A NEW MODEL FOR THE MAGNETOSTRICTIVE GENERATION OF ULTRASOUND IN WAVEGUIDES OF CYLINDRICAL SYMMETRY

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Abstract

In this communication a new model for the generation of ultrasonic waves in waveguides made of a ferromagnetic material is presented. The model comprises three stages: in the first one we solve the purely electromagnetic problem of the distribution of the electromagnetic (EM) field set up in the metallic material by the exciting sources. Then the linearized constitutive equations for the magnetostrictive effect are found in order to compute the mechanical surface stresses and volumetric forces existing in the solid (the terms corresponding to Lorentz forces are also computed for comparison). At this stage, several simplifying assumptions are made: small skin depth penetration of the EM field, strong bias axial field and weak coupling of the EM and elastic fields. The last step of our model is the use of modal analysis to quantify the coupling between the electromagnetic and elastic fields. The excited mechanical wave is computed as an orthonormal expansion of propagating modes in the waveguide, where the amplitude coefficients are dependent on frequency.

The performance of this new method is demonstrated by computing the amplitude of the propagating longitudinal modes (L(0,m)) excited in a ferromagnetic pipe for a given solenoid configuration.

The advantage of this new model is that numerical results can be obtained with greater simplicity when compared with previous schemes found in the literature. Its practical applications include ultrasonic generation in travelling wave non destructive testing (NDT) systems, magnetostrictive delay lines and magnetic-based position sensors.

1 Introduction and state of the art

Generation of ultrasound in waveguides by electromagnetic fields has found many applications in science and engineering. Among them we can cite: magnetostrictive delay lines [1], linear position sensors [2], and non destructive testing (NDT) of structures through propagating waves [3]. In this communication we present a new model for the generation of ultrasound in ferromagnetic pipes. The physical problem that we will describe is shown in figure 1. The ultrasonic wave $\mathbf{u}$ is generated in the pipe by a combination of a bias magnetic field H_0, provided by the magnet, and a dynamic field H_1 caused by the coil:

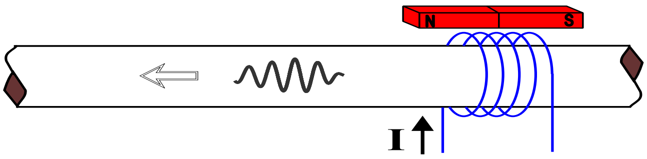

Figure 1: Physical setup for the electromagnetic generation of ultrasound.

$$H(t) = H_0 + H_1 e^{j\omega t} \quad \text{with} \quad H_1 \ll H_0. \qquad (1)$$

The earliest model, developed by Williams [4], deals with generation in wires which have a small diameter compared to the wavelength. Therefore it was enough to solve the simple one dimensional equations; but the results are not applicable to bigger structures like pipes. Boltachev [5] tackled the full three dimensional case, proceeding in three steps: Fourier transform of the coupled magnetoelastic partial derivative equations; solution of the corresponding algebraic problem; and then inverse transform of the equations back into the spatial domain. His model has been recently checked and compared to the experimental results by Sablik [6], but due to its complexity and number of unknown variables, only qualitative agreement has been found.

The outline of this paper is as follows: in the next two sections we will respectively consider the physics of the electromagnetic generation of ultrasound and the propagation of mechanical signals in a waveguide with cylindrical symmetry. Based on this material, the model will be introduced in section 4. Some numerical results from a simulation are shown in section 5. At the end of the paper we draw some conclusions.

2 Electromagnetic generation of ultrasound

With the setup of figure 1, two electromagnetic effects can generate mechanical waves of ultrasonic frequency in metals. The first is the Lorentz force of the bias field acting on the induced (eddy) currents in the pipe. This effect is present in all metals. In ferromagnetic metals we encounter a second source of mechanical motion, magnetostriction [7], which is the deformation and displacement of magnetic domains.

The behavior of ferromagnetic metals where both effects coexist has been described in [8], and depends basically on the bias magnetic field H_0. For low bias, magnetostriction is the stronger effect; however, as field increases, the metal saturates magnetically, and the Lorentz force mechanism, which is linear with frequency, eventually dominates. In this paper we are interested in the study of the first situation.

