ACOUSTIC STREAMING DEVELOPMENT IN ANNULAR RESONATORS

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Abstract

The transient closed-loop acoustic streaming, excited due to nonlinear processes in viscous and thermal boundary layers, is studied numerically in an annular resonator equipped by a short stack. A single purely propagating wave is excited by proper phasing of two localised acoustic sources. The ratios of the viscous boundary layer depth to the resonator transverse dimension are expected to be weak (quasi-adiabatic regime). The Reynolds stresses resulting from the nonlinear processes in boundary layers are incorporated as external sources in a semi-opened numerical code of fluid dynamics. The acoustic streaming is computed by the finite volumes numerical method. The results show that a characteristic time of acoustic streaming stabilization decreases with increase of the volume occupied by the stack and with increase in the number of the stack plates.

Introduction

When an acoustic field is sustained in a resonator, a streaming is excited due to nonlinear phenomena in the viscous and thermal boundary layers [1]. In particular, for an annular resonator, the toroidal geometry may lead to a closed-loop acoustic streaming [2]. The acoustic field can be generated and amplified in an annular resonator by installing a stack (a set of equidistant parallel plates) and imposing a temperature gradient along the latter (annular thermoacoustic prime-mover [3]). The convective heat transport induced by the acoustic streaming modifies the temperature distribution along the stack and disturbs dramatically the amplification process of the acoustic wave [2, 4]. Independently of the generation mode of the acoustic field in an annular resonator (stack heated inhomogenously or piezoelectric sources), an acoustic wave propagating over a stack (initially at constant temperature) pumps heat from one edge towards the other, providing a temperature gradient along the stack [3]. The disturbance induced by the acoustic streaming reduces the efficiency of this annular thermoacoustic refrigerator.

The acoustic streaming development in annular resonators got a particular interest due to its importance in the characterization of the annular thermoacoustic devices. Even though this nonlinear phenomenon was studied analytically [5], no precise solution was obtained and the concerned mode of streaming excitation was only due to the bulk attenuation of acoustic waves. In a recent paper [6], a theory was developed for the transient acoustic streaming in an acoustitron (annular resonator with driven walls). The ratio of the viscous boundary layer depth δ_ν to the waveguide width D was found to be the dominant parameter in the acoustic streaming stabilization.

In the study presented below the acoustic field in the annular resonator is sustained by two sinusoidal localized wave sources. They are characterized by the width d, separated by a distance a, oscillating at the same frequency f with a relative phase shift φ, and inducing normal vibration of the waveguide walls with the velocity amplitude v_0 (see Fig. 1). Although each source excites separately a standing wave, the wave resulting from their simultaneous action can be decomposed into two counter-propagating waves with different (in general) amplitudes. The ratio of the amplitudes can be controlled by varying the parameters a and/or φ.

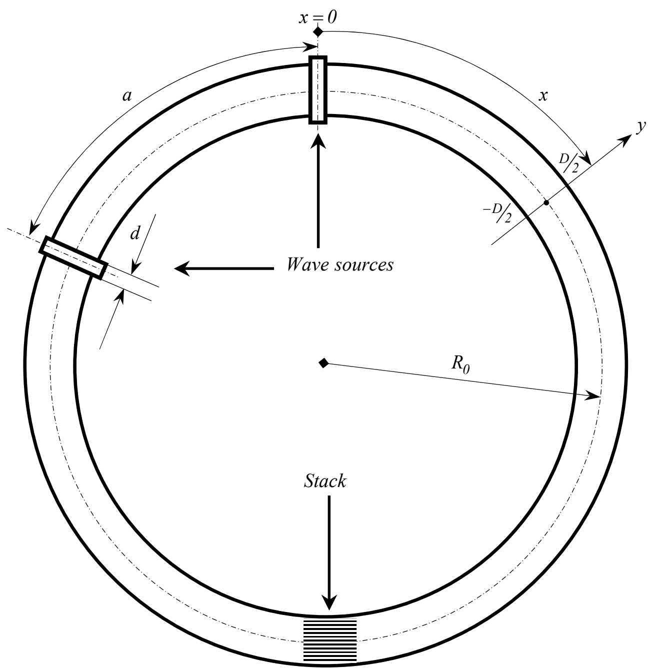

Figure 1 : Two wave sources annular resonator equipped by a stack.

Assuming a waveguide width D much less than the acoustic wavelength λ ($\lambda >> D$), the toroidal geometry of the resonator can be developed in an equivalent straight one [2]. The wave propagation and the acoustic streaming are then studied in a straight channel defined by $0 \le x \le L = 2\pi R_0$ and $-D/2 \le y \le D/2$ in the plane of Figure 1 and with an infinite dimension along the z axis (where R_0 is the radius of the circumference).

