MODELIZATION OF ACOUSTIC PATH OF CANCELLOUS BONE AND QUANTITATIVE ESTIMATION OF BONE DENSITY

T. Otani, R. Takatsu, and S. Tanaka
Department of Electrical Engineering, Doshisha University, Kyotanabe-Shi, JAPAN
totani@mail.doshisha.ac.jp

Abstract
Both fast and slow longitudinal waves were clearly observed in cancellous bone, which correspond to waves of the first and second kind as predicted by Biot’s theory. According to experimental and theoretical studies, the propagation speed of the fast wave increases with the bone density and that of the slow wave remains almost constant. The attenuation constant of the fast wave is much higher than that of the slow wave and is independent of the bone density, but the slow wave is greatly dependent on the bone density and the attenuation constant increases with the bone density. Experimental results on transmitted ultrasonic waves through cancellous bone show that the amplitude of the fast wave increases proportionally with the bone density. The propagation path of ultrasonic waves are modelized and the bone density of cancellous bone is quantitatively given both by the amplitude and by the propagation speed of the fast wave.

1. Introduction
Ultrasonic method is widely used today for the assessment of bone status and osteoporosis diagnosis. Ultrasonic measurements of bone status or bone mass density are generally performed using ultrasonic parameters consisting of the slope of frequency-dependent attenuation (or broadband ultrasound attenuation, BUA) and the speed of sound (SOS). Many results of in vitro laboratory measurements and in vivo clinical trials have shown that ultrasonic parameters, BUA and SOS correlate significantly with bone mass density measured by X-ray method. However, there exists a problem inherent in the ultrasonic method on the reproducibility or the uncertainty of measured ultrasonic parameters. It is generally considered that this problem is caused by the complex geometrical shape of observation site, the transducers positioning and the heterogeneity of cancellous bone.

The aim of this study is to clarify the ultrasonic wave propagation process in the observation site containing cancellous bone and to define the assessment process of the bone density. In our previous study on the acoustic properties of cancellous bone,[1-3] it is shown that both fast and slow longitudinal waves propagate through cancellous bone along the trabecular orientation, which correspond to “waves of the first and second kind” as predicted by Biot’s theory.[4-6] Therefore, the evaluation of bone density based on the acoustic parameters of both fast and slow waves is more reasonable than the present method by BUA and SOS.

2. Ultrasonic wave propagation in cancellous bone
Experimentally obtained results on the ultrasonic wave propagation in cancellous bone are presented here.[1] The propagation speed and the attenuation in cancellous bone were measured in a water tank by use of specimens, 20-30 mm in size and 9 or 7 mm thickness cut from the distal epiphysis of bovine femora with soft tissue in situ. A single sinusoidal pulse wave of 1 MHz, transmitted and received by a pair of wide band PVDF transducers, was used to observe the fast and slow waves separately. Figure 1 shows the received pulsed waveform travelling in water, which is applied to specimens. Figure 2 shows typical waveforms traveling through the cancellous bone in the direction of trabecular alignment. Figure 2(a) is a waveform for a high density ($\rho = 1200 \text{ kg/m}^3$, $V_f = 0.25$) and (b) is a low density ($\rho = 1120 \text{ kg/m}^3$, $V_f = 0.17$). In both Fig.2(a) and (b), the fast and slow longitudinal waves can be clearly observed in the time domain. As the density increases the amplitude of the fast wave becomes greater. At the same time, the amplitude of the slow wave decreases.

The propagation speeds and attenuations of the fast and slow waves were measured using pulse spectrum analysis of received waveforms. Figure 3 shows the propagation speeds of the fast and slow waves in cancellous bone at 1 MHz as a function of bone volume fraction $V_r$. The speed of the fast wave increases as the bone volume fraction (the bone density) increases. The slow wave remains almost constant at about 1400 m/s (or decreases very slightly with the bone density), which is close to the propagation speed of 1450 m/s in bone marrow (soft tissue). The propagation speed 2200-2700 m/s of the fast wave is much slower than the propagation speed 3400-4200 m/s of cortical bone (solid bone). This can be explained by the fact that the cancellous bone is not solid but has a porous network structure. The propagation speeds of both the fast and slow waves are almost constant and nondispersive in

![Fig. 1 Pulse wave with a center frequency of 1.0 MHz transmitted in water by the PVDF transmitter](image-url)
Fig. 2 Pulse waves propagating through bovine cancellous bone with soft tissue in the parallel direction to the trabecular alignment (a) high density ($\rho = 1200$ kg/m$^3$, $V_f = 0.25$), (b) low density ($\rho = 1120$ kg/m$^3$, $V_f = 0.17$) $V_f$: bone volume fraction