Under a sufficiently strong bias field ($H_0 \gg H_1$) it is found that the magnetostrictive behavior of a ferromagnetic metal is both linear and non hysteretic [7], and linear equations couple the mechanical and magnetic variables:

$$\begin{array}{l} \sigma = \mathbf{c^H} \cdot \epsilon - \mathbf{e}^T \cdot \mathbf{H} \\ \mathbf{B} = \mathbf{e} \cdot \epsilon + \mu^\epsilon \cdot \mathbf{H}. \end{array} \quad (2)$$

The notation mirrors the one used in piezoelectricity and is normalized in standard IEEE 319 (1973). The coupling term is given by the magnetoelastic tensor $\mathbf{e}$, whose form depends on the symmetry class of the solid and the direction of the bias field.

3 Mechanical behavior of the waveguide

The solution of the mechanical wave equation for the case of cylindrical symmetry was developed by Pochhammer and Chree in the 19th century, and the first numerical results were obtained by Gazis in 1959 [9].

A solid cylinder or a pipe supports three families of permitted (propagating) modes: torsional T(0,m), longitudinal L(0,m), and flexural F(n,m). The first two have symmetry of revolution around the waveguide's axis. A signal applied to the waveguide will in general excite all the propagating modes within its frequency band.

To find the relative amplitude of the excited modes, we use modal analysis [10], which is based upon the property of orthonormality of the permitted modes. Thus, a generic signal of a given frequency, $\widehat{\mathbf{u}}(r, \theta, z)$, can be decomposed into a finite number of propagating modes and an infinite number of non propagating ones:

$$\widehat{\mathbf{u}}(r, \theta, z) = \sum_p a_p(z) \widetilde{\mathbf{u}}_p(r, \theta) + \sum_{p'} a_{p'}(z) \widetilde{\mathbf{u}}_{p'}(r, \theta). \quad (3)$$

The non propagating modes (denoted with a prime) are needed to satisfy the boundary conditions and can be neglected in points far from the generation region.

To perform the numerical computations involved in the Pochhammer-Chree theory, a set of routines in the Matlab environment were written. The software, named PCDISP, can be used for the following tasks:

- Dispersion curves: phase and group velocity dependence on frequency.
- Mode profiles: distribution of displacement vector and stress tensor.

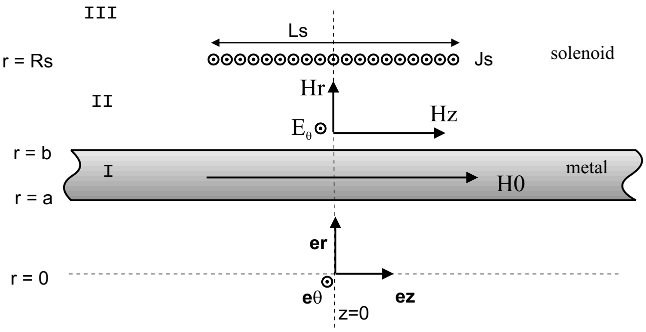

Figure 2: Geometry for the computation of the distribution of the electromagnetic field.

- Modal analysis: computation of modal amplitudes excited by arbitrary external forces.

PCDISP is described in more detail in reference [11].

4 Outline of the model

We will develop the new model in three steps. In the first part the electromagnetic dynamic field created in the pipe by the generating solenoid will be computed. The mechanical forcing terms (forces and stresses) will then be computed in a linear approximation. Finally, the amplitudes of the propagating modes effectively excited in the waveguide are calculated by modal analysis. The complete ultrasonic signal can be formed as an expansion of a finite number of orthonormal modes, with amplitude coefficients dependent on frequency.

4.1 *Electromagnetic field distribution*

The problem of the electromagnetic field distribution is solved in the approximation of weak coupling (i.e., non influence of mechanical deformations on the magnetic field), by considering the wave equation in terms of the potential vector $\mathbf{A}(r, z)$ [12]:

$$\nabla^2 \mathbf{A} - \mu\sigma \frac{\partial \mathbf{A}}{\partial t} = -\mu \mathbf{J_s}. \quad (4)$$

The setup for the problem is given in figure 2.

