EXPERIMENTAL EVALUATION OF Q-VALUES IN A RESONANCE TUBE

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Abstract

The quality factor of resonance is defined as the angular frequency times the energy stored in the system divided by the dissipated power. In this work, we determine the quality factor of the acoustic resonator on the basis of the definition. The obtained quality factor of 88 is significantly different from that (=60) determined by the conventional method using a frequency response curve.

Introduction

Mutual energy conversion between heat flow and acoustic intensity I in a regenerator leads to a rich variety of thermoacoustic phenomena[1-3]. Acoustic intensity I of a wave with an angular frequency ω running through a tube is represented by using the acoustic pressure $P=pe^{i\omega t}$ and the cross-sectional mean acoustic velocity $U=ue^{i(\omega t+\phi)}$ as

$$I=\langle PU\rangle, \qquad (1)$$

where angular brackets (< >) represent time average. In a thermoacoustic Stirling prime mover, the acoustic intensity I is produced in a differentially heated regenerator[4-6]. The produced I can be further amplified through the thermoacoustic energy conversion[7]. It is also known that the thermoacoustic energy conversion makes it possible to pump heat from cold to hot[8]. While the absence of any moving parts in the energy conversion is important, it should be also noted that the produced I is delivered through an acoustic resonance tube without moving parts. In order to gain a further insight into a resonator as an energy transfer tube, we focus on the quality factor Q of the resonance tube.

The quality factor Q is defined as

$$Q=\omega E_s/\dot{E}, \qquad (2)$$

where E_s and $\dot{E}$ represents the stored energy and the dissipated power in the system, respectively. A quality factor Q represents the sharpness of the resonance, and are widely used in many other resonance phenomena such as electrical circuits. However, quality factors Q are often determined from the fitting of the frequency response curve to a Lorentz curve. This conventional method is convenient, but has several disadvantages; Q values thus determined depends not only on the gas column but also on other parts of the resonator such as a driver, and in a system with a very strong damping, the frequency response curve is no longer to fitted to a Lorentz curve . In this work, we determine the quality factor Q of the acoustic resonator based on the definition through the measurement of the acoustic intensity I and the energy density in the resonator.

Experimental

The experimental apparatus in the present work is shown in Fig. 1. A resonator is made of Pyrex glass with length L=0.89 m, and inner radius r=18.5 mm. One end of the resonator is closed by a solid plate, and the other is connected to a woofer speaker through dynamic bellows The resonator is filled with one-bar air and driven at the fundamental frequency. The acoustic pressure P and the core velocity in the tube are measured simultaneously by using pressure transducers and a Laser Doppler velocimeter. We theoretically determined the cross sectional mean velocity U from the core velocity[9,10].

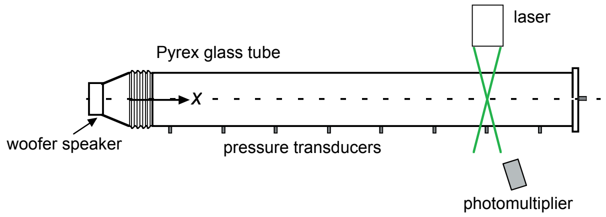

Figure 1 Schematic illustration of the present experimental apparatus.

Results

Figure 2(a) shows the axial distribution of p and u. The coordinate X normalized by the length between the acoustic driver and the closed end is directed toward the closed end. Since we have used the fundamental mode of the present resonator, the pressure amplitude p shows a minimum in the middle of the resonator. On the other hand, the velocity amplitude u shows minimums at the closed end and in front of the driver.

The energy stored in the resonator is obtained by integrating the energy density e given as

$$e=\frac{1}{4}\left\{\frac{p^2}{\rho_m a^2}+\rho_m u^2\right\} \qquad (3)$$

over the volume of the resonator, where ρ_m and a represent mean density and sound speed, respectively.

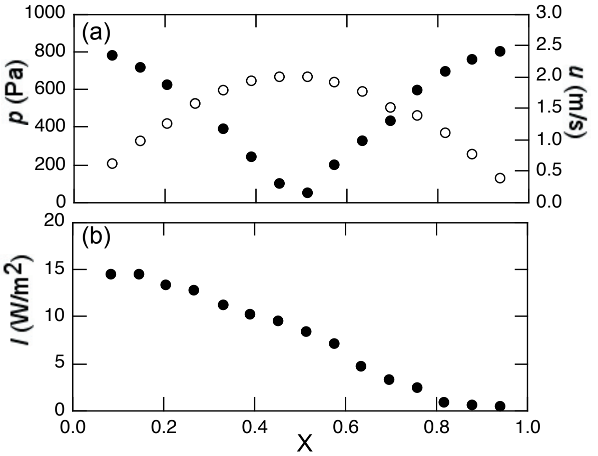

Figure 2 Axial distribution of (a) amplitudes of pressure (solid square) and velocity (open square), and (b) the acoustic intensity *I*.

Assuming a pure standing wave field in the resonator, we obtain

$$E_s = \int e dV = \frac{p^* u^*}{4a} \pi r^2 L \quad , \qquad (4)$$

where p^* and u^* represent the maximum amplitudes of pressure and velocity in the resonator[2]. By inserting the measured data into eq. (4), we determined the stored energy E_s in the present resonator.

Figures 2(b) shows the axial distribution of the acoustic intensity *I* in the resonator. We determined the acoustic intensity *I* using the definition shown in eq. (1). The positive value of *I* represents that it flows from the driver to the closed end. Thus, $I(X=0)$ represents the acoustic intensity emitted from the driver. We also see that the slope of *I* is always negative. This represents the viscous and thermal attenuations accompanied by the oscillating motion of the fluid. Therefore, the total acoustic power emitted from the driver to the resonator is given as

$$\dot{E}=\pi r^2 \cdot I(X=0). \qquad (5)$$

By using eq. (5), we determined $\dot{E}$ as 17W/m^2.

Now we are ready to determine the quality factor *Q* based on the definition. By inserting the evaluated E_s and $\dot{E}$ yields the *Q* of 88 for the present resonator. In comparison, we measured the pressure at the closed end, $p(X=1)$, as a function of the frequency and made the frequency response curve of the present resonator. The estimated peak width yields *Q* as 60. This is significantly different from the *Q* obtained based on the definition.

Furthermore, we also determined the quality factor *Q* in the resonator with a strong damping. A honeycomb ceramic catalyst with many square holes of the sides of 1.03 mm (40 mm in length) is inserted at the velocity loop of the present resonator. As a result of the measurement of E_s and $\dot{E}$, we determined *Q* as 12. We also tried the conventional method, however, it was not applicable, since the frequency response curve in the loaded resonator could not be fitted to a Lorentz curve.

Summary

We experimentally determined the stored energy and the dissipated power of the gas column in the acoustic resonator, and evaluated the quality factor *Q* based on the definition. The obtained quality factor *Q* of 88 was significantly different from that estimated by the conventional method (=60). Our method is applicable to the acoustic resonator with a strong damping.

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