NUMERICAL STUDY OF ACOUSTIC STREAMING IN A RESONATOR WITH LARGE REYNOLDS NUMBER

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Abstract

The large amplitude standing wave excited in a resonator induces acoustic streaming of Rayleigh type outside the acoustic boundary layer on the wall of the resonator. The streaming motion with large Reynolds number is examined numerically in relatively long two-dimensional rectangular boxes. The two-dimensional incompressible Navier–Stokes equations with no external force are used as the governing equations for the streaming velocity. The steady velocity component at the outer edge of the acoustic boundary layer, which induces Rayleigh type streaming, is employed as the boundary condition for the Navier–Stokes equations. By using a finite-difference method, the existence of multiple steady state solutions are shown.

Introduction

Streaming motions induced by acoustic standing waves are classical topics in physics [1,2]. Today, an active control of streaming in resonators becomes an important problem in various applications (e.g., [3]). Some authors have recently carried out accurate measurements for slow streaming motions [4,5]. However, the behavior in the case of large Reynolds number remains unresolved.

Recently, the present author has numerically studied the resonant gas oscillation with a periodic shock wave in a closed tube by solving the system of compressible Navier–Stokes equations [6]. The result has suggested the occurrence of turbulent acoustic streaming when a streaming Reynolds number is sufficiently large. However, the direct numerical simulation of viscous compressible flow is an extraordinarily hard task if one tries to resolve all phenomena from an initial state of uniform and at rest to an almost steady oscillation state throughout the entire flow field including the boundary layer.

In a previous paper [7], we have adopted a simple model based on the linear standing wave solution and a boundary layer analysis. This model employs the incompressible Navier–Stokes equations as the governing equations for the streaming velocity. As a result, we have numerically shown the multiple existence of steady state solutions in a two-dimensional rectangular box of length $L \cong \lambda$, where λ is the wavelength of the standing wave. In the present paper, we shall extend the previous analysis [7] to the cases of higher mode (mainly 5th mode) standing waves in relatively long boxes. Some of our numerical results somewhat resemble experimental ones [8].

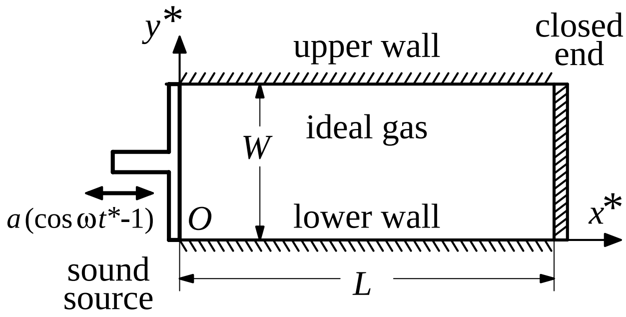

Figure 1: Schematic of model.

Problem

We shall consider the streaming motion induced by resonant gas oscillations in a two-dimensional rectangular box filled with an ideal gas (see Fig. 1). The box, whose length is L and width is W, is closed at one end by a solid plate and the other by a piston (sound source) oscillating harmonically with an amplitude a and angular frequency ω.

We assume that the sound excitation is moderately weak and the dissipation effect is sufficiently small outside the boundary layer; this is assured by

$$M = \frac{a\omega}{c_0} \ll 1, \quad \epsilon = \frac{\sqrt{\nu_0 \omega}}{c_0} \ll w, \qquad w = \frac{W\omega}{c_0} = O(1), \tag{1}$$

where M is the acoustic Mach number, ϵ is a measure of the ratio of the viscous penetration depth to the wavelength, and w is a normalized width of the box. The wave motion in the bulk of the gas can then be regarded as a plane standing wave with small corrections due to nonlinear and dissipation effects.

$$u = M\frac{\sin(x-b)}{\sin b}\sin t + \cdots, \qquad \rho = 1 + M\frac{\cos(x-b)}{\sin b}\cos t + \cdots, \tag{2}$$

where u is the axial component of the fluid velocity normalized by c_0, ρ is the normalized gas density,

$x = x^*\omega/c_0$, and $t = \omega t^*$. We also assume that the gas oscillation concerned is near the nth mode resonance,

$$b = \frac{L\omega}{c_0} = n\pi + \sqrt{M}\Delta, \tag{3}$$

where n is a positive integer and Δ is a nondimensional parameter for a measure of detuning ($\Delta \neq 0$). If $|\Delta| = O(\sqrt{M})$, then the oscillation includes two periodical shock waves as long as the dissipation effect is sufficiently small. The formation of shock waves may induce the turbulent streaming motion as shown in [6]. In what follows we assume $\Delta \approx 1$. Equation (2) can then be rewritten into

$$\begin{aligned} u &= \sqrt{M}\frac{1}{\Delta}\sin x \sin t + \cdots, \\ \rho &= 1 + \sqrt{M}\frac{1}{\Delta}\cos x \cos t + \cdots. \end{aligned} \tag{4}$$