Fig. 3 Propagation speeds of fast and slow waves in cancellous bone at 1 MHz as a function of bone volume fraction $V_f$

Fig. 4 Attenuation of fast and slow longitudinal waves in bovine cancellous bone with soft tissue at three bone volume fractions of $V_f = 0.25$, 0.19, and 0.17, as a function of frequency: (a) fast wave; (b) slow wave in the range 0.5-5 MHz. Figure 4 shows the attenuation for the three specimens of bone volume fraction $V_f = 0.25$, 0.19, 0.17 (density $\rho = 1200$, 1140 and 1120 kg/m$^3$). The attenuation of the slow wave (Fig.4(b)) depends considerably on the bone volume fraction $V_f$ (or the bone density). The attenuation in cortical bone and bone marrow were also obtained experimentally as about $5.0 \times 10^{-2}$ [neper/mmMHz] and $1.3 \times 10^{-2}$ [neper/mmMHz]. Both the fast and slow waves in cancellous bone show much higher attenuation than the bulk wave in cortical bone ($\rho = 1960$ kg/m$^3$) and bone marrow ($\rho = 930$ kg/m$^3$).

The attenuation of the fast wave is almost independent of the bone density as shown in Fig.4(a). However, observed pulse waveforms propagated through cancellous bone shown in Fig.2 show that the amplitude of the fast wave becomes greater as the bone density increases. This fact implies that another acoustic parameter should be introduced to evaluate the bone density. To obtain the dependence of the amplitudes of propagated wave through cancellous bone on the bone density, another experiences were carried out.

A cancellous bone specimen of about 10 mm thickness was cut from the distal epiphysis of bovine femur with soft tissue in situ. The specimen was mounted between a transmitter and a receiver at normal incidence. The focused PVDF transmitter was driven by a single sinusoidal pulse voltage with a frequency of 1 MHz. Pulse waves propagating through the water/specimen/water system was detected by the wide band non focused PVDF receiver. Ultrasonic wave path was scanned in an area of $15 \times 15$ mm and transmitted waves were measured at 1 mm intervals or 225 points. The scanned area was taken in nearly middle region of the specimen, where the trabecular alignment was approximately parallel to the thickness direction or the direction of ultrasonic wave propagation. The local bone density corresponding to the measured points was obtained by use of a micro focus X-ray CT system. The dependence of measured peak to peak amplitudes and propagation speeds is shown for the fast wave in Fig.5 and for the slow wave in Fig.6. The bone density has a strong positive correlation with both amplitude and propagation speed of the fast wave and a clear negative correlation with amplitude of the slow wave.

3. Modelization of propagation path and estimation of bone density

An observation site is set between a transmitting and a receiving transducers in a water tank. When the ultrasonic wave is radiated from a focused transmitter, the plane wave approximation should be reasonably obtained in a range of narrow beam. Then the propagation process through cancellous bone can be considered as one dimensional or the plane wave propagation. Under the condition that the boundaries in
the observation site are perpendicular to the ultrasonic beam, the propagation path can be modelized as shown in Fig.7. The focused ultrasonic wave is radiated by the transmitter $T_r$ to the medium 1 (water), then the ultrasonic wave pass through the medium 2 (soft tissue), the medium 3 (cortical bone), the medium 4 (cancellous bone), the medium 5 (cortical bone), the medium 6 (soft tissue), the medium 7 (water) and arrive at the receiver $R_e$. The ultrasonic wave is separated into two longitudinal waves, the fast and the slow waves during the propagation in the medium 4 (cancellous bone). Received signal voltage $E_2$ of the receiver $R_e$ for the fast wave is expressed as

$$E_2 = [SMA_0][AT][e^{-\alpha_1 x_1}][T_{34}T_{45}]E_1$$

(1)

and received signal voltage $E_2'$ for the slow wave is also expressed as

$$E_2' = [SMA_0][AT]e^{-\alpha_3 x_3}[T_{34}T_{45}']E_1$$

(2)

where

- $S = \frac{P_1}{E_1}$: $S$ is voltage sensibility of the transmitter $T_r$, and $P_1$ is the transmitted acoustic pressure on the radiating surface.
- $M = \frac{E_2}{P_2}$: $M$ is voltage sensibility of the receiver $R_e$, and $P_2$ is the incident acoustic pressure.
- $c_0 = c_1 = c_2$: propagation speed in water
- $c_2 = c_5$: propagation speed in soft tissue (medium 2 and 6)
- $c_3 = c_6$: propagation speed in cortical bone (medium 3 and 5)
- $\alpha_0 = \alpha_1 = \alpha_2$: attenuation constant of water (medium 1 and 7)
- $\alpha_2 = \alpha_6$: attenuation constant of soft tissue (medium 2 and 6)
- $\alpha_3 = \alpha_5$: attenuation constant of cortical bone (medium 3 and 5)

On this experimental trial, thicknesses or distances $x_1, x_2, \ldots, x_7$ is to be experimentally determined by the echo method. When the observation site is removed, the ultrasonic wave propagates only in the water. The received signal voltage $E_0$ is

$$E_0 = SME_0e^{-\alpha_0 x_6} = SMA_0B_0E_1$$

(3)

where $X_0 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$ and $B_0 = e^{-\alpha_0(x_2+x_3+x_4+x_5+x_6+x_7)}$.

