ENHANCED QUANTITATIVE ULTRASOUND TOMOGRAPHY

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Abstract
We propose a reconstruction procedure that deals with broadband ultrasonic signals acquired in the near field domain using probes close to the organ. The tomographic technique based on elliptical back-projection demonstrates high resolution and large contrast imaging capabilities. However, the simulations also delineate some limits. Indeed, the underlying hypothesis of a constant velocity medium (Born approximation), induces geometrical and medium characteristic distortions.
In order to reduce these distortions, we introduce a novel approach that associates transmission and reflection data. In a first step, a transmission tomographic (fan-beam) reconstruction is performed (layer striping technique) to obtain a sound speed map in the organ. In a second step, this map is used for correcting the signals acquired in diffraction before proceeding to tomographic reconstruction in reflection. Numerical results show that this approach enables us to achieve substantial improvement in the reduction of the geometrical aberration, which constitutes a prerequisite for a quantitative diffraction tomography.

Introduction

The general goal of diagnostic ultrasound (US) of the breast aims at performing more specific diagnoses than can be obtained with clinical findings and mammography alone. The specific goals of breast US are to prevent as many unnecessary biopsies as possible and to find cancers missed with mammography. Neither the general nor the specific goals can be achieved completely without the sonographic characterization of solid nodules [1] to distinguish between benign and malignant lesions. The purpose of this study is to contribute to reduce as much as possible, the overlap between the sonographic findings of benign and malignant solid nodules, in order to reduce the rate of current systematic biopsies performed on all solid nodules (X mammographic methodology), regardless of the sonographic findings. This requires firstly to simulate the acoustical behaviour (signature) of such heterogeneities, and secondly to implement an imaging reconstruction procedure being able to deal with these singularities.

The most common ultrasound imaging methods in use today are pulse-echo techniques for which energy is assumed to travel in straight lines at the supposed constant speed of sound. Other ultrasound imaging techniques use both the transmitted and reflected energy to form a two dimensional image of the breast. These techniques include US computed tomography based on the Born approximation [2]. However all of these techniques assume that scattering is weak and do not account for strong diffraction and refraction. Recently, researchers have found that strong refraction can occur [3], which necessitates the development of imaging methods that are not based on a weak scattering assumption.

From a practical point of view, *Ductal Echography* [4] *(DE)* is an new method of breast investigation which attempts to identify the internal mammary structures using the general property of ultrasound to distinguish between epithelium (hypoechogenic) and the connective tissue (more echogenic). However, the technique shows an important drawback as it needs a rigorous training of operators, *i.e.* the operator-dependency remains too high. In order to reduce this operator-dependency, we suggest, as several authors have already done [5], developing a hemi-spherical antenna made of several hundred transducers distributed all over the inner surface of the probe which could surround the mamma (no need for probe orientation, access to non-specular echoes) and operate in real-time. A coherent association of all the contributions (scattered field) produced by a given voxel in the organ will improve the signal to noise ratio and leads to quantitative parametric reconstructions (for instance of compressibility or density).

1. Modeling and reconstruction

Soft biological tissues are classically described, from an acoustical point of view by their density ρ, and their compressibility χ distributions of compact support *D*. Let ρ_0, χ_0 be the acoustic characteristics of the coupling medium, then $\mu = (\chi - \chi_0)/\chi_0$ and $\gamma = (\rho - \rho_0)/\rho_0$ correspond respectively, to the relative compressibility and density fluctuations. We consider as negligible the density term in comparison with the compressibility one as it is usually assumed in the case of soft tissues. We are interested in the inverse problem, i.e., from measurements of the scattered field, our objective is to try to map the values of μ. In order to avoid the difficult 2D derivation of the field, we will conduct the forward problem analysis in 3D space, and from that outcome we will elaborate the 2D reconstruction procedure.

Let us introduce a pressure source into the infinite homogeneous host medium, the scattered field p_d satisfies the classical wave equation based on the assumption that the medium investigated is slightly inhomogeneous. But rather than considering the source $S(\underline{x}, t)$ as generated by the incident field p_i, which is commonly done within the Born approximation, we envisage a more realistic acoustic source term :

$$-\frac{1}{c_0^2}\frac{\partial^2 p_d}{\partial t^2} + \nabla^2 p_d = \frac{\mu}{c_0^2}\frac{\partial^2 p_a}{\partial t^2}. \qquad (1)$$

