PENCIL METHOD FOR ULTRASONIC BEAM COMPUTATION

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Abstract

Ultrasonic beam computation has grown more and more important during the latest years, specially for NDE applications. Indeed, it constitutes a helpful tool for the design of transducers, the definition of testing configurations, or the interpretation of experiments. The method developed at the French Atomic Energy Commission for field computation is based on the propagation of pencils, since this approach provides both numerical efficiency and accuracy, and since it is able to deal with complex configurations: arbitrarily shaped broadband transducers, components with complex shape and constituted of complex media. The derivation of the pencil method, in particular for complex materials, is addressed in this paper. A pencil is used to describe the contribution, at a given observation point, of an elementary wave generated by a point source located on the emitting surface of the transducer. One can demonstrate that the contribution of this elementary wave can be approximated by considering the evolution of a cone of rays, i.e. a pencil, in the vicinity of the geometrical path between this point and the observation point. A pencil is mathematically described *by* a four component vector, at any position along the geometrical path. Consequently, the propagation of a pencil between two points is given by a four by four matrix, linking two of these pencil vectors. This pencil propagation matrix is expressed as a function of the media and the interfaces crossed. When anisotropic media are involved, they are taken into account through their slowness curves that appear in the expression of the pencil propagation matrices. In the same way, curved interfaces can be considered. Thus, components as complex as heterogeneous, anisotropic and with curved inner interfaces can be considered. Inhomogeneous materials can also be treated. After the description of the method, some numerical examples of computed fields into complex media are given.

Introduction

Ultrasonic beam computation is of major interest in non destructive evaluation. Indeed, beam modelling is useful to help the interpretation of NDT experiments, and to assist the design of transducers, the definition of tests, etc. Computed beams are also used as inputs for beam / defect interaction modelling. For these reasons, the French Atomic Energy Commission has been developing a module dedicated to beam computation in its software platform for NDT expertise CIVA. The aim of this paper is to give an overview of the theory of pencils that is used to predict the ultrasonic fields in this module.

To deal with realistic configurations of test, it is required that the method used is able to deal with various kinds of transducers (immersed, contact, phased arrays, etc.) and various kinds of components (anisotropic, heterogeneous, inhomogeneous, etc.). Numerical efficiency is also required for the intensive use of the model. For these reasons, analytical formulations are derived wherever it is possible, i.e. each time it does not imply any loss of generality.

The radiation of the transducer is described mathematically by a diffraction integral (Rayleigh's integral). Since no assumptions are made on the shape of the transducer, this integral is evaluated numerically. The emitting surface of the transducer is thus discretized into point sources, and the global field is a summation of the elementary fields of every point sources. Each point source emits an elementary field that is given by the Green function. This Green function is evaluated by means of the pencil method approximation, that is described in what follows. This method has been developed so that various kinds of components can be taken into account: isotropic, anisotropic, heterogeneous or inhomogeneous can be considered.

The field is computed in terms of impulse responses in order to deal with wideband transducers.

In the first part of this paper, the theory of pencils is described. The method used to evaluate an elementary field due to a point source at a given observation point is derived. In the second part, the propagation of pencils in the different types of materials is addressed. Isotropic, anisotropic, heterogeneous, and inhomogeneous media (for which the wave velocity is varying linearly as a function of the depth) are considered. In the third part, some examples of computed fields are shown. And finally, concluding remarks are given.

The pencil method

It is well known that a spherical wave propagating into an infinite homogeneous and isotropic medium can be approximated by a plane wave which amplitude decreases as a function of the inverse of the distance of propagation. This approximation is valid as soon as the distance between the point source and the observation point is greater than a couple of wavelengths. The pencil method allows to generalize this approximation to the case of complex media.

If one considers a point source immersed in a fluid, it emits a spherical wave that may be distorted when it crosses interfaces of complex shapes or when it propagates into complex media, such as anisotropic media. If an observation point is fixed, the geometrical acoustics principle says that the acoustical energy provided by the source at the observation point mainly propagates along a particular path, called the geometrical acoustics path, or Fermat's path. This path starts at the source point and ends at the observation point, and the Snell-Descartes law applies at each interface along it. This path is also called stationary phase path, since it corresponds to the wave vector for which the phase is stationary when an angular spectrum decomposition is investigated [1, 3].

