Mechanical characterisation of acoustic foams: fractional derivatives approach

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Summary

Porous materials like polymer foam and glass wool are widely used for noise control in several engineering activities such as aeronautics and automotive industries. Their properties are two-fold: sound absorption and damping of the nearby structure.

Generalized constitutive relationships of viscoelastic foams are investigated in which the time derivatives of integer order are replaced by derivatives of fractional order. To this point, the justification of such models has resided in the fact that they are effective in describing the behavior of real materials. In this work, the three-parameter fractional Kelvin Voigt model is compared to the four-parameter fractional Zener model in frequency domain and applied to the prediction of the relaxation function in time domain. These three-parameter and four-parameter models are theoretically analyzed in time domain. The fitted storage and loss moduli over wide range of frequency, up to 10 kHz, can be used in the Biot poroelasticity theory to predict the sound absorption coefficient of the foam.

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1. Introduction

The Biot theory of fluid-saturated porous media provides a description of the waves propagating in soils (water-saturated rocks) [2]. Several authors have extended this theory to sound-absorbing materials, such as glass wool and plastic foams, for noise control applications in engineering activities such as aeronautics and the automotive industries [2]. When the porous material is bounded onto a vibrating structure or is excited at strong sound levels, the skeleton of the foam cannot be considered anymore as rigid and its acoustical behavior can be described by Biot's theory where the elastic properties of the solid phase are needed.

In order to characterize the mechanical behaviour of viscoelastic materials, experiments can be carried out either in frequency or in time domains. In the engineering applications, the relaxation or retardation time function, G(t) or J(t), are extracted from experimental curves of complex moduli versus frequency. The aim of this paper is focused on the experimental determination of the relaxation function G(t) and to the extraction of the complex modulus in frequency domain through a single measurement. This avoids the long and sometimes difficult series of measurements at several temperatures or in a wide frequency range[3,4].

The section 2 is devoted to the presentation of the three-parameter Fractional Kelvin-Voigt Model (FKVM) [5-7] in time domain. Relaxation tests on acoustic foam and FKVM time domain analysis are developed in section 3. The obtained results are presented and discussed in this section.

2. FKVM analysis of relaxation experiments

In the literature of viscoelasticity, mechanical models of springs and dashpots have been used to represent the viscoelastic properties of materials. For instance, the elements of the models are disposed singly and in branches of two (in series or in parallel). Such a model was considered by Zener in 1948 with the denomination of standard linear solid and will be referred here as the Zener model (Fig. 2).

A model with fractional derivatives is a sort of interpolation between viscous and elastic behaviour. It is known that a spring element connects the stress with the zero-order derivative of the strain, while for a viscous element the stress is proportional to the first time derivation of the strain.

Preliminary compression tests were performed by means of a tensile machine where the crosshead speed was fixed at 10 cm.min^{-1}. In static loading, the typical load-displacement curves exhibit a first zone where the slope (dF/dt) is increasing and a second zone having a constant slope. The no-slip boundary conditions imposed by the rigid plates during the beginning of the loading create thin layers close to the boundaries with a complex strain distribution and shear stress where the cells collapse. This collapsing zone is followed by a static compression in the whole sample.

In relaxation tests, a constant strain level (ε_0 = 3%) is applied to the melamine foam sample. While such a strain jump requires infinite strain rate, the loading velocity is limited by the experimental set-up. Therefore, the maximum possible strain rate shall be used ($\dot{\varepsilon}_{max}$ = 0.032 s^{-1}) during t_1= 0.9 s.

The imposed strain is:

$$\varepsilon(t) = \varepsilon_0 H_1(t) \quad (1)$$

where $H_1(t)$ is plotted in Fig. 1. This function is real loading one, which is an approximation

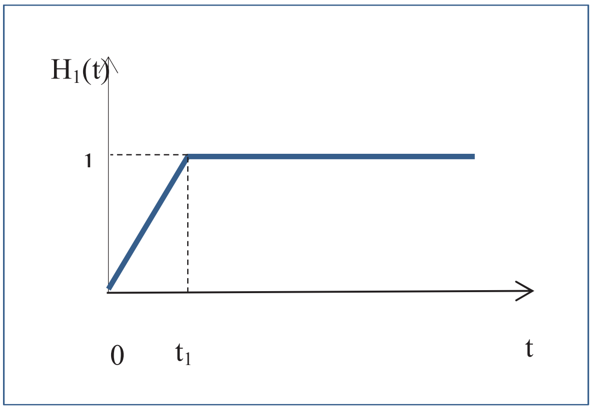

Fig. 1: Shape of imposed strain

of Heaviside function for small value of t_1. This explain why the infinite branch of G(t) is never reached in practice.