Pressure equation

In thermoacoustics linear theory [3, 7] the acoustic disturbances are assumed to be small. The acoustic field variables correspond to the first order expansion (*index 1*) of the pressure p, the axial and transverse velocities v_x and v_y, and the temperature T. An acoustic field variable ψ_1 is expressed through its complex amplitude $\widetilde{\psi}$ as $\psi_1 = \boldsymbol{Re}(\widetilde{\psi}\, e^{-i\omega t})$, where $\boldsymbol{Re}(\cdots)$ denotes the real part, t is the time and $\omega = 2\pi f$ is the angular frequency. A boundary layer approximation of fluid mechanics is achieved under the condition $\lambda >> min(D, \delta_v)$ where $\delta_v = \sqrt{2\nu_0/\omega}$ (ν_0 is the kinematic viscosity at the operating conditions p_0 and T_0) [6]. The variables $\widetilde{v}_x$, $\widetilde{v}_y$ and $\widetilde{T}$ can be presented as the functions of $\widetilde{p}$ only. The pressure $\widetilde{p}$ is given by a homogenous second order differential equation arising from the transverse velocity adherence condition on the walls $\widetilde{v}_y\big|_{y=\pm D/2} = 0$ (see Eq. (54) in Ref. [3]). In the case of the two wave sources resonator under consideration, $\widetilde{v}_y$ is zero on the walls over the entire resonator except at $x=0$ ($\widetilde{v}_y\big|_{y=\pm D/2} = \mp v_0$) and $x=-a$ ($\widetilde{v}_y\big|_{y=\pm D/2} = \mp v_0\, e^{-i\varphi}$). Consequently, the pressure equation becomes inhomogeneous:

$$\frac{d^2\widetilde{p}}{dx^2} + k^2\widetilde{p} = \frac{-2i\omega\rho_0 v_0 d(\delta(x) + \delta(x+a)e^{-i\varphi})}{D(1-f_v)},$$

where δ is the Dirac delta function, $k = k_0(1 + (f_v + (\gamma-1)f_k)/(1-f_v))^{1/2}$ is the acoustic wave number (k_0 is the adiabatic acoustic wave number), γ is the ratio of the specific heats, $f_{v,k}$ are the values of the functions $\Phi_{v,k} = (1+i)\delta_{v,k}\tanh(\eta D/(1+i)\delta_{v,k})/D$ at $\eta = 1$ ($\eta = 2y/D$ is the dimensionless transverse coordinate), $\delta_k = \delta_v/\sqrt{\sigma}$ is the thermal boundary layer depth (σ is the Prandtl number), and ρ_0 is the density at p_0 and T_0.

The δ-localised volume sources can be equivalently described by the discontinuity of the pressure gradients at the transverse cross-sections $x=0$ and $x=-a$. Pressure itself is continuous in these cross-sections. By assuming a quasi-adiabatic regime ($\delta_v/D << 1$) and the resonance condition ($L \cong \lambda$), and fixing the parameters a and φ at $\lambda/4$ and $\pi/2$, respectively, the solution of the pressure equation is obtained. It describes a single wave propagating in the positive direction of the x axis $\widetilde{p} = p_a\, e^{i k_R x}$, with the amplitude $p_a = -2\rho_0 a_0 v_0(d/D)/(k_I L)$, where k_R and k_I are the real and imaginary parts of the wave number k and a_0 is the adiabatic sound speed.

The thickness of the stack installed in the annular resonator is assumed to be much less than the waveguide length and the thickness of the plates is assumed to be much less then the waveguide width. Consequently, no reflection of the purely propagating wave is expected to occur on the edges of the stack. Finally possible modification of the initial homogeneous temperature distribution due to the thermoacoustic effect in the boundary layers is neglected.

Numerical model for the acoustic streaming

A description of the acoustic streaming consists in combining the mass and momentum conservation laws. The resulting equation of motion is averaged over the acoustic wave period τ ($<f> = (1/\tau)\int_t^{t+\tau} f(t')\,dt'$). The system of equations obtained for the acoustic streaming velocity (i.e. average velocity) includes force sources induced by the acoustic field [1, 7]. These forces are quadratic in the amplitude of the acoustic field. Due to the boundary layer approximation $\lambda >> D$ and the toroidal symmetry of an annular resonator "without a stack", this system under the assumption of linear streaming [1] reduces to a one-dimensional equation,

$$\partial v_{xm}/\partial\theta = \partial^2 v_{xm}/\partial\eta^2 + (\tau_c/\rho_0)F_x\,, \quad (1)$$

where v_{xm} is the axial velocity of the acoustic streaming, $\theta = t/\tau_c$ is the dimensionless time, $\tau_c = (D/\pi)^2/\nu_0$ is the characteristic transient time of the acoustic streaming development and F_x is the sum of two bulk forces per unit volume $F_{x1} = -\rho_0(2/D)\partial < v_{1x}v_{1\eta} > /\partial\eta$ and $F_{x2} = b(\rho_0\nu_0/T_0)(2/D)^2\partial < T_1\partial v_{1x}/\partial\eta > /\partial\eta$. The force F_{x2} arises due to the dependence of the viscosity μ on the temperature T via the phenomenological parameter b ($\mu = \mu_0(T/T_0)^b$) [2].

