Sound synthesis using differential geometry: the integrable system of a nonlinear string with large displacements

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Using differential geometry, nonlinear propagation of waves can be formalized in an intrinsic way (avoiding artificial nonlinearities corresponding to a particular choice of coordinates systems). Following Arnold's geometric approach, a dynamical system is considered as a map taking values in an abstract Lie group. For a simple one-dimensional acoustical system, it gives rise to the Reissner beam model for which the motion of each different section, labelled by the arc length s, is encoding in the Special Euclidean Lie group $SE(3)$ - a natural choice to describe motion in 3-dimensional space. It turns out that, fortunately as a map over space-time, this multi-symplectic approach can be related to the study of harmonic maps for which two dimensional cases can be solved exactly. It allows us to identify, among the family of problems, a particular case where the system is completely integrable. Among almost explicit solutions of this fully nonlinear problem, it is tempting to identify solitons, to propose some sound synthesis and to test the known numerical methods on these solutions.