During the tests the normal stress $\sigma(t)$ is measured and the experimental relaxation function is plotted in Fig. 3 and will be compared to the simulated FKVM in the next section. As indicated before, the load-time curve in the beginning of compressive tests must be corrected as illustrated in Fig. 2 where the initial time of loading is shifted to t_0. Then, in the next section, the considered loading duration t_1 is reduced to t_1 - t_0 = 0.5 s. For all these reasons the data in the short loading zone are inaccurate and are not useful in the fitting process.

3. Results and discussion

The relaxation tests were performed in the previous section on three samples with the same experimental conditions.

In the fitting process on relaxation function $G_1(t)$ plotted in Fig. 3 we must take account of the loading zone schematised in Fig. 2; as mentioned above, only the data from t_0 are considered. The three parameters are fitted by a least-mean-square algorithm (m = 164 kPa, b = 44 kPa and α= 0.31) are finally used in frequency domain as illustrated in Fig. 5 for the prediction of storage and loss moduli. In this contest, it may be noted that this fitting process can be simplified with the asymptotic value of $G_1(t)$. If the relaxation test duration is sufficiently large, we obtain [5]:

$$G_1(t) \approx m + \frac{b}{\Gamma(2-\alpha)} t^{-\alpha} \quad (2)$$

If the experiments are performed over a sufficiently longer time, the asymptotic value m of G_1 can be approximated by its final value: $G_1(600)$ in our case (Fig. 4). α and b can be calculated more easily in logarithmic scale of:

$$G_1(t) - G_1(600) = \frac{b}{\Gamma(2-\alpha)} t^{-\alpha} \quad (3)$$

This simple calculation leads to m = 172 kPa, α= 0.34 and b = 41 kPa which are close to the values obtained above.

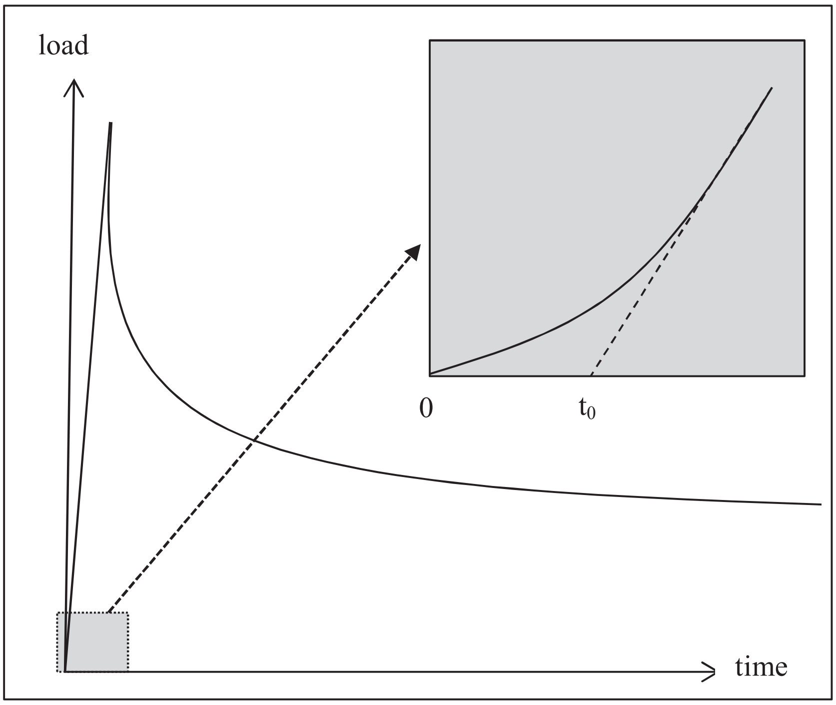

Fig. 2: Typical loading zone in relaxation tests.