For a similar device (acoustitron), Eq. (1) is solved analytically on using Fourier series expansions [6]. A numerical model proposed for the same device provides results in good agreement with the analytical ones [8]. In this latter model, the force components (F_{x1} and F_{x2}) and the initial, symmetry and boundary conditions ($v_{xm}(\theta=0)=0$, $v_{xm}(\eta=\pm 1)=0$, and $\partial v_{xm}(\eta=0)/\partial\eta=0$, respectively) are incorporated in a semi-opened code of fluid dynamics (Fluent 6.0 [9]). A bi-dimensional incompressible flow is considered. Consequently, the numerical code (using finite volumes method) solves the following momentum equation

$$\frac{\partial \boldsymbol{U}}{\partial t}+(\boldsymbol{U}\cdot\nabla)\boldsymbol{U}=-\frac{\nabla p}{\rho_0}-\nu_0\,\Delta\boldsymbol{U}+\boldsymbol{F}\ , \qquad (2)$$

where $\boldsymbol{U}$ is the flow velocity vector and $\boldsymbol{F}$ is the force sources vector. With the boundary layer approximation ($\partial\cdot\cdot/\partial x << \partial\cdot\cdot/\partial y$), the periodicity condition ($p(x=0)=p(x=L)$), and weak velocities (negligible convective terms), the solution of Eq. (2) approximates the solution for Eq. (1).

In this paper, the previous numerical model is applied to the annular resonator and also extended in order to include the stack. The installation of a "short" stack introduces a discontinuity in the axial force distribution F_x. It is important to mention here that the space separating the plates is assumed to be much less than the stack length (boundary layer approximation). Consequently, the acoustic streaming in the stack is one-dimensional (axial). In Eq. (2) the terms $\Delta\boldsymbol{U}$ and ∇p link the axial streamings in the waveguide and the stack which allows the description of the bi-dimensional acoustic streaming occurring in the vicinity of the stack edges (terminations).

Results

In Fig. 2, the axial force profiles F_x along the waveguide and stack cross sections are plotted for air ($\gamma=1.4$ $\sigma=0.71$ and $b=0.77$). These profiles are normalized to the maximum of the force amplitude in the waveguide. The installation of a stack in a given waveguide cross-section (here $x=L/2$, see Fig. 1) is made by adding plates progressively. Consequently, in the indicated cross-section, the ratio of the viscous boundary layer depth to the waveguide width increases up to $\delta_\nu/D=1$ starting from the initial $\delta_\nu/D=0.1$ characteristic to quasi-adiabatic regime.

As expected, the stationary results show no variation of the axial and radial acoustic streaming velocities ($U_x=v_{xm}$ and $U_y=v_{ym}$) along the resonator axis far from the stack and along the central parts of the latter. Furthermore, the radial velocity in these regions approaches zero ($v_{ym}\cong 0$). Consequently, the results here below are presented for the waveguide cross-section $x=0$.

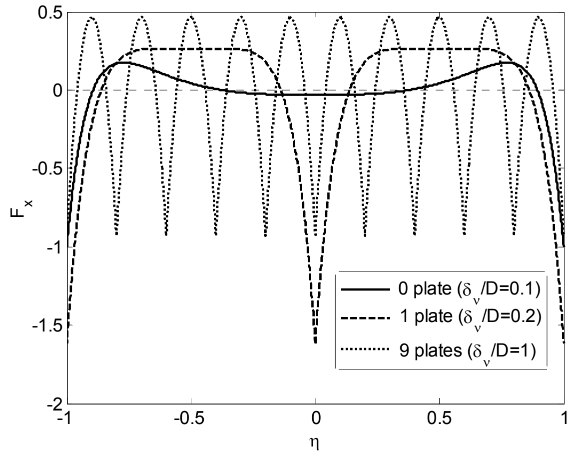

Figure 2 : Force profiles in resonator cross-section where stack is installed.