Then the signal voltage ratio $E_2/E_0$ for the fast wave is given by

$$\left[\frac{E_2}{E_0}\right] = \left[\frac{AT}{B_0}\right]e^{-\alpha_1 x_1}\frac{T_{34}T_{45}}{}$$

(4)

and for the slow wave by

$$\left[\frac{E_2'}{E_0}\right] = \left[\frac{AT}{B_0}\right]e^{-\alpha_3 x_3}\frac{T_{34}'T_{45}'}{}$$

(5)
Thus, the ultrasonic parameters concerning the cancellous bone density is expressed for the fast wave as
\[ T_{34}T_{45} = \frac{B_0}{A_0} \left[e^{-\alpha_1 t_4} \right] \left[ \frac{E_2}{E_0} \right] \] (6)
and for the slow wave as
\[ e^{-\alpha_1 t_4} T_{34}T_{45} = \frac{B_0}{A_0} \left[ \frac{E_2}{E_0} \right] \] (7)
As shown in Fig.5, the parameter \( T_{34}T_{45} \) for the fast wave has a strong positive correlation with the bone density, and the parameter \( e^{-\alpha_1 t_4} T_{34}T_{45} \) for the slow wave has a strong negative correlation with the bone density as shown in Fig.6.

The propagation speed of the fast wave has a strong positive correlation with the bone density and that of the slow wave has almost no correlation. The propagation speed \( c_4 \) is given by
\[ c_4 = x_4/t_4 \] (8)
The propagation speeds, the attenuation constants and the acoustic impedances (or the transmission coefficient \( T_{12}, T_{23}, T_{36}, T_{60} \) of water, soft tissue and cortical bone should be preliminarily obtained. The dependence of the parameter, \( T_{34}T_{45} \) for the fast wave on the bone density and of the parameters \( c_4 \) and \( T_{34}T_{45} \) for the slow wave on the bone density should be experimentally established preliminarily. Thus the bone density of cancellous bone can be evaluated by equations (6), (7) or (8) based on the preliminary obtained ultrasonic parameters.

4. Remarks and conclusions

The propagation phenomena of ultrasonic wave in cancellous bone are experimentally discussed. Two distinct longitudinal waves (fast and slow waves) propagating through cancellous bone can be clearly identified by experimental observation.

The propagation speed and the amplitude of transmitted ultrasonic wave through cancellous bone have a distinct causality both for the fast wave and for the slow wave. Some remarks of the study is listed as follows:

(i) Propagation speed has a clear positive correlation with the density of cancellous bone and is described and formulated by use of Biot’s theory.\(^\text{[4,6]}\)
(ii) The bone density of cancellous bone can be quantitatively evaluated by the propagation speed (Fig.3).
(iii) There exists a possibility to evaluate the trabecular structure based on the propagation speed, because the propagation speed of the fast wave depends on the bone density and the trabecular structure.

2. The slow wave in cancellous bone

The slow wave in cancellous bone is associated with the elastic behavior of the soft tissue (bone marrow) filling the pore spaces. The propagation speed of the slow wave (1400 m/s) is very close to the speed of 1450 m/s in bone marrow.

(2-1) Amplitude of the slow wave

(i) Amplitude increases significantly as the volume of soft tissue increases (as the bone density decreases). This signifies that the amplitude has a strong negative correlation with the bone density.

(ii) Attenuation coefficient for the slow wave increases with the bone density.

(iii) As the amplitude of the slow wave depends on the volume of soft tissue, the bone density can be evaluated indirectly by the volume of soft tissue. (Soft tissue volume fraction (porosity) + Bone volume fraction (bone density) = Total volume)

(2-2) Propagation speed of the slow wave

(i) The propagation speed of the slow wave is almost constant and no information on the bone density is expected. (Strictly speaking, the propagation speed shows extremely slight decrease as the bone density increases (Fig.3).)

(ii) Elastic properties of cancellous bone cannot be evaluated by the propagation speed of the slow wave.

Thus, we come to the conclusion that the bone density and the elastic properties of cancellous bone are able to evaluate based on a clear causality concerning the fast wave.

References