A bootstrap estimation of confidence levels in reverberation time measurements at low frequencies

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Summary
In this paper, a bootstrap evaluation of the confidence intervals in reverberation time measurement is presented and discussed. The confidence interval of a data set is usually calculated assuming the population is normally distributed. In the literature published in the recent years it has been shown and discussed how either multimodal or asymmetric probability density functions are possible in reverberation time measurements at low frequencies. As the assumption of normal distribution is not valid for reverberation time measurements at low frequencies, a bootstrap based method to obtain confidence intervals for reverberation time measurements will be described. The proposed method is validated by simulations and by using an extensive measurement dataset of a room. Reasonably good confidence levels are reached in both cases.

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1. Introduction

In the present paper we describe a Bootstrap approach to evaluate confidence intervals of reverberation time measurements. There are not too many references on the evaluation of the uncertainty of reverberation time measurements. The standard ISO 3382, parts 1 and 2, includes several equations for calculating the standard deviation of measurement of the reverberation time in rooms, based on papers published by Davy in 1979 and 1980 [1, 2]. Later, in 1988, Davy proposed some empirical corrections to these expressions for low frequency measurements [3]. Hagber and Thorsson, [4], during the process of evaluation of uncertainties in standard impact sound measurements, reported an asymmetric distribution functions of the reverberation time measurements at low frequencies. The behavior of the distribution functions given by the measurements performed by these authors matches the description of the distribution functions given by Cabo, Sobreira and Jacobsen [5], where it is shown how not only asymmetrical but also multimodal distribution functions appear in practice. In a previous job, the description of the error introduced by the filter due to its group delay is fully described in Sobreira, Cabo and Jacobsen [6]. Provided the distribution function at low frequencies can be rather complex, the usual assumptions (tendency to normal distributions of the measurement data set) to estimate the uncertainty of the measurement cannot be used at low frequencies. Even the use of Monte Carlo Methods, hereinafter MCM, requires the assumption that the data set has a known distribution, and it should be clear that it is not the case in reverberation time measurement at low frequencies. It should be noted that the meaning of low frequencies range is related to the concept of low modal density where depending on the characteristics of the system under test, the distribution function may become more or less complex.

A common problem in applied statistics is the estimation of an unknown parameter and often it is useful to get an idea of how accurate its estimator is ($\hat{T}_{60}$ in this specific case). Some decades ago, the effort was focused in developing complex (and more often oversimplified) theoretical derivations of the standard error or the variance for most estimators (as those shown in [1, 7]). Nowadays, high-speed computers are accessible for most of the technicians and engineers. Therefore, modern and computer-intensive statistical techniques could be used in order to avoid oversimplified theoretical derivations that sometimes do not fit experimental results. One of this modern statistical methods is the bootstrap, proposed by Efron in 1979 [8]. A good reference to introduce the bootstrap technique to the reader is [9]. Furthermore, the reader can find different methods to obtain confidence intervals in [10] and [11].

2. Methodolody

In practice, the problem is to evaluate the confidence intervals in reverberation time measurements when only a few samples are available and the distribution function of the measurement data is unknown. A previous study with simulated acoustic decays would be useful to obtain large amounts of synthetic acoustic decays simulating different kind of rooms. In order to evaluate and validate the method, the following methodology has been followed:

1. A analytical method to generate acoustic decays has been developed. The energy decay model generated will serve as model function for a Monte Carlo simulation.
2. The Monte Carlo simulation is used to generate a large number of acoustic decays for each modelled room. Different rooms can also be modelled.
3. Bootstrap can be used to estimate confidence intervals using subsets of the acoustic decays generated using the MCM.
4. The method is also applied to evaluate confidence levels with real data.