In Fig. 3, the stationary velocity profiles v_{xm} are plotted for different number of installed plates and different volumes occupied by the stack. These profiles are normalized to the maximum of the velocity amplitude in the waveguide. When the first two plates are installed, the velocity amplitude increases (see curves 0-2 plates in Fig. 3a and Fig. 3b)

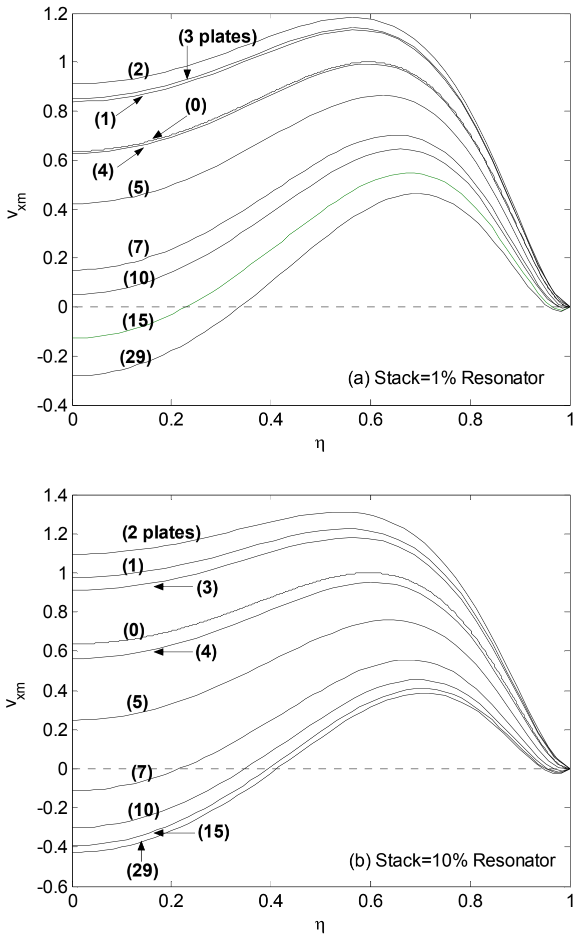

Figure 3 : Stationary velocity profiles in the waveguide cross-section $x=0$.

due presumably to an increase of the force inducing directional streaming. However, subsequent increase in the number of plates leads to the decrease of the maximum streaming velocity (see curves 2-29 in Fig. 3a and Fig. 3b). This latter behaviour of the velocity is assumed to be related to faster increase in the hydrodynamic resistance of the stack in comparison with the increase of the forces promoting directional streaming. It is clear that with increasing number of the plates the unidirectional streaming is gradually transformed in a bi-directional streaming, which carries zero mass flux across the resonator cross-section. Figure 3 also highlights the effect of the stack volume. When the stack length increases, this on the one hand leads to increase of the total force promoting the motion of the fluid (because the volume of the acoustic boundary layers where this force is generated increases). Compare the respective curves 0-3 plates in Fig. 3a and Fig. 3b. On the other hand increasing length of the stack increases hydrodynamic resistance for the directional acoustic streaming. Compare the respective curves 4-29 in Fig. 3a and Fig. 3b.

Averaging the transient velocity profiles over the waveguide cross-section ($\bar{v}_{xm} = (1/2)\int_{-1}^{1} v_{xm}\, d\eta$) allows the investigation of the directional acoustic streaming stabilisation. The dimensionless characteristic time θ_c of streaming stabilisation is obtained by solving the equation $\bar{v}_{xm}(\theta_c) = (1 - e^{-1}) \cdot \bar{v}_{xm}(\infty)$. Figure 4 demonstrates the decrease of θ_c with the increase of the number of plates or the increase of the stack volume.

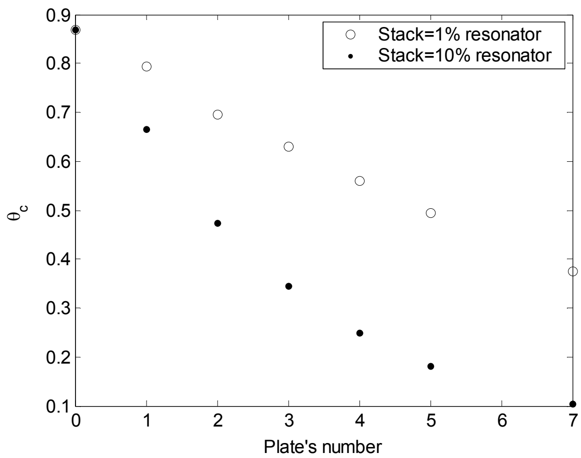

Figure 4 : Dimensionless characteristic time as a function of the number of plates.

Conclusion

The acoustic streaming development in annular resonator (equipped by a short stack) is studied numerically. The numerical results highlight a conflict between two phenomena accompanying the increase in the number of plates composing the stack and the increase in the stack length. In fact the increase in the volume of the boundary layers leads both to the increase of the force, promoting the fluid motion, and the hydrodynamic resistance to directional fluid flow across the stack. As a result the unidirectional streaming transforms with increasing number of stack plates and of their length. The characteristic time of the unidirectional component of streaming, carrying a non-zero mass flux over the resonator cross-section, diminishes with the installation of the stack. Its amplitude diminishes with increasing hydrodynamic resistance of the stack.

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