GYROSCOPIC EFFECT ON SURFACE ACOUSTIC WAVES IN ANISOTROPIC SOLID MEDIA

B. Collet
Laboratoire de Modélisation en Mécanique, UMR - CNRS 7607, Case 162,
Université Pierre et Marie Curie, Paris VI,
8 rue du Capitaine Scott, 75015 Paris, France
Email: bc@ccr.fr

Abstract

The propagation of Rayleigh waves over an elastic anisotropic half-space rotating at a constant angular rate about a symmetry axis is analyzed using a recent approch based on the first intergrals of motion, including Coriolis and centrifugal forces. The secular equation obtained explicitly shows that the wave is dispersive with a normal or an abnormal local dispersion. Numerical results for cubic and tetragonal crystals are presented.

Introduction

The characteristics of ultrasonic waves propagating in solids and their dependence upon various geometric and physical parameters have been under continued study. The effects of electro-mechanic couplings, temperature, pre-stress, acceleration, etc., on phase velocity or frequency provide the foundation for the development of many acoustic sensors. Particularly, frequency or phase velocity shifts due to rotation have been used to make gyroscopes, i.e., angular rate sensors. These sensors may operate with bulk waves in beams, rings, plates, and shells, or with surface waves over a half-space [1-6].

In this paper we show, in the particular framework of the propagation of surface acoustic waves in a linear anisotropic elastic half-space in rotation, the possibilities offered by a novel mathematical formalism based on the method of first integrals of motion [7]. Unlike the classical approaches such as the partial waves method or the Stroh formalism, this new method does not require the successive resolution of two eigenvalue problems for in order obtain the dispersion relation. Here the secular equation is here derived in a direct manner and explicitly. The effect of rotation on the characteristics of guided waves is studied, including Coriolis and centrifugal forces. A selection of numerical results concerning the dispersion curves, sensitivity to direction of rotation axis, and field profiles for cubic $(m3m, \overline{4}3m)$ and tetragonal $(4/mmm)$ crystals are presented and briefly discussed.

Basic Equations

We consider a homogeneous anisotropic crystal occuping a half-space $x_2 \leq 0$, with the plane surface $x_2 = 0$ free of mechanical loads, and rotating at a constant angular velocity Ω_i about x_i-axis. In the frame in co-rotation with the body, the equations of motion, the linear constitutive relations, and the boundary conditions are

$$\begin{aligned}
&\sigma_{ij,j} = \rho\left(\ddot{u}_i + 2\varepsilon_{ijk}\Omega_j\dot{u}_k + \Omega_i\Omega_j u_j - \Omega_j\Omega_j u_i\right), \\
&\sigma_{ij} = c_{ijkl}S_{kl},\ \ S_{kl} = \tfrac{1}{2}\left(u_{k,l} + u_{l,k}\right), \\
&\sigma_{i2} = 0,\ \text{ at }\ x_2 = 0,
\end{aligned} \tag{1}$$

where σ_{ij} is the stress tensor, c_{ijkl} is the elastic stiffness tensor, S_{kl} is the strain tensor, u_i is the mechanical displacement and ρ is the mass density. The second term in the right-hand side of Eq. $(1)_1$ is due to the Coriolis acceleration, both third and four terms are due to the centrifugal acceleration. We study the propagation of Rayleigh surface acoustic wave (SAW) in the x_1-direction, with spatial attenuation in the x_2-direction. To seek solutions, the mechanical displacement is taken in the form of a plane harmonic wave, $u_i(x_k, t) = \overline{u}_i(kx_2)\exp ik(x_1 - vt) = \overline{u}_i(kx_2)\exp i(kx_1 - \omega t)$, with wave number k in the x_1-direction, phase velocity $v = \omega/k$, and frequency ω. The possibility of attenuation in the x_2-direction is introduced through the unknown functions $\overline{u}_i(kx_2)$ which are assumed to satisfy both radiation conditions: $\overline{u}_i(-\infty) = 0$ and $\overline{u}_i^{'}(-\infty) = 0$, where $\overline{u}_i^{'} = d\overline{u}_i/dkx_2$.

