BA THYMETRY RECONSTRUCTION FOR A FREE-TOWED SYNTHETIC APERTURE SONAR

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Abstract
This paper discusses bathymetric image reconstruction for a Synthetic Aperture Sonar (SAS) with an emphasis on interferometric phase errors. The effect of misregistration between the receivers is shown to be the dominant source of error and is corrected by estimating the time delay between receivers at each range bin. Results are shown for a simulated seafloor.

Introduction
This paper describes seafloor bathymetric height estimation using the KiwiSAS Synthetic Aperture Sonar (SAS). The KiwiSAS has been developed by the Acoustics Research Group at the University of Canterbury, New Zealand. This sonar system is designed to provide high resolution imagery of the seafloor in a shallow water environment. The system uses two simultaneous pulsed, linearly chirped FM signals each of 20 kHz bandwidth, each with center frequencies of 30 kHz and 100 kHz. The returned echos are received using three vertically separated hydrophones and stored in baseband form for post-processing. Using standard SAS reconstruction techniques [1] the 2-D imaging resolution is approximately $5 \times 15$ cm.

This paper starts with a brief review of bathymetric image reconstruction and looks at sources of interferometric phase errors in more detail. The problems of echo registration and phase unwrapping are discussed for broadband SAS. Simulation results are then shown to illustrate the importance of echo registration.

Bathymetric image reconstruction
There are a number of approaches used for estimating seafloor bathymetry with a synthetic aperture sonar. In most cases an image is formed using standard phase preserving SAS reconstruction algorithms for each hydrophone [1]. Vertical beamforming can be used to synthesise $H$ independent beams with $H$ hydrophones, however, the angular resolution is poor. The resolution improves with a greater vertical baseline but grating lobe ambiguities occur if the hydrophones are more than half a wavelength apart. In practice, the maximum vertical baseline and the number of hydrophones is limited by the size of the towfish. A better angular precision can be achieved using an interferometer configuration. This assumes that only a single echo wave is incident upon the hydrophones in any given range gate. The time difference of the echoes received by the hydrophones, or equivalently the carrier phase of the complex baseband signals, can be measured and used to estimate the incoming direction of arrival. Provided there is only a single dominant echo signal in each range gate, the accuracy of this method depends on the coherence between the two hydrophone signals.

Interferometric errors
The coherence between echoes measured by an interferometric sonar is limited by noise, baseline decorrelation, and the footprint shift effect [2]. Baseline decorrelation (or speckle decorrelation [3]) is due to coherent interference between multiple scatterers within the sonar footprint [4] while the footprint shift (called pixel misregistration in Synthetic Aperture Radar [5]) results in an interferogram formed from different sections of the seafloor. While baseline decorrelation and the footprint shift effect are both geometry dependent, the footprint shift effect dominates for broadband sonars in a shallow water environment [2]. The greater the bandwidth, the smaller the range resolution, the greater the misregistration, and thus the poorer the coherence. The baseline decorrelation is smaller for an interferometric SAS due to the use of small vertical baselines and broadband signals. It can be compensated by filtering the non-overlapping parts of the two echo spectra [6] provided the seafloor height can be estimated.

The equivalent SNR produced by baseline decorrelation is [4]

$$
\frac{d}{\sqrt{\eta - \sin \eta}},
$$

(1)

where

$$
\eta = \frac{\pi}{a} \frac{f_0}{B} \cos^2 \theta,
$$

(2)

where $a$ is the hydrophone spacing, $H$ is the altitude of the sonar, $f_0$ is the centre frequency, $B$ is the signal bandwidth, and $\theta$ is the angle or arrival (from vertical). With KiwiSAS $B = 20$ kHz, $a = 75$ mm, $H = 5$ m (typically), and $\theta$ ranges from 60 to 85 degrees. At the lower frequency band with $f_0 = 30$ kHz, then $\eta$ ranges from $9 \times 10^{-3}$ to $9 \times 10^{-5}$ with $d$ ranging from $7 \times 10^4$ to $7 \times 10^6$. Thus baseline decorrelation has a negligible effect since the vertical size of the speckle pattern is much larger than the hydrophone spacing. Even in the high frequency band where $f_0 = 100$ kHz, the baseline
decorrelation is negligible since \( \eta \) ranges from \( 3 \times 10^{-2} \) to \( 3 \times 10^{-4} \) with \( d \) values of \( 7 \times 10^3 \) to \( 7 \times 10^7 \).

The equivalent SNR due to the footprint shift is approximately [2]

\[
d \approx \frac{c}{2\alpha B \cos \theta} - 1, \tag{3}
\]
giving values for KiwiSAS in the range 0–9. These figures are poor since the range resolution is smaller than the hydrophone separation. They could be improved slightly by tilting the interferometer axis but this is not often feasible in practice.

Another source of bathymetric error results from roll of the towfish corrupting the phase difference estimates. This can be compensated by instrumenting the towfish with a roll sensor or by estimating the roll from the sonar data [7].

Bathymetry in a shallow water environment also suffers from sea surface multipath. In addition to the direct path echoes, some of the scattered energy from the seafloor is reflected by the sea surface. This interferes with the direct path echoes and corrupts the angle of arrival estimates when using simple interferometric techniques. Unfortunately, when the echoes are of comparable strength, the angle of arrival estimate has a large variance. Moreover, since the hydrophones of an interferometer are typically spaced by more than half a wavelength, there are ambiguities (grating lobes) in the angle of arrival estimates. Thus simple techniques cannot resolve the multipath echoes from the direct path echoes on the basis of angle of arrival. Employing an array of hydrophones can help resolve the multipath problems but current techniques are limited to narrowband signals[8], [9].