2.1. The energy decay model

The model defined by Kob and Vörlander [12] and used by Sobreira, Cabo and Jacobsen [6] has been taken as the starting point. The acoustic decay in a resonant system can be modelled as the superposition of several decaying resonant modes [13]. This transient can be modeled as a weighted sum of decaying cosines, being the weights given by the spatial distribution of the energy per mode. A set of box-shaped rooms are used, with the aim of simplifying the simulations. Thus, given the dimensions and acoustic properties of the walls, it is possible to simulate the energy decay process of every position of the room by using the model shown in Eq. (1),

$$p(x, y, z) = \sum_{i=1,2\cdots\infty} A_i(x, y, z) \cos(2\pi f_i + \phi_i) e^{-\beta_i t} \quad (1)$$

where ϕ_i describes the phase of the modes and $A_i(x, y, z)$ denotes the amplitude of the modes at the measurement point and $t = 0$, being $\beta_i = \frac{-3ln10}{T_i}$ the temporal absorption coefficient of every mode. This model of the temporal absorption coefficient allows us to set the exponential decay of each mode with an attenuation of 60 dB when $t = T_i$ (so T_i is the reverberation time associated with a given normal mode with resonance frequency f_i). The simulations can be notably simplified if the excitation source is placed in a corner of a box-shaped room and a flat response over the frequency range of interest is assumed. In the case of box-shaped rooms, the natural frequencies can be calculated from Eq. (2).

$$f_i = f_{l,m,n} = \frac{c}{2}\sqrt{\left(\frac{l}{Lx}\right)^2 + \left(\frac{m}{Ly}\right)^2 + \left(\frac{n}{Lz}\right)^2}, \quad (2)$$

$$\forall l, m, n = 0, 1, \cdots, \infty,$$

where Lx,Ly,Lz are the room dimensions along the x,y and z axes and c the speed of sound. Thus, it may be considered that all the natural modes of the room are excited by the impulsive source. Under this assumption, considering that every mode is excited with the same amplitude, $A_i(x, y, z) = A_{l,n,m}(x, y, z)$ depends only on the position of every microphone. Thereby, if k_{lx}, k_{ly} and k_{lz} are the wave numbers along the x,y and z directions, the amplitude of every natural mode for every microphone position can be described by Eq. (3).

$$A_{l,m,n}(x, y, z) = \cos(k_{lx}x)\cos(k_{my}y)\cos(k_{nz}z) \quad (3)$$

Note that this model can be a good approximation of the eigenfunctions for a box-shaped room with hard walls, where the reactive part of the impedance of the walls is negligible. In that case the position of the nodes are not notably affected by the absorption of the walls [13].

2.2. The Monte Carlo simulations

The application of the MCM to evaluate of the uncertainty in measurements is the aim of the first supplement of the Guide to the Expression of Uncertainty in Measurement [14], which is known as the GUM. The MCM in acoustics is described in detail and applied to different problems by Rodriguez-Molares [15]. The methodology followed in this work to perform the MCM calculations to evaluate the distribution functions of acoustic decay measurements is described by Cabo, Sobreira and Jacobsen [5]. In the formulation stage of the MCM, the distribution function of every input quantity needs to be defined. Since the goal is to simulate a reverberation time measurement performed by applying the standard ISO 3382, there are some limitations regarding the microphone positions: the minimum distance between microphones must be $\lambda/2$ meters, and none of them can be positioned less than $\lambda/4$ meter far away from the wall [16, 17]. Fulfilling these requirements for every repetition of the experiment, the microphone positions are chosen randomly and sequentially. After a microphone position is chosen a sphere of radius $r = 2$ m is generated containing points that are not allowed for a new microphone position. This process is repeated until the M desired microphone positions are chosen.

2.2.1. Evaluation of the acoustic decay to obtain the reverberation time estimation

Once the acoustic decays are obtained, the reverberation time must be calculated following the method described in the standard. The reverberation time is estimated at the output of the filters by backwards integration of the squared impulse response[16, 18], as shown in Eq. (4).

$$y(t) = \int_t^{\infty} p^2(\tau)d(\tau) = \int_{\infty}^{t} p^2(\tau)d(-\tau). \quad (4)$$

Let us to define the energy decay in the logarithmic scale as $Y_m(t)$ in function of the pressure signal recorded by every microphone.