Under Sommerfeld radiation conditions for p_d we derive the integral expression of the scattered field measured at the receiver location. Let us assume that the wave transmitted in the breast at the source point $\underline{x}_E$ and analysed at an arbitrary location $\underline{x}'$ inside the breast is of infinite bandwidth (short pulse) and is roughly spherical. In fact, due to the weak heterogeneities, which in case of large lesions could be of several ten factors of wavelength, the wave front is distorted. This distortion that we consider as a phase distortion should be rendered by the scattering model. Whereas the effects produced by the (weak) heterogeneities on the amplitude are assumed of less importance and are neglected. Hence, the expression of the *realistic* field (point-like transmitter):

$$p_a(\underline{x}_E, \underline{x}', t) = \delta(t - T(\underline{x}'))\frac{1}{4\pi|\underline{x}' - \underline{x}_E|}, \qquad (2)$$

where, the time of flight (TOF) between the transmitter and the point $\underline{x}'$ is assumed not so different as the TOF along the straight line path $\underline{x}_E\underline{x}'$:

$$T(\underline{x}') \approx T_0(\underline{x}') - \tau_E(\underline{x}'), \qquad (3)$$

where

$$T_0(\underline{x}') = \frac{|\underline{x}_E - \underline{x}'|}{c_0} \text{ and } \tau_E(\underline{x}') = \frac{1}{c_0}\int_{\underline{x}_E\underline{x}'} \frac{c(\underline{x}) - c_0}{c_0}\cdot d\underline{x}$$

The correcting term τ_E is the temporal adjustment one should make to account for large weak heterogeneities. Indeed, the TOF estimated is not exact (straight line and small speed fluctuations hypothesis) but it is a more accurate estimator than is the constant speed of sound approximation usually used within the Born approximation. We deduce the expression of the scattered field for a general waveform $f(t)$:

$$p_d(\underline{x}_E, \underline{x}_R, t) = \left(g_e(\underline{x}, \underline{x}_E, \underline{x}_R, t + \tau) * f''(t), \mu(\underline{x})\right). \qquad (4)$$

where $\tau = \tau_E + \tau_R$ and the ellipsoidal Green function corresponds to :

$$g_e(\underline{x}, \underline{x}_E, \underline{x}_R, t) = \left(16 c_0 \pi^2 |\underline{x}_E, \underline{x}| \cdot |\underline{x}, \underline{x}_R|\right)^{-1} \times \delta\left(c_0 t - |\underline{x}_E, \underline{x}, \underline{x}_R|\right). \qquad (5)$$

The emitter and the receiver are the foci of the ellipsoid $c_0(t + \tau) = |\underline{x}_E, \underline{x}, \underline{x}_R|$ of major axis $c_0(t + \tau)$ where $(t + \tau)$ is equal to the travel time between *E* and *R* via $\underline{x}$. Since τ is a function of $\underline{x}$ (the speed of sound is not constant), the ellipsoid is distorted. Previous works [6] have shown that it is possible to derive approximated 2D solutions valid for limited bandwidth waves, based on Elliptical Back-Projection (EBP) techniques, we adapt this technique to the case of large heterogeneities by the means of the appropriate temporal adjustment:

$$i(\underline{x}) = \left(p_d(\underline{x}_E, \underline{x}_R, t), \vartheta(\underline{x}_E, \underline{x}_R, \underline{x}, t) = \delta(t + \tau(\underline{x}) - |\underline{x}_E, \underline{x}, \underline{x}_R|/c_0)\right)$$

This process backprojects the echo sample over the ellipsoids from which they were originally obtained and provides an approximate solution to the *near field* scattering problem. These adjustments are evaluated using a transmission tomographic technique based on an original “layer stripping” approach that we will describe below.

By analogy with the inverse Radon transform, the filtered EBP image is given by :

$$I(\underline{x}) = \left(p_d(\underline{x}_E, \underline{x}_R, t), \vartheta''(\underline{x}_E, \underline{x}_R, t + \tau)_{\underline{x}}\right). \qquad (6)$$

The filtered EBP image is not the perfect reconstruction of the initial object [7], but, compared with the standard Radon transform, it constitutes a far better approximation for near-field measurements.

Considering the TOF between the transmitter and the receiver, the estimation technique consists in recovering, step by step, from the external up to the inner one, the sound speed map of the medium (hence the *layer stripping* terminology employed). For that purpose, a cylindrical mesh grid of concentric layers of cells is built. The numbers of layers and cells are function of the number *N* of transducers uniformly distributed over a circle all around the breast $\Delta\theta = 360°/N$, and depend also on the minimal angle $\kappa.\Delta\theta$ from which the data are acquired in transmission. The cylindrical mesh is then composed of $N_l = N/2 - \kappa + 1$ layers, and of *N* cells per layers

The *speed reconstruction algorithm* is the following :

Let us note $M(i, j)$ the cell sited in the layer *i*, $i = 1,...,N_l$, $j = 1,...,N$ within the angular limits $[(j-1)\Delta\theta, j\Delta\theta]$, *D*(*i*) the transducer located at the angular position $(i-1)\Delta\theta$. $\Gamma(n, m)$ is the ray between the transmitter *D*(*n*) and the receiver *D*(*m*).

