolerance of 0.3 m/s is used to relax the convergence to the Kutta condition.

4.2. Effect of circulation on the acoustic shielding

The scattering by a NACA 0012 from a monopole source is presented by accounting for a non-uniform mean flow. The unit monopole source is located at $\mathbf{x}_s = [0 - 2]$. An unstructured mesh with linear elements is used by accounting at least for 15 elements per wavelength. Since a 2D problem is investigated, the intensity of the monopole source is frequency dependent; so, we avoid comparing the results for different frequencies in terms of absolute values.

The problem is investigated for different Mach numbers and angles of attack. The flow at $\alpha = 0°$ is directed along the positive x-axis and α is positive counterclockwise. We fix the relative position of the airfoil and the source, whereas the uniform flow is biased

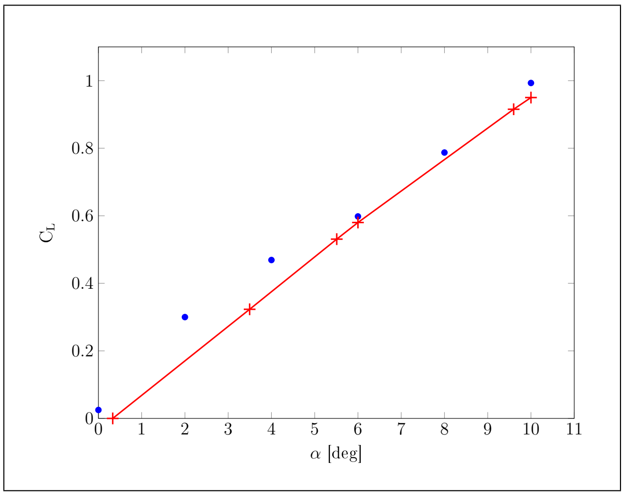

Figure 3. C_L as a function of the angle of attack. Validation against a viscous formulation. Eleni et al. [8] (solid); model proposed (dots).

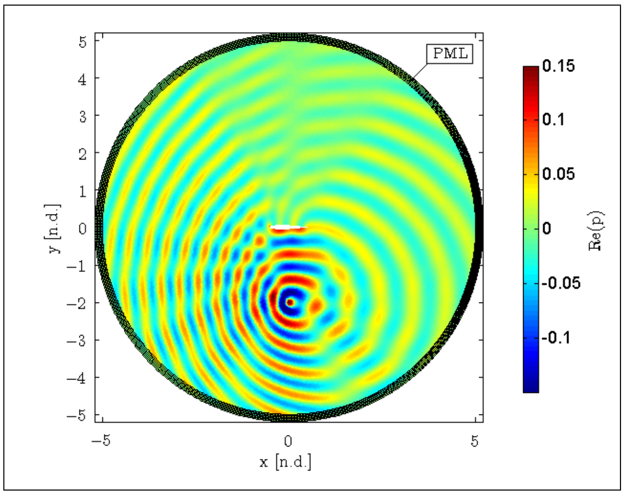

Figure 4. Contour plot: real part of the acoustic pressure. f=500 Hz, $M_\infty = 0.3$ and $\alpha = 10°$.

according to α. The angle θ, used as a reference for the comparison of the results, is zero on the wake line and positive counterclockwise. We want to assess the impact of the circulation over the noise shielding by the airfoil, considering the diffraction from the trailing edge and the refraction by the shear layer. For clarity, noise generation at the shear layer is not represented. Figure 4 shows the real part of the pressure field for $f = 500\ Hz$, $M_\infty = 0.3$ and $\alpha = 10°$.

Figures 5-6 show the Sound Pressure Level (SPL) on a field point circumference centered at the origin of the reference frame and with radius $R = 2.5\ L$, with L the chord length of the airfoil; the results are shown for $f = 100\ Hz$ and $f = 500\ Hz$ for an angle of attack $\alpha = 4°$ and $M_\infty = [0.1, 0.2, 0.3]$. An increase in M_∞ reduces the minimum value of sound pressure level, even though in the region of the leading edge the acoustic pressure raises. For angles corresponding the trailing edge, the decrease in sound pressure level