$$Y_m(t) = 10\log_{10}(y_m(t)) = 10\log_{10}\left(\int_t^{\infty} p^2(\tau)d(\tau)\right)$$
$$= 10\log_{10}\left(\int_{\infty}^{t} p^2(\tau)d(-\tau)\right). \quad (5)$$

As reverberation time measurements are performed using analog to digital conversion (digital sound level meters and analysers), it is possible to derive its discrete version $Y_m[n] = Y_m(nT_s)$, by assuming that the signals are sampled at a frequency rate, $f_s = \frac{1}{T_s}$, and have a finite number of samples, N, as shown in Eq. (6).

$$Y_m[n] = 10\log_{10}(y_m[n]) =$$
$$= 10\log_{10}\left(\frac{\sum_{k=n}^{N} p_m^2[k]}{f_s}\right) \quad dB. \quad (6)$$

The reverberation time of a room can be estimated either by averaging the reverberation times of each individual energy decay or performing a single estimation from the averaged energy decays. If the latter is chosen, the averaged energy decay curve can be written as follows:

$$Y[n] = 10\log_{10}\left(\frac{\sum_{k=n}^{N} \frac{1}{M}\sum_{m=1}^{M} p_m^2[k]}{f_s}\right) \quad dB, \quad (7)$$

where M is the number of acoustic decays and N the number of samples.

Fig. 1 shows an example of energy decay curves simulated with 15 different microphone positions for a given room of dimensions $L_x = 11.3\ m$, $L_y = 10.2\ m$ and $L_z = 3.3\ m$. These decays were obtained at the output of a 50 Hz third-octave band filter. The mean decay curve calculated as defined in equation (7) is shown too. It is interesting to note that the energy decay curve at every single point is less linear than the mean decay curve, therefore the estimation of the reverberation time will be more precise if it is done using the averaged energy decays.

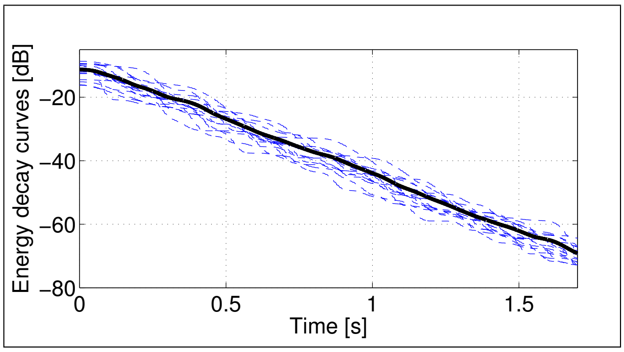

Figure 1. Energy decay curves obtained (in logarithmic scale) for 15 different microphone positions at the output of a 50 Hz third-octave band filters (Actual reverberation time within the band is around 1.7 s)

To minimize the influence of the filters, we are using the time-reversed filtering technique –see ref [6]. The average energy decay is then obtained applying the equation (7). Depending on the dynamic range used in the evaluation of the reverberation time, the EDT, T_{20} or T_{30} are estimated. The averaged slope of the energy decay in this case can be written as:

$$b = \frac{\sum_{k=1}^{N}(t[k]-\bar{t})(Y_{fit}[k]-\bar{Y}_{fit})}{\sum_{k=1}^{N}(t[k]-\bar{t})^2}, \quad (8)$$

where Y_{fit} is the vector containing the samples of $Y[n]$ within the dynamic range of interest $[-5dB, -(R+5dB)]$ (assuming normalization (i,e. $max(Y[n]) = 0dB$)), and t is the vector containing the corresponding times. The estimation of the reverberation time with a dynamic range R from the slope of the averaged acoustic decay can be be written as shown in Eq. (9).

