Abstract

Aerial back sonar for use in a car detects obstacles by the echo pulses reflected from the targets. In order to develop high-performance aerial back sonar, we analyzed waveforms of the received pulses using the finite-difference time-domain (FDTD) method [1]. Since back sonar is generally used outdoors, the change of climate may have a significant effect on the received pulses from the objects. The FDTD method calculates the received waveforms from the object as a function of the object’s height as temperature and relative humidity in the air change.

The result showed that the maximum amplitude of a received pulse had a variation of about 25% with changing relative humidity. The variation of amplitudes also increased with the variation of temperature. If the amplitude of the second pulse is adapted to the threshold of an alarm, we need to consider the variation of relative humidity and temperature.

Introduction

The FDTD method calculates the received waveforms from the object as a function of the object’s height as temperature and relative humidity in the air change.

It is expected the speed of sound is independent of relative humidity at a constant temperature. However, the absorption coefficient should be greatly affected by relative humidity. It is reported that the sound attenuation coefficient of intensity depends on relative humidity.

It is calculated the influence of relative humidity when back sonar placed 0.5 m above the ground projects a Gaussian sound pulse of 40 kHz. The object is placed at \( x = 1.0 \) m and the ground is rigid.

FDTD calculation method

The basic equations of the FDTD method, which is taking account of attenuation, are given as follows:

\[
\begin{align*}
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} & = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\
-\rho \frac{\partial v_x}{\partial t} & = \frac{\partial p}{\partial x} + \eta v_x, \\
-\rho \frac{\partial v_y}{\partial t} & = \frac{\partial p}{\partial y} + \eta v_y,
\end{align*}
\]

where \( p \) is sound pressure, \( v \) is the particle velocity, \( \rho \) is the density and \( t \) is time. The second part of the right hand side in Eqs. (2) and (3) show an attenuation of the medium. If the only plane sinusoidal wave propagates in x-direction, substitutions of two equations into Eq. (1) yields the next equation

\[
\frac{\partial^2 P}{\partial x^2} + \left( \frac{\omega^2}{c^2} - j \frac{\omega \eta}{pc} \right) P = 0
\]

\[
P = P_0 \exp \left[ -j (\gamma_1 - j \gamma_2) x \right]
\]

where \( P_0 \) is the constant and \( \gamma_1 \) and \( \gamma_2 \) are the wave number and attenuation constant, respectively. The speed of sound \( c \) and resistance coefficient are obtained where \( \omega \) is angular frequency:

\[
c = \omega / \sqrt{\gamma_1^2 - \gamma_2^2}
\]

\[
\eta = \frac{2 \gamma_1 \gamma_2}{\sqrt{\gamma_1^2 - \gamma_2^2}} \rho c
\]

The finite differential equations are obtained as a function of discrete positions \( x, y \) in space and a discrete time \( t \) as shown below. [3]

\[
p'(i, j) = p^{i+1/2}(i, j) - C_x[v_x^{i+1/2}(i+1/2, j) - v_x^{i-1/2}(i-1/2, j)] + v_x^{i+1/2}(i, j+1/2) - v_x^{i+1/2}(i, j-1/2))
\]

\[
+v_y^{i+1/2}(i, j+1/2) - v_y^{i+1/2}(i, j-1/2))
\]

\[
v_x^{i+1/2}(i, j+1/2) = C_y[v_x^{i+1/2}(i+1/2, j) - v_x^{i+1/2}(i-1/2, j)]
\]

where \( C_x = c^2 \rho \Delta x / \Delta t \). In these equations, superscripts show the time and \( i \) and \( j \) are the grid-numbers in the x and y directions in space, respectively. For simplification, \( \Delta x = \Delta y \).

Sound speed and attenuation

In general, the speed of sound is independent of relative humidity at a constant
temperature [5]. However, the absorption coefficient is greatly affected by relative humidity. Resistance constant $\eta$ in air is obtained as follows. Evans and Bazley [6] report that the sound attenuation coefficient of intensity $m$, depends on relative humidity,

$$m = (33 + 0.2t)f^2 \times 10^{-12} + \frac{Mf}{k + \frac{2\pi f}{k}},$$

where $f$ is the frequency of the sound, $t$ is the temperature and $M$ is the coefficient given by reference 3. $k$ is given by,

$$k = 1.92h^{1.30} \times 10^5,$$

where $h$ is the absolute humidity and is calculated by the next equation.