We consider the length $L(i,j,n,m)$ of that ray within the cell $M(i,j)$, $L(i,j,n,m)=\int_{M(i,j)}\Gamma(n,m)dl$.

We call $\Omega_{i,j}$ the set of rays crossing the same cell,

$$\Omega_{i,j}=\{(n,m)\mid\Gamma(n,m)\cap M(i,j)\neq\emptyset\}-\{(n,m)\mid q>i,\ \Gamma(n,m)\cap M(q,.)\neq\emptyset\}$$

Assuming that the sound speed values of the cells are known in the external layers $l<i$, the process initiates in the coupling water, the sound speed estimator of the cell $M(i,j)$ is given by:

$$S_{i,j}=\frac{1}{\sum_{\Omega_{i,j}}\Gamma(n,m)}\left(\sum_{\Omega_{i,j}}\Gamma(n,m)\cdot s(n,m)\right) \quad (7)$$

where $s(n,m)$ is the averaged speed of sound calculated from the TOF along $\Gamma(n,m)$ and corrected from the estimations already made on the adjacent external layers.

2. Simulations and performances

In order to evaluate the modified EBP algorithm, we consider a numerical tissue-like phantom, whose response is computed by a Finite Difference Time Domain (FDTD) method. The governing numerical expressions of the FDTD method can be found in references [7,8]. The truncation of the finite simulation domain (1000×1000 cells, $\Delta x=1.6667\,10^{-5}$ m, $\Delta t=$ 5.556 10^{-9} s) to approximate an infinite space is made using a 12 cell Berenger's Perfectly Matched Layer (PML). The residual reflections introduce a noise of –80dB in the computation region, or in other words, a nominal SNR on the scattering data of 40 dB. The cylindrical academic phantom simulates a fluid object whose characteristics are : density 1000 kgm^{-3}, velocity 1,650ms^{-1} (figures 1 & 4). The transducers (transmitter and receivers, 2.25 MHz) move along a circle of radius *R*, where *R* = 6.7 10^{-3}m is the external radius of the phantom. The radii of the holes filled with aqueous solution (celerity 1500ms^{-1}) are respectively r_0= 1.12 mm r_1= 0.55 mm, r_2= 0.275 mm... r_5= 0.07 mm. The antenna is composed of 120 transducers. The minimal angle $\kappa.\Delta\theta$ equals 33°, the cylindrical mesh is composed of 50 layers.

Figure 2 illustrates the geometrical distortions induced by the EBP reconstruction when no temporal adjustment is performed. Figure 3 shows the modified EBP reconstruction based on analytical temporal adjustments. Figure 5 depicts the celerity map obtained using the layer-stripping technique applied on the phantom presented figure 4. One can observe that the small holes are not visible since they do not induce strong delays on the expected TOF, which is rather different when the defects are large compared with the wavelength. Figure 6 allows us to detect objects of one fifth the wavelength and to discriminate defects of one half the wavelength. Figure 7 introduces a more difficult experiment on a solid speculated nodule (velocities 1,520 within the spicules and 3,000 ms^{-1} in the core of the lesion, standard deviation of 2%, 0,3 mm of correlation length). Figure 8 illustrates the EBP reconstruction which provides a rather accurate image .

Conclusion

In this paper, we have shown that when using the Born approximation, special attention should be paid to both the amplitude of the contrast functions associated to the physical parameters of the tissue-like model and to the size of the defects which should remain within the range of the wavelength. In practice, breast lesion may be of 0.5 to 1cm diameter or may be solid (calcified) nodule, thus in order to obtain an accurate imaging, one has to temporally adjust the echoes so as to preserve their mutual spatial coherence. The FDTD numerical simulations show the benefit obtained once the corrections are made : the geometrical distortions are removed, a speed map is provided. This information is crucial as far as solid nodules are concerned, indeed quantitative estimation will be relevant for computer assisted diagnosis.

Acknowledgement

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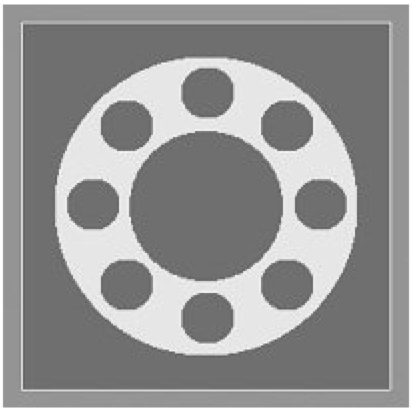