If this path is known, a wave vector, and thus a plane wave, is selected. A time of propagation t_p can be evaluated along this path. If the point source is excited by a Dirac δ-function at time t=0, its contribution at the observation point will appear at time $t= t_p$. The theory of pencils is used to evaluate the amplitude of this contribution.

Definition of a pencil

In order to evaluate the amplitude of the contribution, one studies the divergence of a pencil, following Deschamps [2] who developed the method in electromagnetics. A pencil is a cone of rays located around the geometrical acoustics path. Let the axial ray be the ray along this path, and the paraxial ray belonging to the envelope of the pencil (see figure 1).

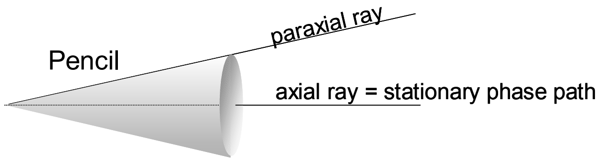

Figure 1: Definition of the axial and paraxial rays.

The vertex of the cone is located at the source point. The shape of the pencil can be described mathematically at any position along the stationary phase path if the position of the paraxial ray with respect to the axial ray is known. One then defines dx and dy, being the coordinates of the paraxial ray in a plane perpendicular to the axial ray. If $dx=dy=0$, the axial and paraxial rays coincide. If the direction of the paraxial ray is also known, one can evaluate the evolution of the pencil with the propagation. This direction is given by the projection of the slowness vector associated with the paraxial ray on the plane previously defined, say ds_x and ds_y. One then defines a four-component vector, called the pencil vector, constituted of these four quantities.

To describe the propagation of a pencil, one uses a 4x4 matrix **L**, called pencil propagation matrix, that sets a linear relation between two pencil vectors ψ and ψ' describing the same pencil, but at different times. One has:

$$\boldsymbol{\psi}' = \mathbf{L} \cdot \boldsymbol{\psi} \qquad (1)$$

The components of **L** will be defined in the next parts of this paper, as a function of the media and interfaces through which the pencil propagates. One also defines four 2x2 sub-matrices A, B, C and D, so that:

$$\mathbf{L} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \qquad (2)$$

The evolution of the amplitude of the elementary wave radiated by the point source is given by the evolution of the section of the pencil.

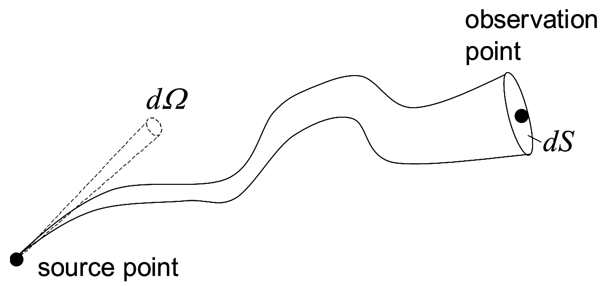

Figure 2: Evolution of a pencil from the point source to the observation point.

Let $d\Omega$ be the solid angle covered by the pencil at the point source and let dS be the section of the pencil at the observation point. The divergence factor is defined by $DF^2= d\Omega\,/\,dS$. It describes the divergence of the wave emitted by the point source and, consequently, the amplitude of this wave at the observation point. Since both $d\Omega$ and dS can be expressed with the help of ψ and ψ', the divergence factor is also written as:

$$DF^{-2} = s_1^2 \det \mathbf{B} \qquad (3)$$

where s_1 is the slowness in the medium of the source.

This means that one just needs to establish the formulation of the pencil propagation matrix L in the different configurations to be treated (isotropic, anisotropic, heterogeneous and inhomogeneous media) to derive the amplitude of the contribution of the point source at the observation point. The different cases are addressed in what follows.

Propagation of a pencil in an isotropic medium

Let one considers a pencil propagating inside a homogeneous and isotropic medium. The pencil propagates from point **M** to point **M'**, and the corresponding pencil vectors are respectively ψ and

ψ'. The relation between these two quantities is merely given by a homothetic transformation, since dx' and dy' can be evaluated if ds_x and ds_y are known. The pencil propagation matrix $\mathbf{L}_{iso}$ in this case is written as:

$$\boldsymbol{\psi}' = \mathbf{L}_{iso} \cdot \boldsymbol{\psi} = \begin{bmatrix} \mathbf{1} & \frac{r}{s}\mathbf{1} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \boldsymbol{\psi} \qquad (4)$$

where r is the distance between M and M', and s is the slowness of the wave under study, longitudinal or transversal, **1** is the 2x2 identity matrix and **0** the 2x2 zero matrix.

