PIEZOELECTRIC VIBRATING BEAM RESONATORS REVISITED: FLEXURAL AND TORSIONAL MODES OF GALLIUM ORTHOPHOSPHATE GAPO$_4$

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Abstract

Flexural modes of Quartz crystal beams are widely used for wrist watch resonators and vibrating beam resonators for sensors. Torsional modes of quartz have also been studied even if less applications have been developed. We present here a first evaluation of both flexural and torsional modes of rectangular cross-section vibrating bars of Gallium Orthophosphate ($GaPO_4$). An analytical model of vibrating beam resonators in flexural and torsional modes is built and validated by comparison with Quartz data on similar structures. Temperature effects are introduced by the approximate but classical method of varying effective material constants, beam dimensions and crystal mass density versus temperature. Temperature-compensated $GaPO_4$ orientations of vibrating beam in torsional modes are predicted.

Introduction

The piezoelectric crystal of gallium orthophosphate ($GaPO_4$) is new quartz homeotypic crystal. Due to the lack of the $\alpha-\beta$ phase transition most material properties are stable up 970°C. Its excellent thermal stability opens new field of applications for high temperature sensors. Flexural modes of quartz crystal beams are widely used for wrist watch resonators and vibrating beam resonators for sensors. Torsional modes of quartz have also been studied even if less applications have been developped.

This paper reports a first evaluation of both flexural and torsional modes for rectangular cross-section vibrating beams of $GaPO_4$. An analytical model of vibrating beam resonators in flexural and torsional modes is built and validated by comparison with quartz data on similar structures.

Temperature effects are introduced by the approximate but classical method of varying effective material constants, beam dimensions and crystal mass density versus temperature. Temperature-compensated cut for $GaPO_4$ in vibrating beam in torsional modes are predicted.

I. Analytical model of flexural modes

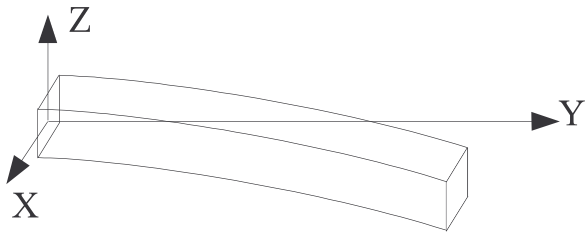

Figure 1 : Geometry of beam in flexural modes

The geometry of the rectangular beam is represented in figure 1. As a first approach, the analytical model of flexural modes is built without taking into account the piezoelectric effects and mass loading [1]. Thus, the equation of motion (1) for this vibrating beam model is:

$$\frac{1}{s_{22}} I \frac{\partial^4 w}{\partial y^4} - \rho A \frac{\partial^2 w}{\partial t^2} = 0 \quad (1)$$

where: I: Inertia of the beam
A: Area of the beam's section
ρ: Material's density
s_{22}: Compliance constant for Y axis
(x, y, z): Particules coordinates
u, v, w: Mechanical displacements
X, Y, Z: Axes

With (1), the resonant frequencies (2) are determined:

$$f = \frac{\lambda^2}{2\pi L^2} \sqrt{\frac{I}{\rho A s_{22}}} \quad \text{with } \lambda = \alpha L \quad (2)$$

L: Length of the beam

$$\alpha^4 = \frac{\rho S_{22} A \omega^2}{I}$$

However, the $GaPO_4$ is an anisotropic material then these properties depend on the orientation. Therefore the frequency depends on the cut angle [1][2]. In this study, we use only the rotated X-cut defined in figure 2.

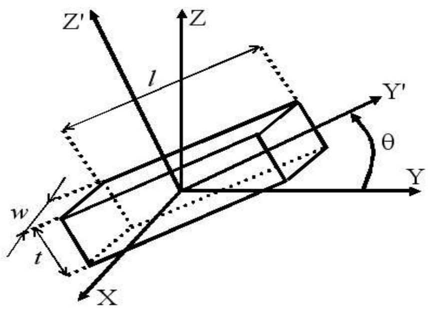

Figure 2 : Rotated X-cut

II. Temperature effects of flexural modes

The dependance of the frequency versus the temperature is an essential characteristic to resonators or sensors which control the frequency.

The dependance of resonant frequencies on temperature is defined in the equation (3):

$$f(T)=f(T_o)(\alpha(T-T_o)+\beta(T-T_o)^2 \ldots + \gamma(T-T_o)^3) \quad (3)$$

with

$$\alpha=\frac{1}{f}\frac{\partial f}{\partial T}; \quad \beta=\frac{1}{2!\,f}\frac{\partial^2 f}{\partial T^2}; \gamma=\frac{1}{3!\,f}\frac{\partial^3 f}{\partial T^3}$$

The first three Temperature Coefficients of Frequency (TCF)

The Studies of temperature effects on vibrating beam resonator in flexural modes are made in [4]. Contrary to quartz, the 1st TCF of $GaPO_4$ has two zero values for particular angles θ = -16.7° and θ = -48°. Hence compensation of temperature effects to the first order exists. Temperature compensated cuts for flexural modes are very else to what is found for lengt h extensional modes.