**Registration**

The misregistration between the interferometer signals due to the footprint shift effect can be corrected by shifting and resampling one of the echoes provided the seafloor height is known for each range. Care has to be taken with the resampling step to avoid interpolation errors introducing additional phase noise[10], [11].

The problem with registration is that an estimate of the seafloor height at each range is required. Typically the height estimate needs to be known so that the misregistration is less than a tenth of the range resolution. This is equivalent to a height accuracy of a tenth of the height depth of focus.

One approach is to reconstruct a height image by back-projecting the data over multiple heights and then choosing the most likely surface using a priori surface statistics, for example using Belief Propagation [12]. The cost function is based on the consistency of the back-projected measurements and the expected variance. While belief propagation is a form of maximum a posteriori (MAP) estimation, it is computationally expensive.

A direct method to the registration problem is to estimate the time delay between the two signals by correlating windowed sections of the echo data around the range of interest. The position of the correlation peak can then be used to estimate the misregistration. The phase of the correlation peak gives a more accurate but ambiguous estimate of the shift between the two signals. However, as explained in the following section, the ambiguity number can be determined from the shift of the correlation peak if the signals are broadband. An estimate of the coherence between the two signals and thus the accuracy of the phase difference can be found from the amplitude of the correlation peak.

**Phase unwrapping**

The interferometric phase difference produces an ambiguous height estimate due to the mod \( 2\pi \) phase wrapping. The height ambiguities at a range \( r \) are separated by \( H_{\text{amb}} \), given by

\[
H_{\text{amb}} = \frac{rc}{f_0d} = \frac{r\lambda}{d}. \tag{4}
\]

This is the height that produces a range difference equal to the wavelength \( \lambda = c/f_0 \).

The height ambiguity can be resolved using phase unwrapping techniques, based on estimates of the seafloor height or surface continuity arguments. Iterative schemes have also been proposed that consider the number of residues in the interferogram [13].

The number of ambiguous heights can be reduced by employing broader bandwidth signals since this reduces the region where the signals are coherent. This region is called the height depth of focus, \( H_{\text{DOF}} \), and is given by

\[
H_{\text{DOF}} = \frac{rc}{2Bd}. \tag{5}
\]

This is the height that produces a range difference equal to the range resolution, \( c/(2B) \), where \( B \) is the system bandwidth. The height depth of focus can be related to the height ambiguity by

\[
H_{\text{DOF}} = \frac{f_0}{2B} H_{\text{amb}} = \frac{Q}{2} H_{\text{amb}}, \tag{6}
\]

where \( Q \) is the ratio of the signal centre frequency to bandwidth. Note that if \( Q \) is less than 2 then the height depth of focus is less than the height ambiguity and thus phase unwrapping is not required since there is no ambiguity [14]. The KiwiSAS lower frequency band has a \( Q \) of 1.5 while the upper frequency band has a \( Q \) of 5.
Figure 1: Bathymetry of a ±1 m sinusoidal seafloor reconstructed using (a) interferogram with phase unwrapping, (b) interferogram with registration to -10 m using linear interpolation, (c) interferogram with registration to -10 m using an 8-term truncated sinc interpolator, and (d) time delay estimation using a 16 point FFT correlator with sub-sample quadratic interpolation.

Thus the lower frequency band can give an unambiguous phase estimate to help unwrap the upper frequency band. In practice, additional filtering is required since the lower frequency band is more susceptible to sea surface multipath.

Results

To illustrate bathymetric height reconstruction using an interferometric SAS, a seafloor with a sinusoidal height variation and a Gaussian surface roughness was modeled using an ensemble of point scatterers. The mean water depth was 10 m and the sonar with the KiwiSAS parameters was positioned mid-water at -5 m. The simulated echoes for a linear FM chirp were pulse compressed and the range sidelobes were removed by deconvolving with the chirp autocorrelation function and convolving with a Blackman-Harris window function. A bathymetric image was then formed as shown in Figure 1(a) using the phase of the Hermitian product of the two signals. Phase unwrapping was performed around the expected depth of 10 m. As can be seen, the height estimates are very noisy due to the poor coherence resulting from the signal misregistration.

Figure 1(b) and Figure 1(c) show reconstructed height images after the echoes have been registered assuming a depth of 10 m. Figure 1(b) used a linear interpolator while Figure 1(c) used an 8-term truncated sinc interpolator to demonstrate the importance of the interpolator on the phase error. Note that the height variance degrades with the height difference from the expected seafloor height.
An image formed by estimating the echo misregistration using correlation of the echoes is shown in Figure 1(d). The window length was 16 samples and the correlation was formed using a FFT correlator. The correlation peak was determined to sub-sample accuracy using complex quadratic interpolation. The phase at the interpolated correlation peak was then unwrapped using the shift of the correlation peak to determine the ambiguity number. Note, the resulting height image is smoother due to the correction of the echo registration and averaging by the correlator. However, some artifacts have been introduced and also some distortion of the original sinusoidal seafloor, especially in the areas of positive gradient.

Conclusion
This paper has shown that pixel misregistration is the most dominant interferometric error in bathymetry reconstruction. This can be corrected if a good estimate of the seafloor height is available. Alternatively, the average time delay between the echoes can be estimated using a short term correlation at each range bin. This gives good results where the seafloor height is slowly changing.

References