$$T_{R,2} = \frac{-60}{b}. \quad (9)$$

2.3. A Bootstrap based method to obtain confidence intervals for reverberation time measurements

In many practical situations the central limit theorem can be invoked to assert that a given estimator is *asymptotically* distributed as a Gaussian random variable. This can be true for some estimators, if the sample $\mathbf{X} = \{X^{(1)}, X^{(2)}, ..., X^{(n)}\}$ is large enough. But this is not the case for all the reverberation time measurements, mainly at low frequencies. Due the notable variability of the energy decay curves over the room, filtering and the method used to estimate the reverberation time, the straight line that best fits the decay curve has a random slope b, following some unknown distribution, let say $\mathcal{F}_b$. The distribution of the slope is unknown: it depends on the number of modes within the filter band, which implies that it depends on the kind of room (volume and room shape) and the modal density – see [5]. Hence, any assumption about $\mathcal{F}_b$ will lead to non-generalizable results. In fact, the standard [16, 17] recommends to increase the number of microphone positions or divide the measurement in different decoupled spaces that may potentially have a different reverberation time, for some complex geometries. Note that for that kind of enclosures the distribution of the measured slope $\mathcal{F}_b$ over the whole volume will be multimodal, so any estimation of the uncertainty based on the assumption of normality will lead to a completely biased results.

Regarding the expression of the estimator, given by Eq. (8), there is no single expression to assert the standard error of it. Because of that, and due the small number of independent source/microphone positions that are usually available in a practical measurement, the bootstrap principle can be used to derive the standard error and to get confidence intervals. Let us to rewrite the expression of the energy decay curve $y[n]$, which is the sampled version of the energy decay process, $Y[n]$, Eq. (7), in natural units. Let $y[n]$ be the effective energy decay curve, obtained by averaging the squared pressure signals, as follows,

$$y[n] = \frac{1}{Mf_s}\sum_{m=1}^{M}\sum_{k=n}^{N} p_m^2[k] = \frac{1}{Mf_s}\sum_{m=1}^{M} y_m[n], \quad (10)$$

where the only modification with respect Eq. (7) is the change in the order of the integral and the sum (and the absence of the logarithm). Eq. (10) means that averaging of the squared pressure signals is equivalent to averaging the energy decays. This fact allows to use the bootstrap principle for an application in which the resampling with replacement technique is physically justified. Due the periodicity of the eigenfunctions for most of the rooms, even for some complicated geometries, it can be proven that the probability of placing two microphones in two acoustically symmetric positions for some frequency bands is not negligible, mainly at low frequencies. It means that for some points of the room, the decay curve within a given frequency band will have a similar shape. The replacement during the resampling can model this effect. Thus, the classical bootstrap technique by resampling (with replacement) the M energy decay curves can be applied to obtain an estimate about the uncertainty or the accuracy of our estimator, as shown in table I. The resampling is performed over measurement data and the desired averaged slope is computed by applying Eq. (10) and Eq. (8). The set $\{y_1[n], \cdots, y_M[n]\}$ is the sample of M decays and the set $\{y^*_{k,1}[n], \cdots, y^*_{k,M}[n]\}$ is the kth bootstrap sample obtained by resampling with replacement.

Table I. A bootstrap-based method to obtain confidence intervals for Reverberation Time Measurements

Step 0: **Measurement:** Collect the data $p_m[n] = p_m(nT_s)$ and compute the energy decay curves for all the pressure signals Eq. (4) $\{y_1[n], y_2[n], ..., y_M[n]\}$.
Step 1: **Resampling:** Draw a sample, $\{y^*_{k,1}[n], y^*_{k,2}[n], ..., y^*_{k,M}[n]\}$, by resampling with replacement (of the same size M).
Step 2: **Effective decay curve:** Average the energy decay curves to obtain the effective decay curve in the logarithmic scale, Eq. (7). $Y^*_k[n] = 10\log_{10}\left(\frac{1}{M}\sum_{m=1}^{M} y^*_{k,m}[n]\right)$.
Step 3: **Bootstrap estimation:** Then, from $Y^*_k[n]$, obtain the vectors Y_{fit} and t and calculate b^*_k, by aplying Eq. (8).
Step 4: **Repetition:** Repeat steps 1-3 many times to obtain a total of B bootstrap statistics $\{b^*_1, b^*_2, ..., b^*_k, ..., b^*_B\}$.
Step 5: **Ranking:** Sort the bootstrap estimates of the slope b^*_k into increasing order, $[b^*_k]_1 \leq [b^*_k]_2 \leq ... \leq [b^*_k]_B$, where $[b^*_k]_j$ is the $j-th$ smallest value of b^*_k.
Step 6: **Confidence interval:** Thus, if $[[b^*_k]_{q_1}, [b^*_k]_{q2}]$ is an interval containing $(1-\alpha)B$ of the estimates, being $q_1 = \lfloor B\alpha/2 \rfloor$ and $q_2 = B - q_1 + 1$, then the interval $([b^*_k]_{q_1}, [b^*_k]_{q_2})$ is a $(1-\alpha)100\%$ confidence interval for b.