III. Analytical model of torsional modes

The other vibrating beam resonator studied is vibrating beam in torsional modes [5][6]. The figure 3 shows the geometry of the rectangular cross-section of the beam in torsional modes.

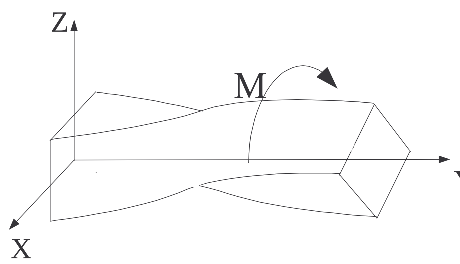

Figure 3 : Geometry of the beam in torsional modes

The mechanical displacements u, v, w along the axes X, Y, Z are:

$$\begin{cases} u(y,t)=-\tau\, y\, z \\ v(y,t)=\tau\,\phi(x,z) \quad (4) \\ w(y,t)=\tau\, x\, y \end{cases}$$

where ϕ(x,z) is the St Venant's torsion function and $\tau=\frac{\partial\alpha}{\partial y}$ is the torsional angle per unit length with α rotation angle.

The strains in the beam are S_4 and S_6 and the corresponding stresses, assuming τ constant are:

$$\begin{cases} T_4=\tau[C_{44}(\frac{\partial\phi}{\partial z}+x)+C_{46}(\frac{\partial\phi}{\partial x}-z)] \text{ (5.a)} \\ T_6=\tau[C_{46}(\frac{\partial\phi}{\partial z}+x)+C_{66}(\frac{\partial\phi}{\partial x}-z)] \text{ (5.b)} \end{cases}$$

In the rotated X-cut, the elastic constant C_{46} is equal to zero, then the function ϕ(x,z) is given by (6):

$$\phi(x,z)=-xz+\frac{8z_o^2}{\pi^3}\sqrt{(\frac{C_{66}}{C_{44}})}\sum_{n=0}^{\infty}\{\frac{(-1)^n}{(2n+1)^3}\ldots$$

$$\ldots\frac{\sinh(\frac{(2n+1)\pi x}{z_o}\sqrt{(\frac{C_{44}}{C_{66}})})}{\cosh(\frac{(2n+1)\pi x_o}{2z_o}\sqrt{(\frac{C_{44}}{C_{66}})})}\}\ldots$$

$$\ldots\sin(\frac{(2n+1)\pi z}{z_o}) \quad (6)$$

The torque M_t is related to the torsional angle τ by the torsional constant C_t:

$$M_t=C_t\tau=\tau\iint(C_{44}x^2+C_{66}z^2+C_{44}x\frac{\partial\phi}{\partial z}\ldots$$

$$\ldots -C_{66}z\frac{\partial\phi}{\partial x})\,dx\,dz \quad (7)$$

Using (7), C_t is then expressed as:

$$C_t=\frac{C_{66}\,x_o z_o^3}{3}(1-\frac{z_o}{x_o}\sqrt{(\frac{C_{66}}{C_{44}})}\frac{192}{\pi^5}\ldots$$

$$\ldots\sum_{n=0}^{\infty}\{\frac{1}{(2n+1)^5\,\pi^5}\ldots$$

$$\ldots\tanh(\frac{(2n+1)\pi x_o}{2z_o}\sqrt{(\frac{C_{44}}{C_{66}})})\}) \quad (8)$$

The equilibrium equation (9) combined with stress equations (5.a & 5.b) lead to the equation of motion (10):

$$T_{ij,j}=0 \quad (9)$$

$$\rho I_y \frac{\partial^2 \alpha}{\partial t^2} - C_t \frac{\partial^2 \alpha}{\partial y^2} = 0 \quad (10)$$

where

$$I_y = \frac{x_o z_o (x_o^2 + z_o^2)}{12}$$

moment of inertia in rectangular cross-section

C_t: torsional constant

α: ratation angle versus Y axis

The solution of the equation (10) is expressed as:

$$\alpha(y,t) = [C\sin(a\,y) + B\cos(b\,y)]\cos(\omega t) \quad (11)$$

The constants *C, B, a* and *b* are determined by the boundary conditions.

The resonant frequencies are determined by using the principle of energy's conservating. Hence, for a beam which is fixed-free, the resonant frequencies are (12) and for a fixed-fixed beam they are expressed in (13):

$$f = \frac{n}{4\ y_o}\sqrt{\frac{C_t}{\rho I_y}} \quad (12)$$

$$f = \frac{n}{2\ y_o}\sqrt{\frac{C_t}{\rho I_y}} \quad (13)$$

In table 1 are represented the resonant frequencies for the two boundary conditions expressed before and beam dimensions are: 1 mm of thickness, 2 mm of width and 15 mm of length :

n	***1***	***2***	***3***	***4***	***5***
Fixed-free	40142	80285	120429	160572	200715
Fixed-fixed	20071	40142	60214	80285	100357

Table 1.a : Quartz's frequencies for the five first modes

n	***1***	***2***	***3***	***4***	***5***
Fixed-free	26770	53540	80311	107081	133851
Fixed-fixed	13385	26770	40155	53540	66925