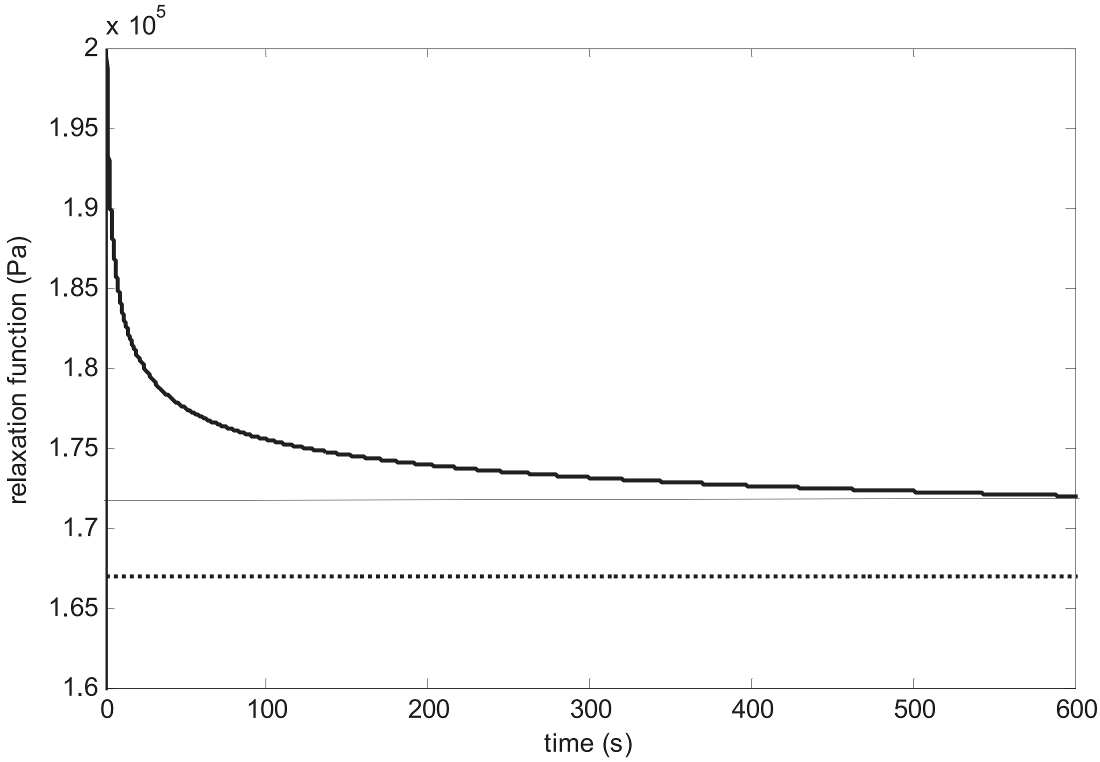

Fig. 3: Experimental relaxation function with the calculated asymptote (dotted line).