3. Results and discussion

3.1. Measurements

In order to validate the conclusions of this investigation and the proposed method, a set of 41 source/microphone positions were used to perform a reverberation time measurement in a real classroom of the Telecommunications Engineering School – University of Vigo. Concretely the classroom T-104. A model of such room is shown in Fig. 2. It's shape does not match a shoe-box: one of the side walls has an oblique angle and, on the other hand it contains several furniture elements (desks, windows, a blackboard, etc). Therefore, natural modes are quite different from those corresponding to a box-shaped room, as it was shown in [19] by Finite Element Method – FEM – simulations. The classroom has a Volume V=332.67 m^3, a heigth H=3.20 $m$an active surface S= 472.76 m^2 and 80 desks. A subsampling technique can be applied to obtain subsamples of $M < 41$ source/microphone positions to simulate measurements with less points i.e. sets of measurement with samples of the same size

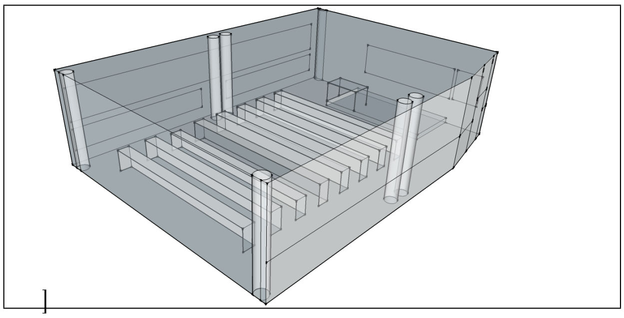

Figure 2. Model of the measured classroom (using Google SketchUp)

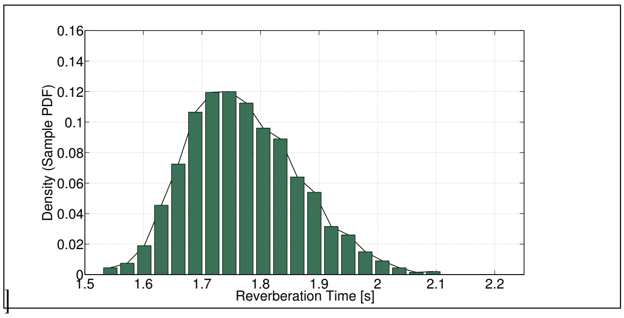

Figure 3. Sample distribution of T20 at the 63 Hz third-octave band for the measured classroom using M=15 source/microphone positions.

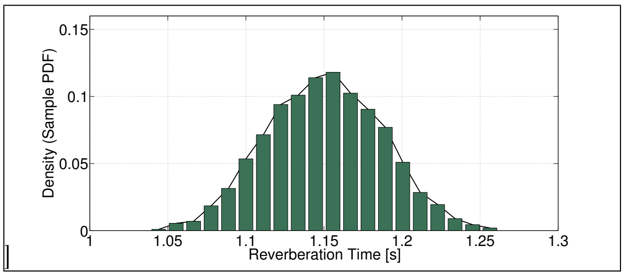

Figure 4. Sample distribution of T20 at the 100 Hz third-octave band for the measured classroom using M=15 source/microphone positions.

as the usual number of measurements taking according to the standards. Thus a large number of measurements of the reverberation time in the room can be simulated without limiting the generalization of results. In order to validate the conclusions and the proposed method using the maximum number of independent signals a dynamic range $R = 20\ dB$ has been taken because in some of the measurements the 45 dB dynamic range needed to evaluate T_{30} according to the standard was not available. Fig. 3 and 4 show the PFDs of the sample at the 63 Hz and 100 Hz frequency band respectively. It can be observed how the PDF of the 63 Hz frequency band is strongly asymmetrical, as it has also been described in the literature [4, 5]. This result justifies by itself the use of bootstrapping to calculate the confidence levels in reverberation time measurements at low frequencies.

