New ways of lumped parameter analysis in an enclosed environment.

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Summary

When the prediction of acoustics needs to be done accurately, and fast. It is good to use a lumped parameter approach. Nowadays Sabine's equations are leading when making lumped parameter analysis, but Eyring already proposed to skip some of the simplifications in order to analyze anechoic rooms. In this paper new equations for lumped parameter room acoustics are derived, that could be used regardless of the type of room. The mathematical simplification of Sabine turns out to lead to high inaccuracies at higher absorption values in anechoic rooms, this is consistent with experience in the field.

In case of the determination of the sound reduction index, both the new, and Sabine equation work well in laboratory conditions. In a room with more sound absorption the use of the new equation proposed in this paper is a necessity. The new equation has considerable consequences throughout the field of room acoustics. For the reverberant distance, a more general equation is proposed.

1. Introduction

Practice shows a difference in the sound reduction index between highly reflective rooms, and anechoic rooms. In absorbent rooms the sound reduction index seems to decrease without a logical explanation. This phenomenon needs to be explained in order to make accurate predictions in room acoustics. Generally the evaluation of the sound insulation value is done by the equations of Sabine [1].

A possible cause for the negative effect on the sound reduction index could be the simplifications that are involved in the analysis of the room acoustics based on Sabine. In the current paper the mathematical lumped parameter analysis of room acoustics is expressed in Chapter 2.1. The expressions gained differ from Sabine [1], and Eyring [2]. The difference between these expressions, and previous work is quantified in Chapter 2.2. In chapter 3.1, the influence of the direct field is discussed as a second possible cause for a lower reduction index in anechoic rooms. In chapter 3.2 the influence of the lumped parameter analysis leads to a new equation for the reverberant distance.

2.1. Lumped parameter room acoustics

When an acoustic source is reflected in an enclosed area, not only the source, but also the reflections of the source can be heard. In theory the number of reflections of the source are endless, and the acoustic energy that remains in the room after a certain number of reflections equals:

$$E_a = E_{a,0} * (1-\alpha)^n \qquad (1)$$

Where $E_{a,0}$ is the acoustic energy introduced to the room in a timeframe till the first reflection, α is the sound absorption coefficient, and n is the (average) number of reflections. For noise control it is interesting to know how the infinite number of reflections influence our hearing, or a measurement. Therefore first the cumulated sound power that is preserved in the room is determined:

$$E_{a_{total}} = E_{a,0} * \sum_{n=0}^{\infty}(1-\alpha)^n$$
$$= E_{a,0} * \left(1 + \frac{1}{\left(\frac{1}{1-\alpha}\right)-1}\right) = E_{a,0} * \left(\frac{1}{\alpha}\right) \qquad (2)$$

Thus the infinite sum of reflections can be simplified dramatically. This also implies room acoustics analysis can be done rather easily, as long as the average absorption value of a room can be determined. Instead of measuring the absorption value, the absorption value is determined from measurements of the reverberation time. The reverberation time is the time it takes to reduce an initial sound level by 60 dB.

$$t = T \quad when \quad \frac{E_a(t)}{E_a(0)} = \frac{1}{10^6} \tag{3}$$

Where t is the time, T is the reverberation time, and L_p is the sound pressure level. Note that:

$$\frac{10^{0.1Lp(t)}}{10^{0.1Lp(0)}} \approx \frac{10^{0.1Lw(t)}}{10^{0.1Lw(0)}} \approx \frac{E_a(t)}{E_a(0)} = (1-\alpha)^{\frac{ct}{p}} \tag{4}$$

Where c is the velocity of sound, L_w is the sound power level, and p is the mean free path. Since:

$$(1-\alpha)^{\frac{ct}{p}} = 10^{-6} \tag{5}$$

,When t = T, the average absorption value can be found as a function of the reverberation time:

$$\alpha = 1 - 10^{-\left(\frac{6p}{cT}\right)} \tag{6}$$

And vice versa:

$$T = \frac{-6p}{c * \log(1-\alpha)} \tag{7}$$

The mean free path according to Sabine [1], and Eyring [2]:

$$p = 0{,}62 * V^{0{,}33} \tag{8}$$

$$p = \frac{4V}{S} \tag{9}$$

Where V is the volume of the room, and S is the internal boundary area. They have also adopted the assumption that the sound field in a room is omnidirectional homogeneous. This assumption will be adopted in this paper, so their equations for the mean free path can be used.