The solution is further simplified if the skin depth penetration $\delta = (2/\omega\mu\sigma)^{1/2}$ is small compared with the thickness of the tube, $b - a$. In this case there is an approximate solution:

$$H_z(r, z) = H_{zb}(z) e^{-(1+j)(b-r)/\delta}, H_r(r, z) \simeq 0, \quad (5)$$

where $H_{zb}(z)$ is the axial field in the outer surface ($r = b$) of the pipe. This situation where the radial field can be neglected compared with the axial field also simplifies the description of the magnetostrictive behavior of the pipe.

4.2 Coupling terms

Once the distribution of the field is known, the magnetostrictive stress tensor in the pipe can be computed by use of equation 2. At this point, two more assumptions are made: first, we regard the metal as elastically isotropic, and simplify the compliance tensor in equation 2; second, the application of a strong bias field H_0 in the axial direction reduces the matrix of piezomagnetic coefficients to the following form [7]:

$$\mathbf{d} = \begin{bmatrix} & & & & d_{15} & 0 \\ & & & d_{15} & & 0 \\ d_{31} & d_{31} & d_{33} & & & 0 \end{bmatrix} \quad (6)$$

The stress in the metal caused by the magnetostrictive effect can be given as:

$$\sigma_{\text{ms}} = -\mathbf{e}^T \cdot \mathbf{H}, \quad (7)$$

where $\mathbf{H}$ is the magnetic vector field computed before and $\mathbf{e} = \mathbf{d} \cdot \mathbf{c}^H$. The volume force field is found as the divergence of the stress tensor:

$$\mathbf{f}_{\text{ms}} = \nabla \cdot \sigma_{\text{ms}}.$$

4.3 Modal analysis

The final step of our model is computing the amplitudes of the propagating modes generated in the waveguide. With the arrangement of figure 1, it is clear that only modes with cylindrical symmetry will be excited. Further, torsional modes are excluded because the forcing terms have no azimuthal ($\mathbf{e}_\theta$) component. Therefore we will only be concerned with the longitudinal modes L(0,m), which have a displacement vector with the general form:

$$\widehat{\mathbf{u}}_p(r,\theta,z) = \mathbf{u}_p(r)e^{-j\xi_p z} = [u_{rp}(r), 0, u_{zp}(r)]e^{-j\xi_p z}. \quad (8)$$

The amplitudes of the propagating modes in equation 3 can be found by performing an integral which extends to the generation area R_g [10]:

$$a_p(z) = \frac{e^{-j\xi_p z}}{4P_p} \int_{R_g} e^{j\xi_p z'} [f_p^s(z') + f_p^v(z')] \, dz'. \quad (9)$$

The term f_p^s is the coupling of the surface stress to a given mode, and is found by integrating $\widehat{\sigma}_{\text{ms}}$ around the surface of the tube ∂D:

$$f_p^s(z) = -j\omega \oint_{\partial D} [\mathbf{u}_p^*(r) \cdot \widehat{\sigma}_{\text{ms}}(r,\theta,z)] \cdot \mathbf{e_n} \, dl. \quad (10)$$

Similarly the coupling of the volume force $\widehat{\mathbf{f}}_{\text{ms}}$ is determined by taking the integral in the cross section of the tube D:

$$f_p^v(z) = -j\omega \iint_D [\mathbf{u}_p^*(r) \cdot \widehat{\mathbf{f}}_{\text{ms}}(r,\theta,z)] \, dS. \quad (11)$$

The displacement profile of the mode, $\mathbf{u}_p(r)$, as well as the numerical integrals of equations 9-11 are computed with the PCDISP package described in section 3.

5 Numerical results

The behavior of model is checked with a simulation. Generation of ultrasound takes place in an iron pipe with the setup shown in figure 2 and the data of table 1. The magnetic and magnetostrictive parameters for iron are taken from reference [13].

Table 1: Data of the iron tube used in the simulations.