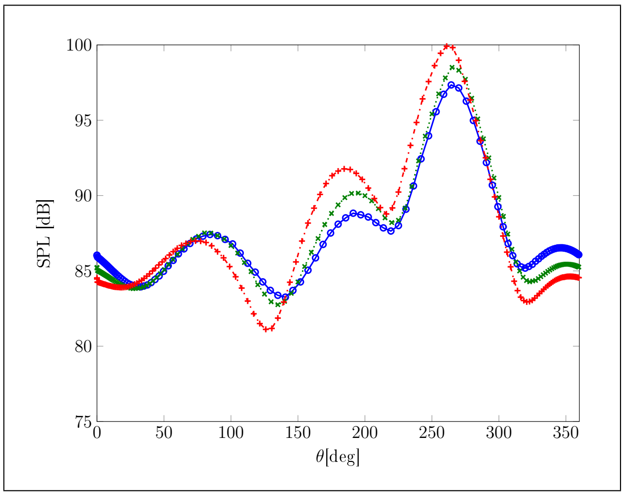

Figure 5. Sound pressure level. $f = 100\ Hz$, $R_{fp} = 2.5L$. $M_\infty = 0.3$ (solid - marker: +), $M_\infty = 0.2$ (dotted - marker: x) and $M_\infty = 0.1$ (dashed - marker: ∘). Scattering by a NACA 0012 from a monopole source with a non-uniform mean flow, dependency on the Mach number.

with an increase in Mach number suggests a reduction of the refraction effect with the velocity. At the same time, it is worth noticing that the directivity of the source is influenced by the flow: with an increase in speed, a reduction of the directivity downstream is observed.

Figures 7-8 show the SPL on the same field point circumference as in Figures 5-6 for $M_\infty = 0.3$ respectively at $f = 100\ Hz$ and $f = 500\ Hz$; the figures are shown for a number of angles of attack, i.e, $\alpha = [0°, 4°, 10°]$. For large angles of attack, the deep in the SPL plummets of 10-15 dB. In the shadow zone, a phase shift towards the trailing edge is shown when the angle of attack is increased. The phase shift is not constant because of the local effects of the mean flow velocity gradients around the airfoil.

In the high pressure region, the SPL is almost independent of the angle of attack. On the other hand, an increase of the peak value of 3-4 dB is shown when the dependency on the Mach number is investigated. These effects are not frequency related: the SPL amplification due to the uniform mean flow is the dominant effect.

5. Conclusions

A simplified approach to study noise propagation in presence of a lifting body is presented. The method is based on a quasi-potential flow formulation; it allows the application of the finite element method both to the mean flow and the aeroacoustic problem. The method assumes a fixed straight wake whose extent is tuned to satisfy the Kutta condition in the mean flow problem; wave refraction is imposed by means of a penalty integral approach based on the continuity of the acoustic pressure.

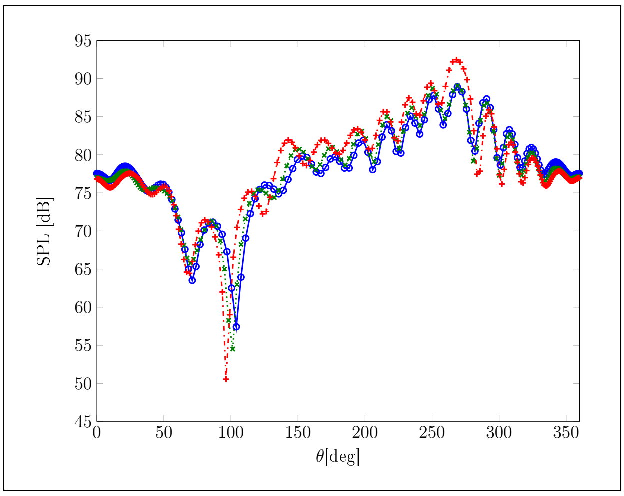

Figure 6. Sound pressure level. $f = 500\ Hz$, $R_{fp} = 2.5L$. $M_\infty = 0.3$ (solid - marker: +), $M_\infty = 0.2$ (dotted - marker: x) and $M_\infty = 0.1$ (dashed - marker: ∘). Scattering by a NACA 0012 from a monopole source with a non-uniform mean flow, dependency on the Mach number.