Explicit dispersion equations

Using the inhomogeneous harmonic plane waves introduced above, the equations of motion are reduced to ordinary differential equations with respect to dimensionless variable $x = kx_2$. In the present study only two cases are examined:

Case 1- $\Omega = \Omega_1\mathbf{i}$ *(Rayleigh wave propagation in tetragonal crystal 422, 42m, 4mm, 4/mmm rotating about the* x_1*-axis)*

$$\begin{aligned}
&\alpha_{11}\overline{u}_1^{''} - \gamma_{11}\overline{u}_1 + i\beta_{12}\overline{u}_2^{'} = 0, \\
&\alpha_{22}\overline{u}_2^{''} - \gamma_{22}\overline{u}_2 + i\beta_{21}\overline{u}_1^{'} - i\delta_{23}\overline{u}_3 = 0, \\
&\alpha_{33}\overline{u}_3^{''} - \gamma_{33}\overline{u}_3 + i\delta_{32}\overline{u}_2 = 0,
\end{aligned} \tag{2}$$

where

$$\begin{aligned}&\alpha_{11}=c_{66},\alpha_{22}=c_{11},\alpha_{33}=c_{44},X=\rho v^2,\\&\gamma_{11}=c_{11}-X,\gamma_{22}=c_{66}-X(1+\varepsilon_1^2),\\&\gamma_{33}=c_{44}-X(1+\varepsilon_1^2),\varepsilon_1=\Omega_1/\omega,\\&\beta_{12}=\beta_{21}=c_{12}+c_{66},\delta_{23}=\delta_{32}=2X\varepsilon_1.\end{aligned}\tag{3}$$

Case 2- $\Omega=\Omega_2\mathbf{j}$ *(Rayleigh wave propagation in tetragonal crystal 422, 42m, 4mm, 4/mmm rotating about the* x_2*-axis)*

$$\begin{aligned}&\alpha_{11}\overline{u}_1''-\gamma_{11}\overline{u}_1+i\beta_{12}\overline{u}_2'+i\delta_{13}\overline{u}_3=0,\\&\alpha_{22}\overline{u}_2''-\gamma_{22}\overline{u}_2+i\beta_{21}\overline{u}_1'=0,\\&\alpha_{33}\overline{u}_3''-\gamma_{33}\overline{u}_3-i\delta_{31}\overline{u}_1=0,\end{aligned}\tag{4}$$

where

$$\begin{aligned}&\alpha_{11}=c_{66},\alpha_{22}=c_{11},\alpha_{33}=c_{44},X=\rho v^2,\\&\gamma_{11}=c_{11}-X(1+\varepsilon_2^2),\gamma_{22}=c_{66}-X,\\&\gamma_{33}=c_{44}-X(1+\varepsilon_2^2),\varepsilon_2=\Omega_2/\omega\\&\beta_{12}=\beta_{21}=c_{12}+c_{66},\delta_{13}=\delta_{31}=2X\varepsilon_2.\end{aligned}\tag{5}$$

In both cases, the boundary conditions Eqs $(1)_3$ take the form

$$\begin{aligned}&\overline{u}_1'(0)+i\overline{u}_2(0)=0,\\&ic_{12}\overline{u}_1(0)+c_{11}\overline{u}_2'(0)=0,\\&\overline{u}_3'(0)=0.\end{aligned}\tag{6}$$

As integrating factors for Eqs.$(2)_{1,2,3}$ and Eqs.$(4)_{1,2,3}$, it is convenient to use the following functions

$$\begin{aligned}&\overline{u}_1',i\overline{u}_2,\overline{u}_3,\ \text{for Eq. }(2)_1,\\&\overline{u}_2',i\overline{u}_1,i\overline{u}_3',\ \text{for Eq. }(2)_2,\\&\overline{u}_3',i\overline{u}_2',\overline{u}_1,\ \text{for Eq. }(2)_3,\\&\overline{u}_1',i\overline{u}_2,i\overline{u}_3',\ \text{for Eq. }(4)_1,\\&\overline{u}_2',i\overline{u}_1,\overline{u}_3,\ \text{for Eq. }(4)_2,\\&\overline{u}_3',i\overline{u}_1',\overline{u}_2,\ \text{for Eq. }(4)_3.\end{aligned}\tag{7}$$

Multiplying Eqs.(1) and Eqs.(4) by integrating factors, integrating by parts over $x\in[0,-\infty[$, and taking in account the boundary and radiation conditions at $x=0,-\infty$, we obtain for each case a non-homogeneous system of nine linear algebraic equations which include three integrable terms and six nonintegrable terms given by respectively:

Case 1

$$\begin{aligned}&F_1=\tfrac{1}{2}\overline{u}_1^2(0),\quad F_2=\tfrac{1}{2}\overline{u}_2^2(0),\\&F_3=\tfrac{1}{2}\overline{u}_3^2(0),\end{aligned}\tag{8}$$

$$\begin{aligned}&F_4=i\int_{-\infty}^{0}\overline{u}_1'\overline{u}_2'dx,\quad F_5=i\int_{-\infty}^{0}\overline{u}_2'\overline{u}_3dx,\\&F_6=i\int_{-\infty}^{0}\overline{u}_1\overline{u}_2dx,\quad F_7=\int_{-\infty}^{0}\overline{u}_1\overline{u}_3dx,\\&F_8=i\int_{-\infty}^{0}\overline{u}_2''\overline{u}_3'dx,\quad F_9=\int_{-\infty}^{0}\overline{u}_1'\overline{u}_3'dx.\end{aligned}\tag{9}$$

Case 2

$$\begin{aligned}&G_1=\tfrac{1}{2}\overline{u}_1^2(0),\quad G_2=\tfrac{1}{2}\overline{u}_2^2(0),\\&G_3=\tfrac{1}{2}\overline{u}_3^2(0),\end{aligned}\tag{10}$$

$$\begin{aligned}&G_4=i\int_{-\infty}^{0}\overline{u}_1'\overline{u}_2'dx,\quad G_5=i\int_{-\infty}^{0}\overline{u}_1'\overline{u}_3dx,\\&G_6=i\int_{-\infty}^{0}\overline{u}_1\overline{u}_2dx,\quad G_7=\int_{-\infty}^{0}\overline{u}_2\overline{u}_3dx,\\&G_8=i\int_{-\infty}^{0}\overline{u}_1''\overline{u}_3'dx,\quad G_9=\int_{-\infty}^{0}\overline{u}_2'\overline{u}_3'dx.\end{aligned}\tag{11}$$

These systems of equations at $x=0$ may be written in the form

$$\begin{aligned}&A_{mn}F_n=i\overline{u}_2(0)\,\overline{u}_3(0)\,B_m,\ m,n=1,2,...9,\\&C_{mn}G_n=i\overline{u}_1(0)\,\overline{u}_3(0)\,D_m,\ m,n=1,2,...9,\end{aligned}\tag{12}$$

where are $A_{mn},\ C_{mn},\ B_m$, and D_m are found explicitly in terms of coefficients of Eqs.(2), (4). The size 9×9 of the matrices in Eqs.$(12)_{1,2}$, is large, but the elements are simple (many zero elements). From Eqs.(12), one finds

$$\begin{aligned}&\overline{u}_2^2(0)=2(\Delta_2/\Delta)\,i\overline{u}_2(0)\overline{u}_3(0),\\&\overline{u}_3^2(0)=2(\Delta_3/\Delta)\,i\overline{u}_2(0)\overline{u}_3(0),\end{aligned}\tag{13}$$

$$\begin{aligned}&\overline{u}_1^2(0)=2(\Pi_1/\Pi)\,i\overline{u}_1(0)\overline{u}_3(0),\\&\overline{u}_3^2(0)=2(\Pi_3/\Pi)\,i\overline{u}_1(0)\overline{u}_3(0),\end{aligned}\tag{14}$$

where $\Delta=\det A_{mn}$, $\Pi=\det C_{mn},\Delta_2,\ \Delta_3$, are the determinants of matrices obtained by replacing corresponding columns of A_{mn} by B_m, and Π_1,Π_3 are the determinants of matrices obtained by replacing corresponding columns of C_{mn} by D_m.

The explicit secular equations sought are the compatibility condition of nonlinear equation systems. From Eqs.(13)-(14), we obtain for:

Case 1- $\Omega=\Omega_1\mathbf{i}$

$$\Delta^2+4\Delta_2\Delta_3=0,\tag{15}$$

Case 2- $\Omega = \Omega_2 \mathbf{j}$

$$\Pi^2 + 4\Pi_1\Pi_3 = 0. \tag{16}$$

The case of a half-space rotating about the x_3-axis ($\Omega = \Omega_3 \mathbf{k}$) has been recently solved for tetragonal $422, 42m, 4/mmm$ and orthorhombic crystals using the polarization vector method [8].