Under the assumption that the sound path is evenly distributed, or well averaged by the measurement procedure, the measured value could be predicted. The reflective sound will after each reflection, since the sound is assumed to be evenly distributed over the boundary surface, occupy a volume that equals the absorption area multiplied by the mean free path, this will be called the fictitious volume. If the fictitious volume is relatively high to the real volume, a higher sound pressure level is to be expected.

For example a room of 3 by 4 by 5 m. has a surface area = 94 m^2, while the volume equals 60 m^3. The mean free path is either 2.43 or 2.55 m, for respectively equation 8 or 9. The fictitious volume is equal to the volume of the room multiplied by respectively 3.8 or 4. So roughly 4 times more sound will be measured as impacts the boundary area. This is logical when you consider the boundary area cannot, in contrast to a random point within the room, be approached from all directions. While the measured sound pressure is influenced by pressure fluctuations coming from all directions.

This may seem like the sound in the receiving room is overvalued by a factor 4, and the Sound insulation value in a laboratory is thus underestimated by 300%, but then one would forget that on the sending room the same effects hold, and only about $1/4^{th}$ of the sound pressure level measured in the sending room radiates the separating element. This relation forms the basis behind the theory behind the commonly used diffusivity correction.

Note that when there is only direct radiation on the separating element, the reduction index could increase due to the lack of flanking sound.

2.2 Quantification of the difference between the proposed equations, and the equations based on Sabine.

Now the theoretical model is complete to evaluate the room acoustics and compare this mathematical model to the theory based on Sabine [1]. The origin of the differences between Sabine [1] follows from a simplification. Eq. 7 can be rewritten as:

$$T = -\frac{6\ln(10)}{c} * \frac{p}{\ln(1-\alpha)} \quad (10)$$

Then when eq. 9 is used to determine p, and:

$$\ln(1-\alpha) = -\alpha - \frac{\alpha^2}{2} - \frac{\alpha^3}{3} - \frac{\alpha^4}{4} \ldots \approx -\alpha \quad (11)$$

Sabine [1] can be derived:

$$T = -\frac{24\ln(10)}{c}\frac{V}{S\alpha} = 0.16\frac{V}{S\alpha} = 0.16\frac{V}{A} \quad (12)$$

Where A is Sα. In other words Sabine [1] neglecting all but the first term of the Taylor expansion. When the number of α is very low the simplification works pretty well, but for higher values of α the simplification of Sabine causes a higher defect. When the velocity of sound is 340 m/s, eq. 7 and 9 can be used to derive:

$$T = \frac{-24V}{S*340*log(1-\alpha)} = -0.07\frac{V}{S*log(1-\alpha)} \quad (13)$$

This equation is valid for any type of room. Because the relation between T and α is different from the relation stated by Sabine[1], and Eyring [2], it would be more clear to call α in eq. 2, 4, 5, 6, 7, 10, and 13 α_{vDijk}, and α in eq. 12 α_{Sabine}.
These differences can be seen more clearly in figure 1. Where the deviation is a function *y(x)*:

$$y = \frac{T(\alpha_{Sabine})}{T(\alpha_{vDijk})} - 1 = \frac{0.16\frac{1}{x}}{-0.07\frac{1}{log(1-x)}} - 1 \quad (14)$$

Eyring [2[has made a different relation between reverberation time, and sound absorption for dead rooms [2]:

$$T = \frac{-0.05V}{S\,log(1-\alpha)} \approx 0.05\frac{V}{S\alpha} \quad (15)$$

The full equation has a different constant than eq. 13. The deviation becomes a plotted in green in figure 2:

$$y_2 = \frac{T(\alpha_{Eyring})}{T(\alpha_{vDijk})} - 1 = \frac{0.05}{0.07} - 1 \quad (16)$$

The simplified equation of Eyring [2] is shown in red in figure 2, based on the following equation:

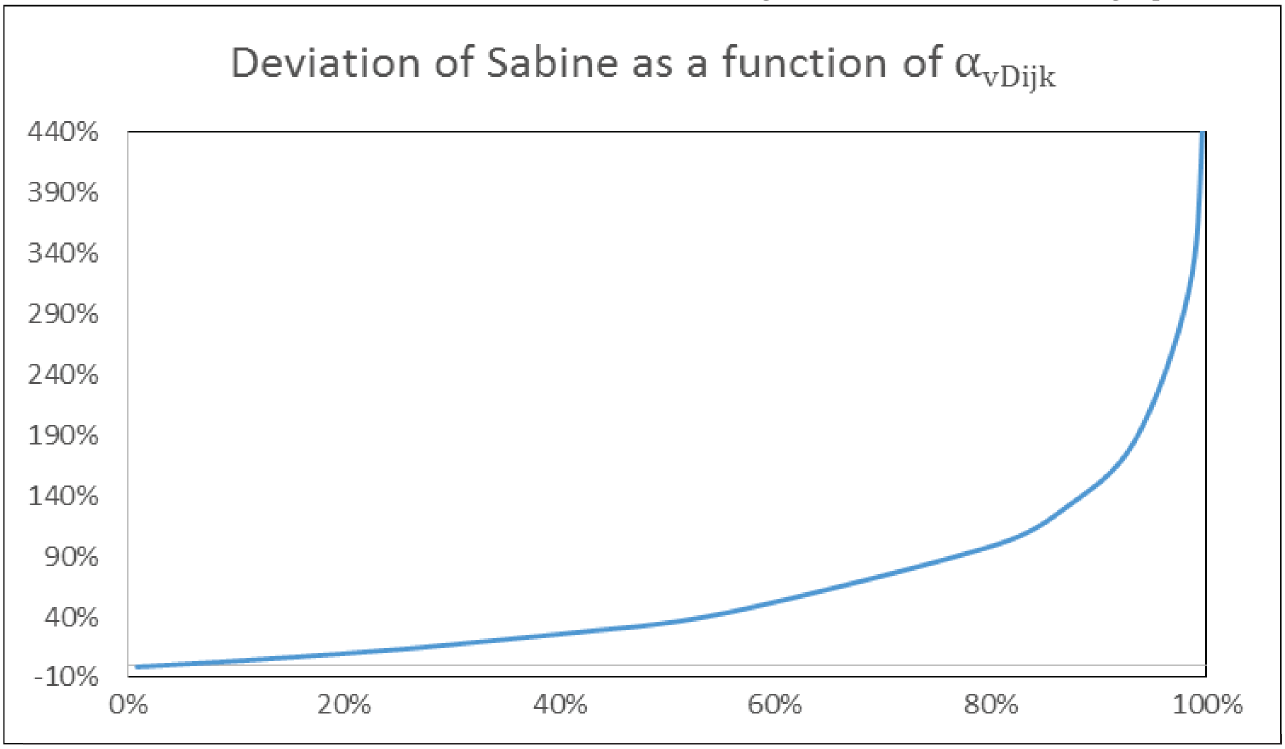

Figure 1. Deviation of Sabine from the mathematical model as a function of the real average α value.

$$y_3 = \frac{0.05\frac{1}{x}}{-0.07\frac{1}{log(1-x)}} - 1 \tag{17}$$

These relations state the error that occurs when the reverberation time or other properties of the room, that are related to the reverberation time are calculated from a value of α, according to eq. 7. Note that this error is also to be expected for the sound pressure level.

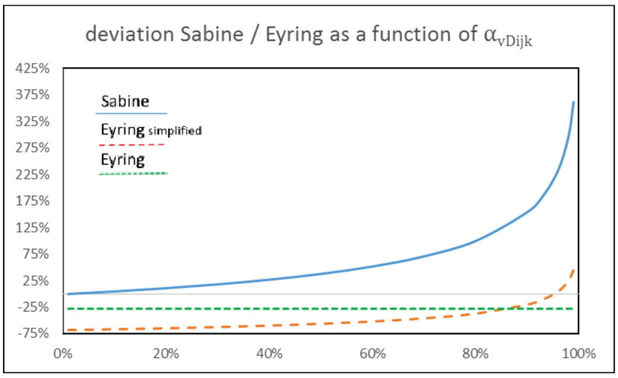

Figure 2 Deviation of the predictions made by Eyring, and Sabine as a function of the mathematical average absorption coefficient α_{vDijk}

When the properties of a room are not predicted, the absorption coefficient can be determined from the reverberation time. The error in the determined value of α as a function of α_{vDijk} is the same as in figure 1 and 2. However it is common practice to determine α_{Sabine}, therefore the deviation as a function of α_{Sabine} in figure 3.