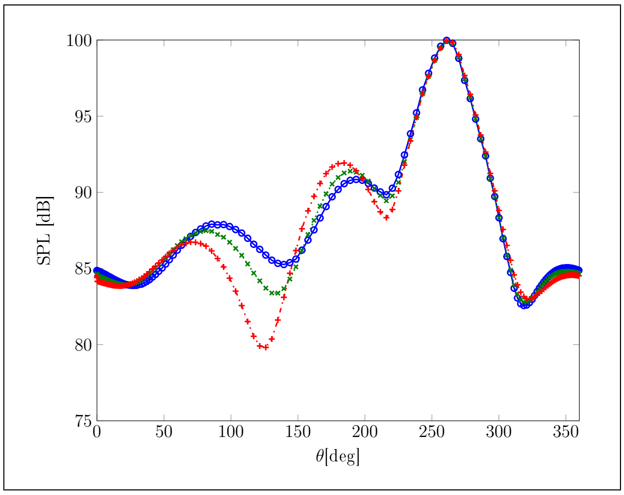

Figure 7. Sound pressure level. $f = 100\ Hz$, $R_{fp} = 2.5L$. $\alpha = 10°$ (solid - marker: +), $\alpha = 4°$ (dotted - marker: x) and $\alpha = 0°$ (dashed - marker: ∘). Scattering by a NACA 0012 from a monopole source with a non-uniform mean flow, dependency on the angle of attack.

The method is applied to study the effect of the generation of lift on the acoustic shielding by an airfoil. In the shaded area, the circulation induces a phase shift of the acoustic pressure towards the trailing edge whereas it has a small effect in the zone were the waves are mainly refracted. On the other hand, in the high pressure zone an increase of 3-4 dB is obtained by accelerating the mean flow from $M_\infty = 0.1$ to $M_\infty = 0.3$.

Acknowledgement

The authors gratefully acknowledge the European Commission for the support of the project

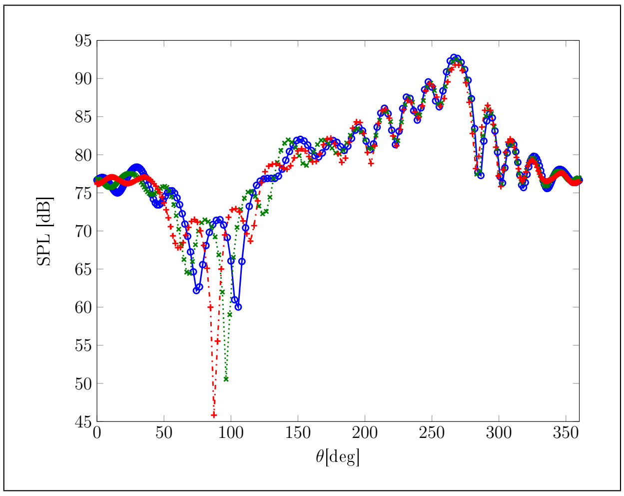

Figure 8. Sound pressure level. $f = 500\ Hz$, $R_{fp} = 2.5L$. $\alpha = 10°$ (solid - marker: +), $\alpha = 4°$ (dotted - marker: x) and $\alpha = 0°$ (dashed - marker: ∘). Scattering by a NACA 0012 from a monopole source with a non-uniform mean flow, dependency on the angle of attack.

FP7-PEOPLE-2013-ITN CRANE, Grant Agreement 606844.

References

[1] R.J. Astley: Numerical methods for noise propagation in moving flows, with application to turbofan engines. Acoustical Science and Technology 30 (2009) 227–239.

[2] E. Baskharone, A. Hamed: A new approach to cascade flow analysis using the finite element method. Proc. AIAA 18th Aerosp. 1980, 1-9.

[3] W. Eversman, D. Okunbor: Aft fan duct acoustic radiation. Journal of Sound and Vibration 2 (1998) 235-257.

[4] E.O. Suciu, L. Morino: Nonlinear steady incompressible lifting-surface analysis with wake roll-up. AIAA Journal 15 (1977) 54-58.

[5] S.R. Sipcic, L. Morino: Wake dynamics for incompressible and compressible flows. In: L. Morino (Ed.). Computational Methods in Potential Aerodynamics, Springer Verlag, Berlin, 1986.

[6] R.J. Astley: A finite element, wave envelope formulation for acoustical radiation in moving flows. Journal of Sound and Vibration 3 (1985) 471-485.

[7] A. Bermudez, L. Hervella-Nieto, A. Prieto, R. Rodriguez: An optimal perfectly matched layer with unbounded absorbing function for time-harmonic acoustic scattering problems. Journal of Computational Physics 2 (2007) 469-488.

[8] D. C. Eleni, T. I. Athanasios and M. P. Dionissios: Evaluation of the turbulence models for the simulation of the flow over a National Advisory Committee for Aeronautics (NACA) 0012 airfoil. Journal of Mechanical Engineering Research 4 (2012) 100-111.