Inner radius a	3 mm
Outer radius b	4 mm
Bias magnetic field H_0	37 000 A/m
Magnetization $\mu_0 M_0$	2.07 T
Rel. permeability μ_{ri}	4.4
Magnetostr. coeff. d_{33}	-2.8×10^{-10} m/A
Conductivity σ	$9.9 \times 10^6\,\Omega^{-1}\text{m}^{-1}$
Shear modulus G	$7.7 \times 10^{10}\,\text{N/m}^2$
Coil length L_s	10 mm
Coil radius R_s	9 mm

The result of the simulation of the magnetostrictive generation is shown in figure 3, where the transfer function:

$$\frac{|a_p(z,\omega)|}{I(\omega)},$$

for the two longitudinal modes L(0,1) and L(0,2) is plotted. It can be checked with PCDISP that these are the only permitted modes below 1 MHz. The largest generation efficiency occurs at low frequencies (between 20 kHz and 200 kHz), when the wavelength of the ultrasonic wave is bigger than the length of the solenoid ($\lambda > L_s$), so that constructive interference happens. As we approach the cutoff frequency of mode L(0,2), the phase velocity of mode L(0,1) decays quickly, and destructive interference (because $L_s > \lambda$ no longer holds), causes a decrease in the amplitude of the ultrasonic wave. Above the cutoff frequency of L(0,2), this mode dominates. This behavior agrees well with the results from other researchers ([4] for example).

6 Conclusions

In this communication, a new model for the magnetostrictive generation of ultrasound in cylindrical waveguides has been presented. The model is valid under the assumptions of strong axias bias field, small penetration depth of the magnetic field in the metal, and mechanical isotropy. The method involves the following steps: (a) computation of the electromagnetic field; (b) finding the the coupling mechanical forces and stresses due to the magnetostrictive effect; and (c) computation of the amplitude of excited modes by modal analysis. This model has the advantage of providing analytical (closed) solutions, with computations considerably simpler than other existing techniques.

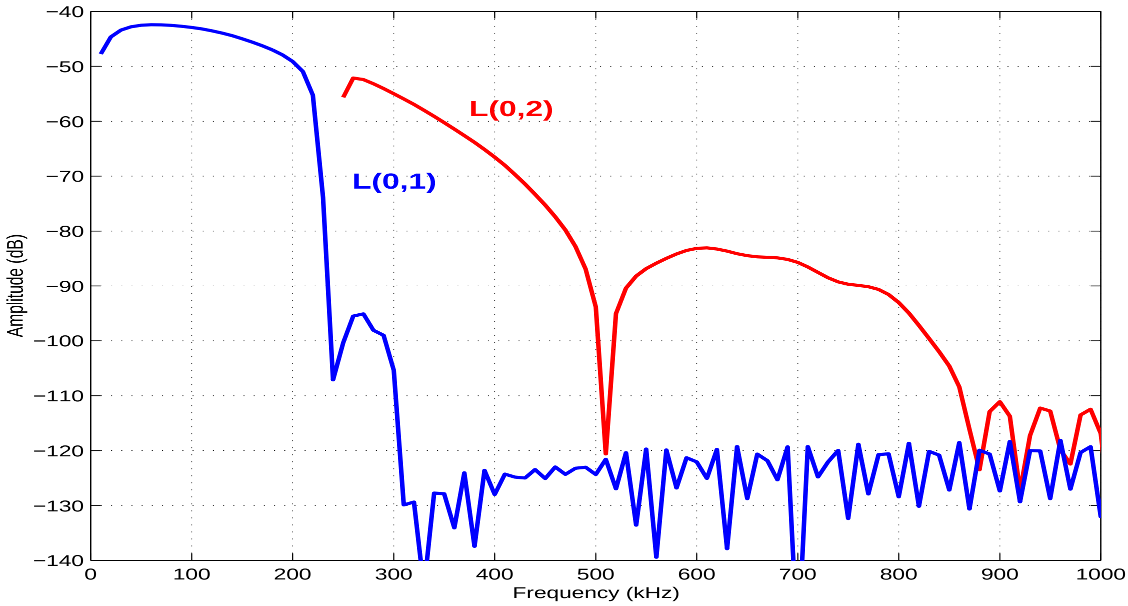

Figure 3: Relative amplitudes of modes L(0,1) y L(0,2) generated in the waveguide by the magnetostrictive effect up to a frequency of 1 MHz.

Acknowledgments

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