Propagation of a pencil in an anisotropic medium

In an anisotropic component, the energy direction differs from the wave vector direction. The slowness surface, related to a type of wave, gives the evolution of the phase velocity as a function of the direction. Let $s_z=g(s_x, s_y)$ be the function defining this surface. For a given wave vector on the slowness surface, the corresponding energy direction is given by the normal to this surface.

The axial ray of a pencil is located along the geometrical acoustics path, or stationary phase path, as mentioned previously. This path is defined by the energy direction in an anisotropic medium. The evolution of the pencil is then given by the evolution of the normal to the slowness surface, so that the second partial derivatives of g, denoted as g_{mn}, are involved in the pencil propagation matrix. One has:

$$\boldsymbol{\psi}' = \mathbf{L}_{ani} \cdot \boldsymbol{\psi} = \begin{bmatrix} 1 & 0 & -r_k g_{20} & -r_k g_{11} \\ 0 & 1 & -r_k g_{11} & -r_k g_{02} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \boldsymbol{\psi} \; (5)$$

with g being defined in a basis so that $s_x=s_y=0$ corresponds to the wave vector of the axial ray, and

$$g_{nm} = \frac{\partial^{n+m} s_z}{\partial s_x^n \partial s_y^m} \qquad (6)$$

and r_k is the projection of the distance of propagation on a unit vector parallel to the wave vector.

Propagation of a pencil in a heterogeneous medium

A heterogeneous medium is described by a set of contiguous homogeneous media, possibly anisotropic. The propagation of a pencil in this kind of material is a succession of elementary propagations in homogeneous media. In other words, the propagation of the pencil can be described by the product of pencil propagation matrices for the different media. One has:

$$\boldsymbol{\psi}' = \ldots \mathbf{L}_{prop3} . \mathbf{L}_{inter2} . \mathbf{L}_{prop2} . \mathbf{L}_{inter1} . \mathbf{L}_{prop1} . \boldsymbol{\psi} \qquad (7)$$

where $\mathbf{L}_{prop1,2,3,\ldots}$ are pencil propagation matrices related to the propagation in different media and $\mathbf{L}_{inter1,2,\ldots}$ are pencil matrices that describe the evolution of a pencil when reaching an interface. Indeed, refraction or reflection of the pencil leads to a deformation of the pencil. The curvature of this interface, mathematically described by the 2x2 curvature matrix **C**, is involved in the evolution of the pencil, through the pencil propagation matrix for an interface, that writes:

$$\boldsymbol{\psi}' = \begin{bmatrix} \boldsymbol{\Theta}'\boldsymbol{\Theta}^{-1} & \mathbf{0} \\ h\boldsymbol{\Theta}'^{-T}\mathbf{C}\boldsymbol{\Theta}^{-1} & \boldsymbol{\Theta}'^{-T}\boldsymbol{\Theta}^{T} \end{bmatrix} \cdot \boldsymbol{\psi} \qquad (8)$$

where the 2x2 matrices Θ and Θ' represent the projection of the pencil on the interface in the direction of the axial ray, respectively before and after interaction with the interface, and $h=k'\cos\theta'-k\cos\theta$, with θ and θ' the incident and refracted (or reflected) angles.

For each reflection or refraction on an interface, one can calculate the reflection or transmission coefficient, related to the plane wave associated to the geometrical acoustics path. This quantity is not involved in the shape of the pencil, but it has to be considered as a global factor in the contribution of the point source.

Propagation of a pencil in an inhomogeneous medium

Attention is now paid to inhomogeneous media, that is to say media which mechanical characteristics are smoothly varying as a function of the position considered. Here, the study is focused on media for which the celerity of the wave varies linearly with respect to the depth. One has:

$$c(x_g, y_g, z_g) = a + b z_g \qquad (9)$$

a and b being constants.