Table 1.b : $GaPO_4$'s frequencies for the five first modes

IV. Temperature effects of torsional modes

Using the approximate method, we obtain an analytical model of the first two TCF:

$$\alpha = \frac{1}{2}[-\frac{\dot{y}}{y} + \frac{\dot{C}_{66}}{C_{66}} + 3\ \frac{\dot{z}}{z} - \frac{h}{1-h}\frac{\dot{h}}{h} \ldots$$

$$\ldots\text{-}2\ \frac{x^2}{x^2+z^2}\frac{\dot{x}}{x} - 2\ \frac{z^2}{x^2+z^2}\frac{\dot{z}}{z}] \quad (14.a)$$

$$\beta = \frac{1}{2}[(\log(f))^{(2)} + \alpha^2] \quad (14.b)$$

with $\frac{\dot{h}}{h} = [\frac{\dot{z}}{z} - \frac{\dot{x}}{x} + \frac{1}{2}\frac{\dot{C}_{66}}{C_{66}} - \frac{1}{2}\frac{\dot{C}_{44}}{C_{44}}](1 \ldots$ $\ldots\text{-}\sum_{n=0}^{\infty}\frac{1-\tanh^2 A}{\tanh A}A)$ and

$$A = \frac{(2n+1)\pi\, x}{2z}(\frac{C_{44}}{C_{66}})^{\frac{1}{2}}$$

In figure 4.a and 4.b are drawn the evaluation of the first TCF versus rotated X-cut for $GaPO_4$ and quartz.

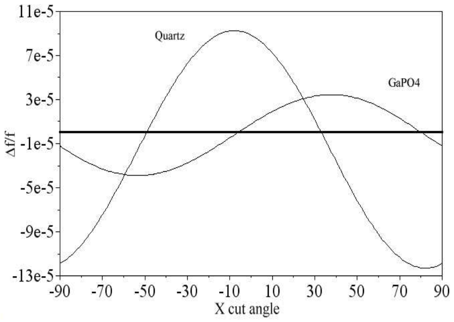

Figure 4: 1st TCF for quartz and $GaPO_4$ in torsional modes

The temperature-compensated cut is found again for quartz at θ = -34° (φ = 60°) and θ = 32° (φ = 0°). $GaPO_4$ curve has a zero value for theta equal to -7° then it does exist a temperature-compensated cut for $GaPO_4$ in torsional modes.

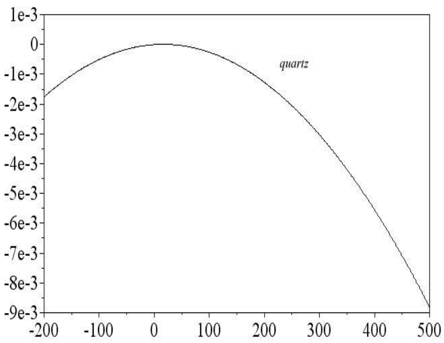

Figure 5 : Δf/f versus temperature for quartz's compensated cut

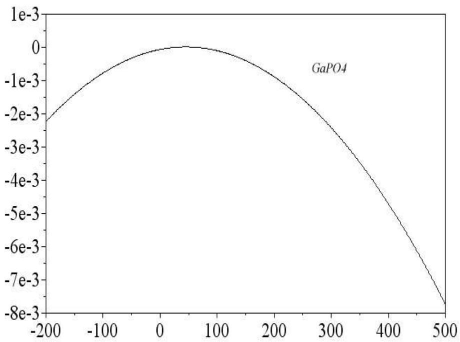

Figure 6 : Δf/f versus temperature for $GaPO_4$'s compensated cut

In figure 5, we notice that the temperature effects have the same consequence on $GaPO_4$ and quartz for their temperature compensated cut. However, the excellent stability of $GaPO_4$ lets it to have a temperature compensated cut at high temperature. In figure 6, it is reported the frequency variations versus temperature for $GaPO_4$ for a cut ($\theta = 19.5°$) which is compensated at 480 °C.

Conclusion

As a conclusion, we found again the cut angle where occurs the minimization of the temperature effects in flexural modes for quartz [2]. However for $GaPO_4$, a temperature compensated cut exists in flexural modes at about -16.7°, piezoelectric effects and mass loading need to be taking into account and its realize a little offset of the cut angle.

In torsional modes, quartz has got a temperature compensated cut at $\theta = -34°$ ($\phi = 60°$) and $\theta = 32°$ ($\phi = 0°$) [3]. $GaPO_4$ has also a temperature compensated cut at -7°.

For $GaPO_4$, by slightly adjusting the cut angle a temperature compensated cut exists at high temperature in torsional modes like quartz but material properties of quartz limit the temperature using at high temperature (<530 °C where $\alpha - \beta$ transition occurs). Whereas the excellent thermal stability of $GaPO_4$ lets using at high temperature (>530 °C). In flexural modes, for $GaPO_4$ temperature compensated cut exists at high temperature by slightly adjusting the cut angle too.

Acknowledgements

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