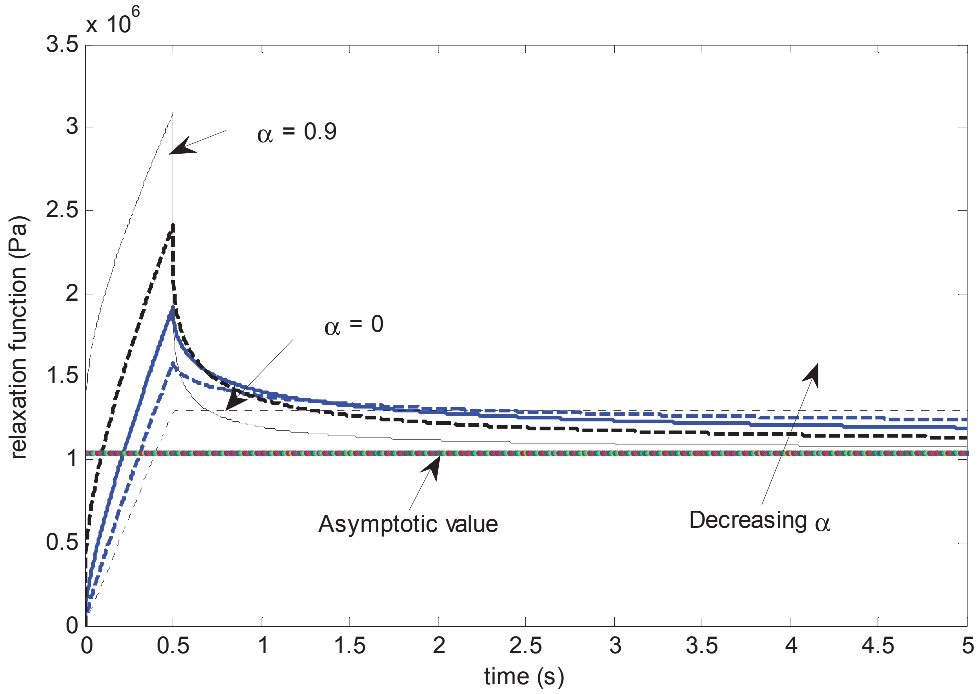

Fig. 4: Relaxation function $G_1(t)$ for $\alpha = 0; 0.3; 0.5; 0.7$ and 0.9; the Bagley parameters m = 1.05 MPa and b = 0.24 MPa.

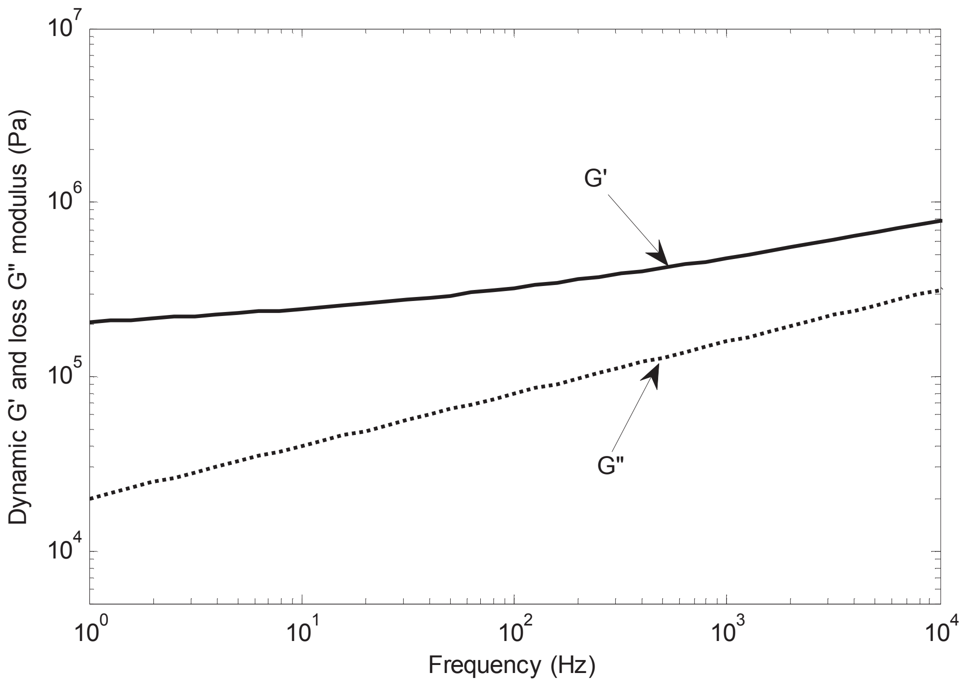

Fig. 5: Frequency dependences of the dynamic and loss moduli of melamine foam

4. Conclusion

The purpose of this work was to investigate the conditions when the three-parameter fractional Kelvin Voigt model can be used the mechanical characterisation of acoustic foams. By using bibliographic data, a comparison in the frequency domain shows a good agreement between this model and fractional Zener model for frequencies ranging up 10 kHz. These models was analysed in time domain and their mathematical and physical background have been outlined.

In order to characterise the viscoelastic material properties of melamine foams an experimental relaxation test with compressive specimens has been carried out. The FKVM is capable of describing with an efficient method the variation of the dynamic and loss moduli over a range of frequencies used in sound absorption applications.

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