Table II. Estimated confidence level after some iterations for the estimators T_{20} and T_{30} - nominal level of 95% - Room of dimensions $L_x = 11.3\ m$, $L_y = 10.2\ m$ and $L_z = 3.3\ m$

Band	Parameter	Realizations 100	200	500
50 Hz	T_{20}	0.94	0.925	0.926
	T_{30}	0.98	0.990	0.986
63 Hz	T_{20}	0.94	0.945	0.940
	T_{30}	0.94	0.935	0.932
80 Hz	T_{20}	0.93	0.965	0.966
	T_{30}	0.94	0.940	0.944
100 Hz	T_{20}	0.98	0.985	0.988
	T_{30}	0.93	0.935	0.960

3.2. Convergence of the estimation of confidence intervals

In order to validate the proposed method the MC simulation presented in section 2.2 was performed to generate the acoustic decays. These decays has a know slope and a known true value for the reverberation time can be defined, which is not possible in the case of real measurements. $M = 15$ acoustic decays have been used for every simulated room to get confidence intervals following the method described in in Table I. An method to get a nominal confidence of $p\%$ for a given parameter should contain its expected value the $p\%$ of the times. By applying this idea to our problem, if the reverberation time measurement were repeated many times in a room, the confidence interval given by the proposed method should contain the expected value of the estimator the $p\%$ of the times. Then, assuming that its expected value is known for every room (obtained as a result of the previous MCM simulation), a simple way to validate the method is to calculate the confidence interval for each repetition of the reverberation time measurement. Thus, the confidence level reached by the method can be estimated by measuring the sample probability of obtaining a confidence interval that contains the expected value of the estimator. This validation process was done for several rooms with different dimensions and different reverberation times. Table II shows the confidence levels reached for the T_{20} and T_{30} after different numbers of repetitions of the MCM simulation for the same enclosure. As it can be seen, the mean estimated confidence levels tend to the nominal confidence level (95%) for both estimators (T_{20} and T_{30}), meaning that the proposed method works reasonably well for this room.

Finally, in order to obtain the minimum number of microphone-source positions needed to obtain a convergence of the confidence level to the nominal value of 95 % a computationally expensive simulation has been carried out. The resampling has been performed 200 times and for each frequency band 500 bootstrap repetitions has been carried out to obtain the confi-

Table III. Convergence of the confidence levels as function of the number of source-microphone positions (number of decays) – Nominal confidence level 95 %

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Number of decays	Frequency Band			
	50 Hz	**63 Hz**	**80 Hz**	**100 Hz**
4	87.2 %	88.7 %	89.1 %	83.0 %
5	91.2 %	89.2 %	91.7 %	84.3 %
6	89.8 %	92.1 %	89.5 %	84.7 %
7	92. 3 %	91.4 %	92.8 %	91.3 %
8	94.7 %	95.9 %	95.4 %	93.9 %
9	95.1 %	94.8 %	94.3 %	95.8 %

dence levels. To avoid well known problems related to the filtering [20, 6, 12] the results are referred to the use of time-reversed filtering of the acoustic decays, so the results are considered applicable to situations with BT>4 AND time-reversed filtering. The Table III shows that a minimum of 8 independent acoustic decays are needed to guarantee the convergence to a nominal confidence level of 95%. The selection of microphone-source positions should avoid equivalent positions (i.e. microphone positions that are symmetric).