Numerical results

In this section we present a selection of numerical simulations which illustrate the gyroscopic effects on the linear dispersion spectra and on the profiles versus depth of the mechanical displacement components of generalized Rayleigh waves propagating along $[100]$ axis in the plane (010) for Si (silicon), GaAs (gallium arsenide) and TiO_2 (rutile). The Rayleigh wave velocity is normalized by the shear bulk wave velocity $v_6 = (c_{66}/\rho)^{1/2}$. Figures 1-3 give the variation of the phase velocity of the rotation-perturbed Rayleigh wave versus rotation rate about two symmetry axis. The dispersion is always stronger for the rotation axis aligned with the propagation direction. We observe, for three selected materials a normal dispersion i.e the phase velocity decreases in function of rotation rate. The weak range of variation of rotation rate is fixed in view of applications, consequently, $\varepsilon_{1,2}$ can be considered a small parameter and this suggests that one may tackle approximately Eqs.(15)-(16) via a perturbation technique. The influence of centrifugal force on dispersion spectra may be significant on the range of rotation rate ($\simeq 18\%$).

In order to obtain the profile of the three components of the field of displacement, it is supposed that the functions $\overline{u}_i$ are in exponential form $\overline{u}_i = A_i \exp(\eta k x_2)$,

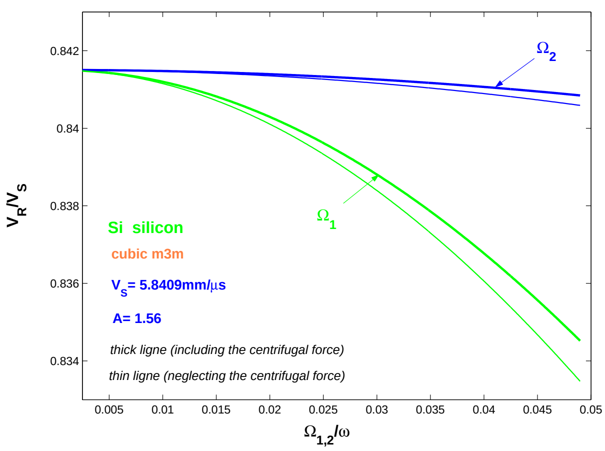

Figure 1: Phase velocity dispersion curves for Si crystal as a function of rotation rate.

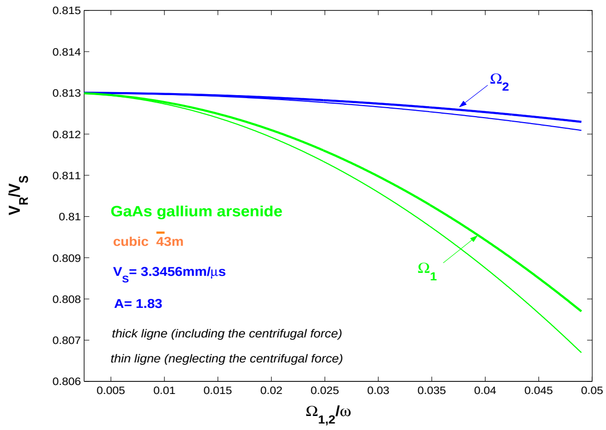

Figure 2: Phase velocity dispersion curves for GaAs crystal as a function of rotation rate.

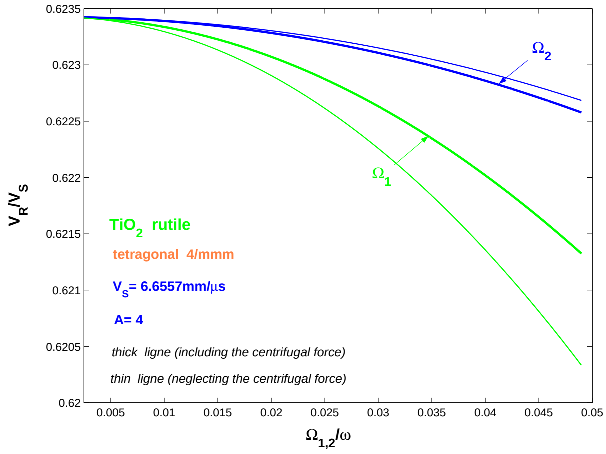

Figure 3: Phase velocity dispersion curves for TiO_2 crystal as a function of rotation rate.

where η is a decay constant. By substituting the displacement field $u_i = A_i \exp(\eta\omega x_2/v) \exp i(\omega/v)(x - vt)$ into the boundary conditions Eqs.$(1)_3$ and by using the known expression the velocity of phase of Rayleigh waves $v_R = v_R(\omega)$ one obtains a homogeneous system which admits a non trivial solution, if the decay constant (or attenuation factor) η satisfy a characteristic equation of the third degree in η^2. As in the usual procedure, a sorting (Re $\eta_\alpha > 0,\ \alpha = 1, 2, 3,$) among the six roots must be made to satisfy the radiation conditions *(eliminating the case of leaky surface waves)*. Finally by setting an amplitude on the surface or the acoustic energy flow density vector, one can find explicitly the fields and plot the profiles.