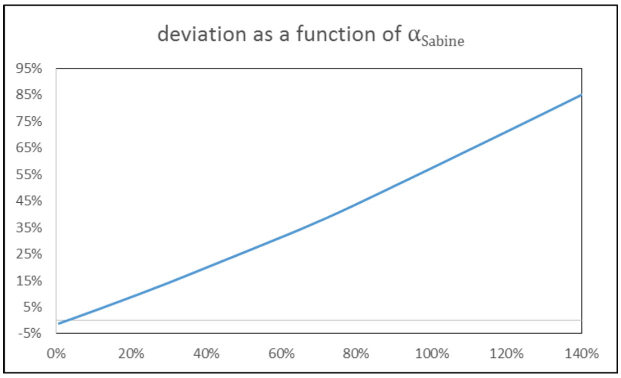

Figure 3 deviation as a function of α_{Sabine}

Note that in theory there is no limit to the value of α_{Sabine}, whereas α_{vDijk} cannot exceed unity.

One of the basic equations in the determination of the sound reduction index states the relation between the average α-value of the room multiplied by the total room surface, the boundary surface, and two sound pressure levels [3]:

$$R = L_S - L_R + 10 * \log\left(\frac{S_c}{A}\right) \tag{18}$$

Where L_S, and L_R are respectively the sound pressure level in the send room, and the adjacent receiving room, and S_c is the contact area between both rooms.
Note that part of eq. 10 can be rewritten as:

$$10\log\left(\frac{S_c}{A}\right) = 10\log(S_c) - 10\log(S) - 10\log(\alpha) \tag{19}$$

So there are three corrections done on the raw measured sound pressure level difference. The correction for the contact surface is a consequence of the definition of the sound reduction index, which is defined per area. The other two corrections are to correct for the accumulation of sound as proven in eq.2, and the geometry where the internal boundary area gets evenly irradiated by the sound as any other receiving surface. Eq. 19 can be applied in combination with eq. 7 or 13.

3.1 Influence of direct field

In this part, the contribution of the direct field in the appointed measurement positions is reviewed. In reverberant laboratory rooms international standard requirements are stated for the reverberation time:

$$1s \leq T \leq 2\left(\frac{V}{50}\right)^{\frac{2}{3}} s \tag{20}$$

In an example would be a room of 5 by 4 m, with a height of 3 m, the range of acceptable values of α could be determined:

$$\alpha_{max} = 1 - 10^{-\left(\frac{24V}{cS}\right)} = 1 - 10^{-\frac{24*60}{340*94}} = 0.099 \tag{21}$$

$$\alpha_{min} = 1 - 10^{-\left(\frac{24V}{2cS\left(\frac{V}{50}\right)^{\frac{2}{3}}}\right)} = 1 - 10^{\frac{-24*60}{2*340*94*\left(\frac{60}{50}\right)^{0.67}}} = 0.045 \tag{22}$$

So the direct filed only contributes to $\frac{1}{\frac{1}{0.045}}, or\ \frac{1}{\frac{1}{0.099}}$, which is between 4.5 and 9.9% of the total sound power level present in the room. Also the low value of α implies that the deviation from using Sabine is less than 10%. So for small laboratories the sound reduction index can be determined with Sabine within a mathematically acceptable accuracy.

In an anechoic room however, the sound reduction is much higher, than the apparent sound reduction index that is determined in the field. Because the value of A in eq. 18 can be highly overestimated, when using eq. 12, as shown in figure 1 and 3. This means that a field measurement of the apparent sound reduction index, does not necessarily reflect on the quality of the installed element. Unless eq. 12 gets replaced by eq. 7.

In an anechoic room the contribution of the direct field is relevant. However the direct field is not irradiated from all sides. So the contribution of the direct field is highly dependent on the measurement positions, and the geometry of the source.

When a sound reduction index is determined there are speaker positions in one room, and an adjacent room, where the only relevant sound source is the separation element. In a field measurement where the direct field is relevant, it is interesting to know how the microphone positions influence the measured values of the direct field. Therefore four examples are chosen. In two cases (B, and D) a point sound source is in the middle the separating wall, and in the other cases (A, and C) the separating wall is the sound source itself. The size of the separating wall is the width, times the height, as mentioned in table 1.