Fig. 1. Tissue-like phantom

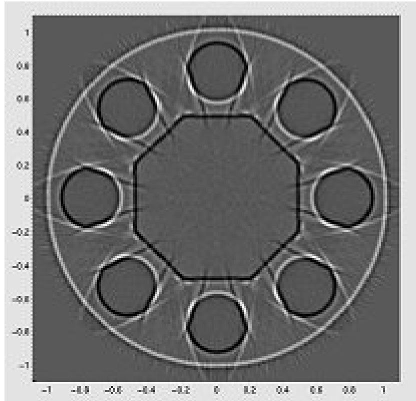

Fig. 2. EBP Reconstruction

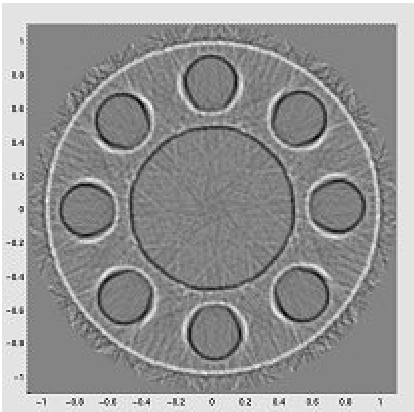

Fig.3. EBP and temporal correction

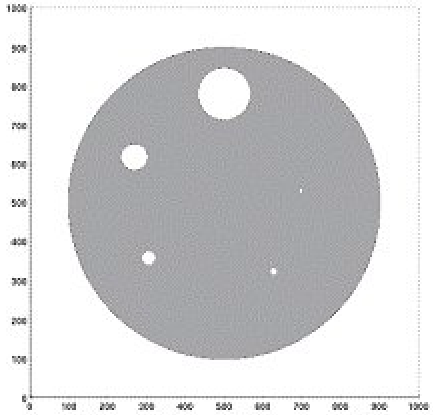

Fig. 4 water holes : r_{min}=0,07 mm

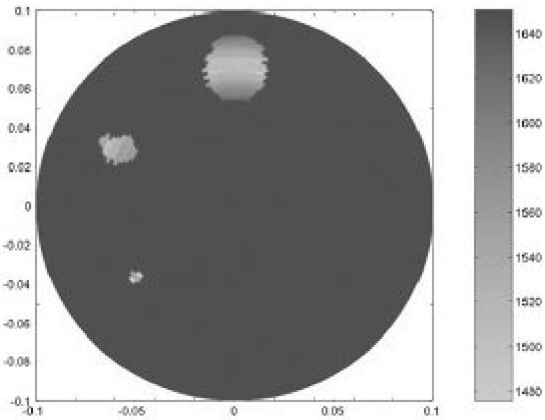

Fig.5. Speed map (layer stripping)

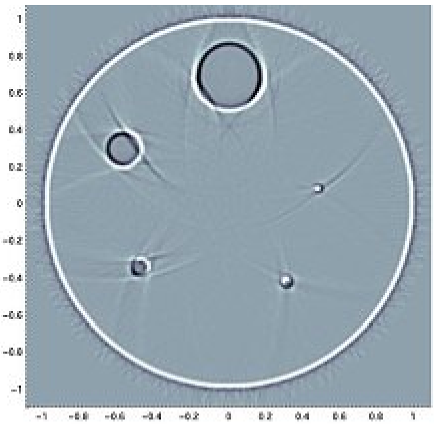

Fig. 6. EBP reconstruction

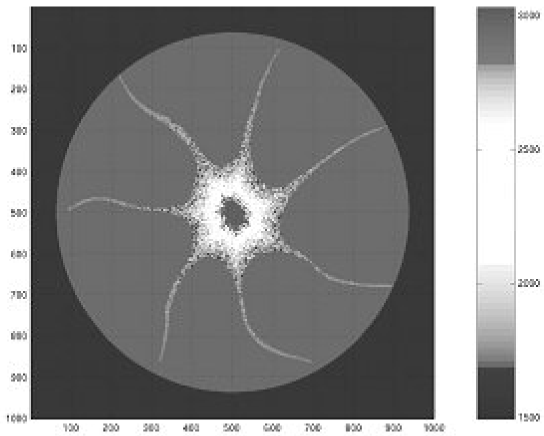

Fig 7 spiculated lesion

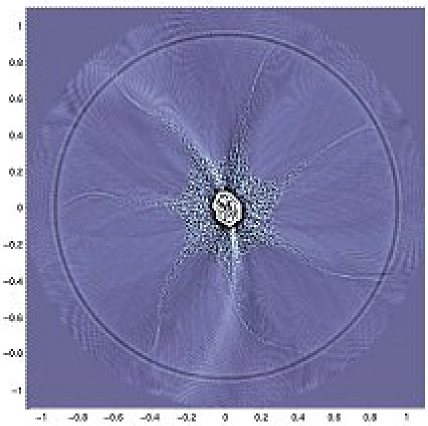

Fig 8 EBP reconstruction