The distributions of the normalized displacement components according to the normalized distance (with wave length) from the free surface are shown in Figures 4-6. For two rotation axes, the effects of rotation rate on

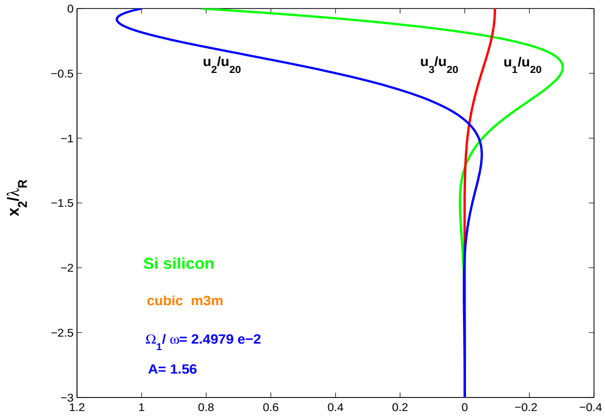

Figure 4: Normalized components of mechanical displacement for Si crystal as a function of depth (in wavelengths).

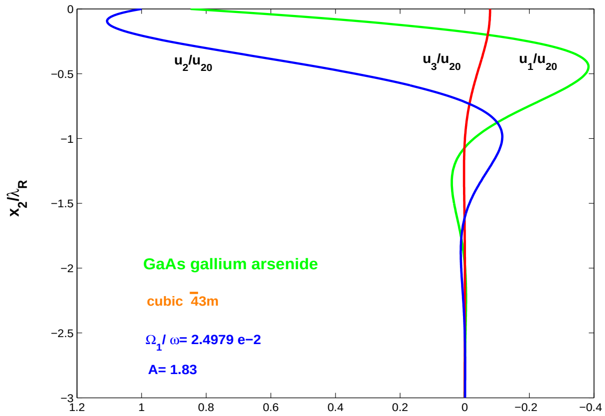

Figure 5: Normalized components of mechanical displacement for GaAs as a function of depth (in wavelengths).

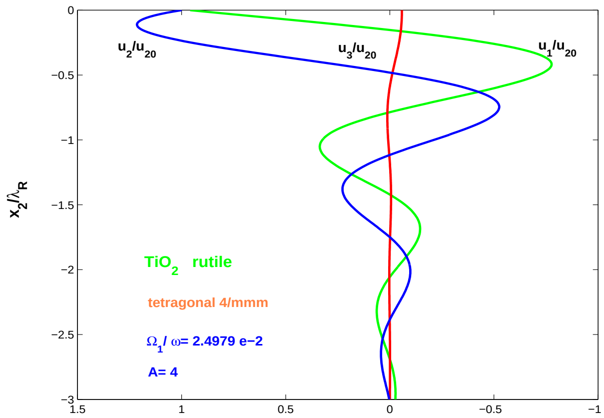

Figure 6: Normalized components of mechanical displacement for TiO2 as a function of depth (in wavelengths).

the penetration depth of the generalized Rayleigh waves are insignificant and as for non rotating crystals, the larger anisotropy factor $A = 2c_{66}/(c_{11} - c_{12})$ decreases less the damping of the oscillations. The displacement components are normalized by u_{20} the amplitude of u_2 at the free surface. We note the presence of a small u_3 is caused by rotation.

For the three crystals chosen to illustrate the results, the dispersion is locally normal, however some other simulations based on Stroh formalism (not reproduced here) have shown for commonly used materials such as PZT-5H piezoelectric ceramic, an abnormal dispersion i.e a Rayleigh wave velocity increasing locally when the rotation ratio increases. This observation is in accord with results found in Refs. [5, 6].

Finally note that in general, a linear rotation-sensitivity is researched in design of gyroscope devices. Conversely the insensitivity to the rotation is sometimes desirable for the frequency stability of sensors mounted on rotating structures.

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