4. Conclusions

A bootstrap based method to calculate confidence intervals for reverberation time measurements has been proposed and validated. Results of the validation process are reasonably good for box-shaped rooms with different reverberation times. Furthermore, the method has been tested against real measurements performed in a real classroom with a more complex geometry that contains a notable amount of active surface, obtaining similar results. Therefore, this method is expected to work well for other shapes due the correspondence between the bootstrap principle and the periodic nature of the problem. The proposed method works pretty well for most kind of box-shaped rooms and reverberation times and it is expected to work well for other geometries. It has been calculated that a minimum of 8 *independent* acoustic decays are needed to guarantee the convergence to a nominal confidence level of 95 %. Independent acoustic decays could be considered if during the placement of microphone source, symmetric situations are avoided in the set of positions. It has been shown, by observation of the convergence of the calculations of confidence levels, that the set of measurements taken should provide at least 8 independent acoustic decays to guarantee a confidence level of 95 %.

Future work should be oriented to test the proposed method in real measurements in other kind of rooms, in order to validate the conclusions of this investigation and the proposed method for other kind of enclosures. It would be interesting to test the method for lower BT products, because then the error due the filtering becomes the dominant factor.

References

[1] Davy J. The variance of decay rates in reverberation rooms. *Acustica*, 43(1):12–25, August 1979.

[2] J.L. Davy. The variance of impulse decays. *Acustica*, 44:51–56, 1980.

[3] J. L. Davy. The variance of decay rates at low frequencies. *Appied Acoustics*, 23:63–79, 1988.

[4] Pontus Thorsson Klas Hagberg. Uncertainties in standard impact sound measurement and evaluation procedure applied to light weight structures. In *Proceedings of 20th International Congress on Acoustics, ICA 2010*.

[5] D. P. Cabo, M. A. Sobreira, and F. Jacobsen. A monte-carlo investigation of the uncertainty of acoustic decay measurements. In *Proceedings of Euronoise*, Prague, 2012.

[6] Sobreira M., Cabo D. P., and Jacobsen F. The influence of the group delay of digital filters on acoustic decay measurements. *Applied Acoustics (2012).*, 73(9):827–833, September 2012.

[7] J. L. Davy. The variance of impulse decays. *Acustica*, 44(1):51–56, January 1980.

[8] B. Efron. Bootstrap methods: Another look at the jackknife. *The Annals of Statistics*, 7(1):1–26, 1979.

[9] B. Efron and R.J. Tibshirani. *An Introduction to the Bootstrap*. Chapman and Hall, New York, 1993.

[10] D.N. Politis. Computer-intensive methods in statistical analysis. *IEEE Signal Processing Mag*, 15(1):39–55, January 1998.

[11] A. M. Zoubir and B. Boashash. The bootstrap and its application in signal processing. *IEEE Signal Processing Mag*, 15(1):56–70, January 1998.

[12] M. Kob and M. Vörlander. Band filters and short reverberation times. *Acustica united with Acta Acustica*, 86:350–357, 2000.

[13] Alan B. Coppens Lawrence E. Kinsler, Austin R. Frey and James V. Sanders. *Fundamentals of Acoustics*. John Wiley and Sons, fourth edition, 2000.

[14] ISO IEC guide 98-3. Guide to the expression of uncertainty in measurement, suplement 1: Propagation of distributions using a Monte Carlo method., 2008.

[15] Rodriguez Molares A. *A Monte Carlo approach to the analysis of uncertainty in acoustics*. PhD thesis, Escuela de Ingeniería de Telecomunicación. University of Vigo, 2011.

[16] ISO 3382-1:2009. Acoustics –Measurement of room acoustic parameters – part 1: Performance spaces.

[17] ISO 3382-2:2008. Acoustics –Measurement of room acoustic parameters – part 2 reverberation time in ordinary rooms.

[18] M.R: Schroeder. New method of measuring reverberation time. *Jounal of The Acoustical Society of America*, 37:409–412, 1965.

[19] I. Jáuregui Novo. Modelado y validación de acústica en aulas. Master's thesis, E.T. de Telecomunicación, Vigo, 2014.

[20] F. Jacobsen. A note on acoustic decay measurements. *Journal of Sound and Vibration*, 115(1):163–170, May 1987.