The direct sound pressure originated from a point source is:

$$L_{p,direct} = 10 * \log\left(\frac{1}{2Л(x^2+y^2)}\right) + L_w \qquad (23)$$

The sound pressure originated from a separating wall is:

$$L_{p,direct} = 10 * \log\left(\frac{1}{bh+\frac{2}{3}Лy^2+Л(b+h)y}\right) + C_{prox} + L_w \qquad (24)$$

Where b is the width, h is the height, and y Is the distance from the separating element, and x is the distance from the source in the orthogonal direction. C_{prox} is the proximity correction [4], which is 1 dB as long as:

$$\frac{bh}{bh+\frac{2}{3}Лy^2+Л(b+h)y} \geq 0.4 \qquad (25)$$

Theoretically the lumped value

$$L_{p,direct,lumped} = 10 * \log\left(\frac{4}{S}\right) + Lw \qquad (26)$$

Table 1: Direct field approaches in four cases

Case	average	lumped	length	width	Height	D/E
A	2,21E-02	4,26E-02	5 m	4 m	3 m	52%
B	3,43E-02	4,26E-02	5 m	4 m	3 m	81%
C	1,06E-02	1,85E-02	6 m	10 m	3 m	57%
D	1,38E-02	1,85E-02	6 m	10 m	3 m	74%

Table 1 shows *D/E*, which is the contribution of the direct field divided by the expected contribution of the direct field. The direct field is likely to contribute less to the measured value than expected. This phenomenon thus contradicts that in anechoic rooms lower sound reduction index values are found. The D/E value in table 1 is low, partly because of the asymmetric boundary conditions for measurement positions. This reduces the apparent defects in the use of Sabine. When a critical part in the separating element is responsible for the sound reduction index value (case B and D), the contribution of the direct sound is relatively accurately predicted, and a lower sound reduction index is to be expected.

3.2 Reverberant distance

In a concert hall, it is interesting to know at what radius the direct field has an equal contribution to the sound that is heard as the indirect field. This is called the reverberant distance, defined by Kuttruff [3]:

$$r_h = 0,056\sqrt{V/T} \qquad (27)$$

The direct field sound pressure level in a room originated from a point source is:

$$L_{p_d} = L_w - 10 * \log(4Лr^2) \quad (28)$$

Where r is the distance from the receiver till the source. The reverberant field sound pressure level at a random point in the room is:

$$L_{p_r} = L_w + 10 * \log\left(\frac{4\left(\frac{1}{\alpha}-1\right)}{S}\right) \quad (29)$$

Wherein the number 4 comes from the use of eq. 9 for p as described at the end of chapter 2.1. Note that now instead of 1/α, as in eq. 2, 1/α-1 is used, because of the missing contribution of the direct field.

When $L_{Pd} = L_{Pr}$, then $r = r_h$. So the reverberant distance becomes:

$$r_h = \sqrt{\frac{S}{16\pi * (1-\alpha)}} \quad (30)$$

Eq. 7,9, and 27 can be combined to form:

$$r_h = 0{,}056\sqrt{c}\sqrt{\frac{S\, log(1-\alpha)}{-24}} \quad (31)$$

Then it is immediately clear that eq. 30, and 27 differ in their relation to the term α. However when eq. 27 is combined with eq. 9 and 12 the following equation can be stated:

$$r_h = 0{,}056\sqrt{\frac{S\,\alpha}{0{,}16}} \quad (32)$$

Note that when the direct field is not neglected in eq. 29:

$$L_{p_r} = L_w + 10 * \log\left(\frac{4\left(\frac{1}{\alpha}\right)}{S}\right)$$

$$\frac{1}{4Лr^2} = \frac{4\left(\frac{1}{\alpha}\right)}{S},\ r^2 = \frac{S\alpha}{16Л}$$

$$r_h = \sqrt{\frac{S\alpha}{16\pi}} \quad (33)$$

Which is the same as eq. 32. In other words, the misinterpretation of the defined phenomenon in the field of room acoustics that follows from the simplification of Sabine is widespread. A great number of general relations may need to be reviewed.

4. Conclusions

The relation between the absorption coefficient α in a room, and the reverberation time should be written as: $\alpha = 1 - 10^{-\left(\frac{6p}{cT}\right)}$
Where p is the mean free path, and c the speed of sound, this equation is suitable for any type of room. This has consequences on a lot of derived equations. The use of this equation enables the accurate determination of α, and the prediction of sound pressure levels in anechoic rooms. The reverbarent distance should be expressed as: $r_h = \sqrt{\frac{S}{16\pi(1-\alpha)}}$

Acknowledgement

This project has been supported by Alara-Lukagro. Special thanks go out to Theo Campmans, Michiel Bos, and Wout van Dijk, whom have supported this work in different stages of the process.

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