In this kind of material, one can demonstrate that the ray paths are arcs of circles, the centre of the circles being located at a depth $z_c = -a / b$. Figure 3 shows an illustration of the propagation of rays in an inhomogeneous material, for which the celerity increases with depth. The angle θ_T is defined as the tangent to the transmitted ray at the interface between the inhomogeneous medium and the medium where the wave previously propagated.

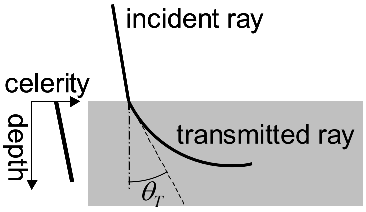

Figure 3: Ray path in an inhomogeneous medium.

The time of propagation from the interface to the point on the curve ray to be considered is denoted as *t*. One wishes to evaluate a pencil vector at this position (say ψ') with respect to the pencil vector at the interface (say ψ). The pencil propagation matrix, that links these two vectors, is written as:

$$\boldsymbol{\psi}' = \mathbf{L}_{inh} \cdot \boldsymbol{\psi} = \begin{bmatrix} \mathbf{A}_{inh} & \mathbf{B}_{inh} \\ \mathbf{C}_{inh} & \mathbf{1} \end{bmatrix} \cdot \boldsymbol{\psi} \qquad (10)$$

with

$$\mathbf{A}_{inh} = \begin{bmatrix} \dfrac{1 - e^{2bt}\tan^2\dfrac{\theta_T}{2}}{\cos\theta_T\left(1 + e^{2bt}\tan^2\dfrac{\theta_T}{2}\right)} & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{B}_{inh} = \begin{bmatrix} \dfrac{a^2 p^2}{b\tan\theta_T} & 0 \\ 0 & \dfrac{a^2\left(e^{bt} - 1\right)}{b} \end{bmatrix},$$

$$p = \frac{\left(1 + \tan^2\dfrac{\theta_T}{2}\right)\left(1 - e^{2bt}\tan^2\dfrac{\theta_T}{2}\right)}{\tan\dfrac{\theta_T}{2}\left(1 + e^{2bt}\tan^2\dfrac{\theta_T}{2}\right)} - \frac{1}{\tan\theta_T},$$

$$\text{and } \mathbf{C}_{inh} = \begin{bmatrix} -\dfrac{b\sin^2\theta_T\left(1 + e^{2bt}\tan^2\dfrac{\theta_T}{2}\right)^2}{4a^2 e^{2bt}\tan^2\dfrac{\theta_T}{2}\cos\theta_T} & 0 \\ 0 & 0 \end{bmatrix}$$

Examples

Heterogeneous and anisotropic component

The component considered now is constituted of different volumes made of the same austenitic stainless steel (anisotropic), but with different crystal orientations. Two other volumes (on the left and right in figure 5) are constituted of isotropic ferritic steel. This is a typical description of weld [4]. The field plotted in figure 4 corresponds to the longitudinal (or quasi-longitudinal) waves transmitted into the component. One can compare this field with that radiated by the same transducer into a component made of ferritic steel (right hand of figure 4.). Some beam distortions due to the heterogeneous and anisotropic structure are emphasized.

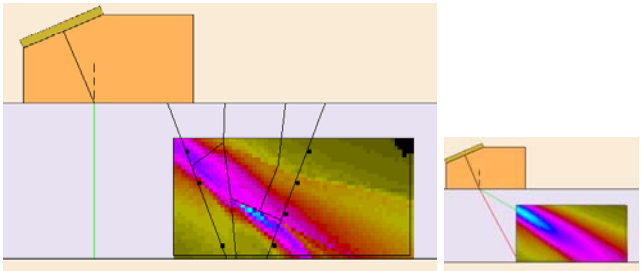

Figure 4. Field radiated into a heterogeneous anisotropic component (left) and in an isotropic component in the same conditions.

Other examples of fields in complex components computed with Civa can be found in [5,6]. Some validations have been presented in [4] (comparisons with experiments) and [1] (comparison with finite element computation).

Conclusion

The pencil method used in the computation of ultrasonic fields in Civa software is presented in this paper. The method has been developed in order to enable the modelling of fields radiated inside anisotropic, heterogeneous components, and inhomogeneous components, possibly with complex geometries.

References

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