Boundary layer analysis

Equation (4) suggests that the physical quantities such as u and ρ may be expanded in powers of $\sqrt{M}$ and the method of matched asymptotic expansions can be applied for the analysis inside the acoustic boundary layer. Using the method of matched asymptotic expansions, we have, in the first order,

$$\begin{aligned} u_1 = &-\sqrt{M}\frac{1}{\Delta}\sin x\Big[\cos t \\ &- \exp\Big(-\eta\sqrt{\frac{\text{Pr}}{2}}\Big)\cos\Big(t - \eta\sqrt{\frac{\text{Pr}}{2}}\Big)\Big], \end{aligned} \tag{5}$$

$$\begin{aligned} \rho_1 = &\sqrt{M}\frac{1}{\Delta}\cos x\Big[\sin t \\ &+(\gamma-1)\exp\Big(-\eta\sqrt{\frac{\text{Pr}}{2}}\Big)\sin\Big(t - \eta\sqrt{\frac{\text{Pr}}{2}}\Big)\Big], \end{aligned} \tag{6}$$

where the subscript 1 denotes the first order term and $\eta = y/\epsilon$ is the coordinate normal to the boundary (we here concentrate on the lower wall $y = 0$). In the second order, we have to solve

$$\frac{\partial^2 \overline{u_2}}{\partial \eta^2} = -\overline{\frac{v_3}{\alpha}\frac{\partial u_1}{\partial \eta}} - \overline{u_1\frac{\partial u_1}{\partial x}} - \overline{\rho_1\frac{\partial u_1}{\partial t}} - \frac{\partial \overline{p_2}}{\partial x}, \tag{7}$$

where the subscript 2 denotes the second order term and the bar denotes the time average, e.g.,

$$\overline{u_2} = \frac{1}{2\pi}\int_0^{2\pi} u_2(x,\eta,t)\,\mathrm{d}t. \tag{8}$$

Solving Eq. (7) and taking the limit $\eta \to \infty$, we obtain

$$\lim_{\eta\to\infty}\overline{u_2} = -\frac{M}{\Delta^2}\sin 2x\Big[\frac{3}{8} + \frac{\sqrt{\text{Pr}}(\gamma-1)}{4(\text{Pr}+1)}\Big], \tag{9}$$

and $\overline{\rho_1 u_1} \to 0$. That is, we have the time-averaged mass flux $\overline{u_2}$ at the outer edge of the boundary layer.

Governing equation for streaming

Then, the governing equations for the steady streaming velocity (U, V) outside the boundary layer are [7]

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \tag{10}$$

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + \frac{\partial p}{\partial x} = \frac{1}{\text{Re}}\Big(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\Big), \tag{11}$$

$$U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + \frac{\partial p}{\partial y} = \frac{1}{\text{Re}}\Big(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\Big), \tag{12}$$

where all variables are nondimensionalized and a streaming Reynolds number Re is defined as

$$\text{Re} = \frac{U_0}{\epsilon^2},\ U_0 = \frac{M}{\Delta^2}\Big[\frac{3}{8} + \frac{\sqrt{\text{Pr}}(\gamma-1)}{4(\text{Pr}+1)}\Big], \tag{13}$$

The boundary condition on the upper and lower walls is given as

$$U = -\sin 2x,\ V = 0, \tag{14}$$

and the non-slip condition is applied at $x = 0$ and $x = n\pi$. We here notice some important features of Eqs. (10)–(14): (i) the streaming velocity (U, V) is the divergence free and rotational (vortical) vector field; (ii) the momentum equations (11) and (12) have no driving force term, because the wave field may be described by the plane standing wave (4); (iii) since the boundary layer is sufficiently thin, we can neglect its thickness and impose the boundary condition (14) at $y = 0$ and $y = w$; (iv) the length of the box b can also be approximated as $n\pi$.

Previous results

In Ref. 7, we have analyzed the steady streaming motions in the case of $n = 2$ (second mode), and thereby we have demonstrated that (i) the classical flow pattern, which has a rotational and reflectional symmetry, ceases to be a stable solution beyond $\text{Re} = 240$, (ii) new flow pattern, which has reflectional symmetry about $x^* = L/2$, exists in a wide rage $140 < \text{Re}$, (iii) an asymmetric flow pattern appears at $\text{Re} = 180$, (iv) 180-degrees rotational symmetric pattern also appears at $\text{Re} = 700$.