$$h = \frac{P_{w0} \times h_r}{P_m},$$

where $P_{w0}$ is the saturated vapor pressure at the measured temperature, $P_m$ is the measured atmospheric pressure and $h_r$ is the relative humidity. When $m$ is used in FDTD, half of $m$ is substituted to the attenuation coefficient $\gamma_2$ of sound pressure [3]. It is assumed that a constant value of $m$ at 40 kHz is used for calculating, because the frequency bandwidth of the projected Gaussian pulse is very narrow. The relation between attenuation coefficient $m$ and relative humidity $h_r$ is shown in Fig.1, when the frequency is 40 kHz [4]. The maximum values of the attenuation coefficient increase with the relative humidity.

**Simulation**

Back sonar placed 0.5 m above the ground projects a Gaussian sound pulse of 40 kHz as shown in Fig. 2. It is assumed that the sound source is composed of 45 discrete point sources in the calculation grids. A Gaussian weighting function is also assumed for the vibration velocity of the sound source. The object is placed at $x = 1.0$ m and the ground is rigid. To satisfy Courant’s equation for stability, calculation increments are chosen in space, $\Delta x = \Delta y = 0.4$ mm, and in time $\Delta t = 0.8 \mu$s. Mur’s first-order absorbing boundary conditions [8] are provided to eliminate the reflection wave from the outer boundary of the calculation space. The atmospheric pressure is assumed to be 1 atm. in the analysis domain.

When the temperature is constant at 20 °C, the received echo signals from the object at a height of 0.5 m is shown in Fig. 3 at relative humidity of 0 and 50 %. The attenuation coefficient has a minimum value at 0% relative humidity and a maximum value at a relative humidity of 50 %. It is clearly shown that the received pulse is composed of two pulses. The first pulse reflected from the upper-left corner of the object arrives at 6.0 ~ 6.2 ms after the start of projection, and the second pulse is reflected from the corner of the object and the ground and arrives at about 6.7 ms.

![Fig.1 Intensity absorption coefficient $m$](image1)

![Fig.2 Block-diagram of calculation](image2)
Fig. 3 Received pulse from object at no humidity

The amplitude of the received pulse at zero relative humidity is reduced by about 20% compared to the amplitude at relative humidity of 50%.

Fig. 4 Maximum amplitude of 1st and 2nd pulse.

Figure 4 shows that the maximum amplitudes of the first and the second pulses increase with incremental changes of the object’s height at humidities of 0 and 50%. If 0 dB is defined as the source pressure, the amplitude of the second pulse is almost -30 dB. We can adapt this amplitude level to the threshold of an alarm, because this level is almost constant. Amplitudes of the first and the second pulses are -2.15 and -2.41 dB at relative humidity of 0 and 50%, respectively.

Fig. 5 2nd pulse vs. relative humidity

Figure 5 shows the normalized maximum amplitude of the second pulse versus relative humidity at temperatures of 10, 20 and 30 °C. At zero relative humidity, all amplitudes are almost the same.

Table I shows the maximum amplitude when the temperature changes from 10 to 30 °C. The result shows that an increase of temperature enlarges the variation of amplitudes. The relative humidity at the minimum amplitude decreases monotonically with increases in the temperature. The FDTD method was used to calculate the received wave-forms of aerial back sonar while the relative humidity and temperature changed. The result showed that the maximum amplitude of a
received pulse had a variation of about 25% with changing relative humidity. The variation of amplitudes also increased with the variation of temperature. If the amplitude of the second pulse is adapted to the threshold of an alarm, we need to consider the variation of relative humidity and temperature.

### Conclusion
The FDTD method calculated the acoustical characteristics of aerial back sonar. It projected a 40 kHz pulse whose pulse-width was about 0.15ms. The sound pressure field of sonar was almost the same in various temperatures from 10 to 30 degrees in Celsius. We also calculated the reflected echo signal from the target as a function of its height. The amplitude and propagation time of the reflected echo pulse changed a little with temperature variation. However, the amplitude of the received pulse at zero relative humidity was reduced by about 20% compared to the amplitude at relative humidity of 50%. The relative humidity at the minimum amplitude of pulse decreased monotonically with increases in the temperature. It also, was shown that the maximum amplitude of a received pulse had a variation of about 25% with changing relative humidity. These results show the validity of the FDTD method.

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### REFERENCES
6) E.J.Evans and E.N.Bazley: Acustica vol.6, p.238, 1956