The multiple existence of steady streaming has thus been demonstrated in the case of $n = 2$ in Ref. 7. In the following, we shall present the numerical results mainly for the case of $n = 5$ (fifth mode) in a relatively long boxes.

Results

The steady state solutions for the boundary value problem (10)–(14) are numerically obtained with a finite-difference method, by solving initial value problems starting from some appropriate initial conditions.

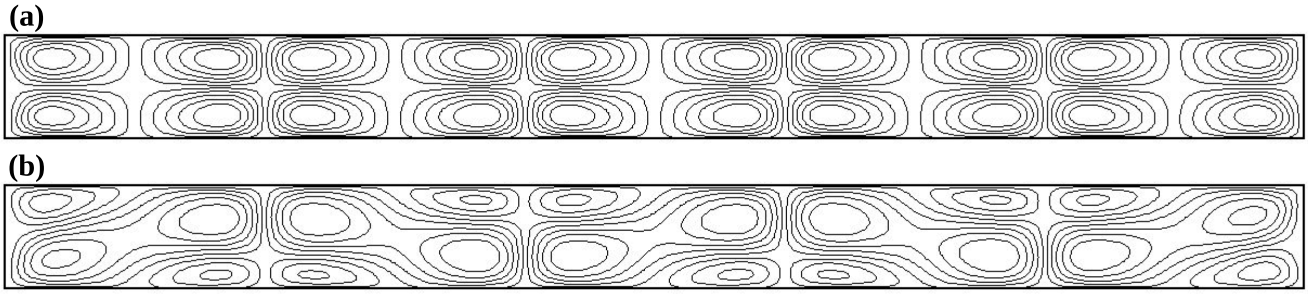

Figure 2: Streamlines for Re $= 140$. (a) classical Rayleigh streaming. (b) symmetric flow.

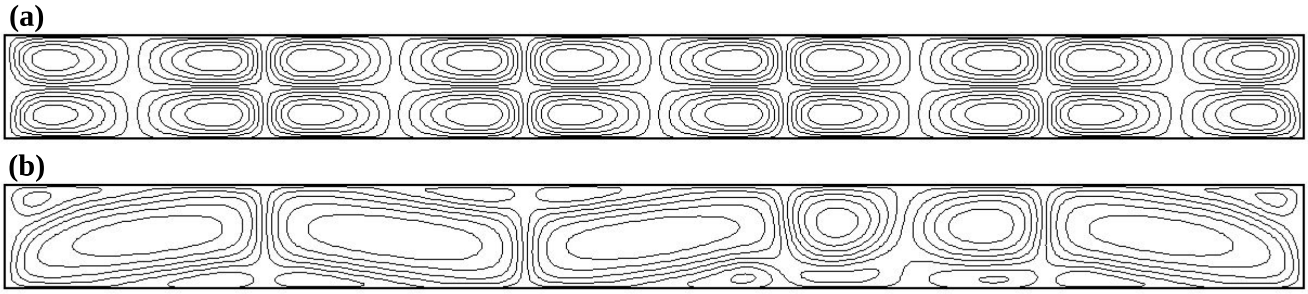

Figure 3: Streamlines for Re $= 300$. (a) classical Rayleigh streaming. (b) asymmetric flow.

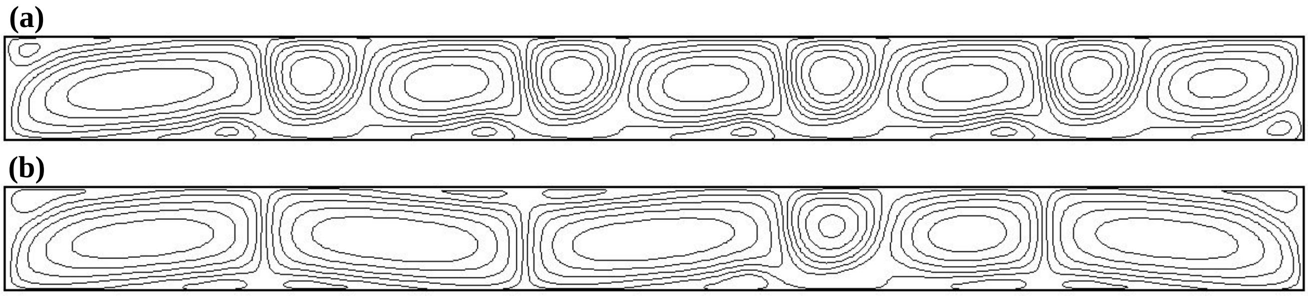

Figure 4: Streamlines for Re $= 600$. Different type symmetric flows.

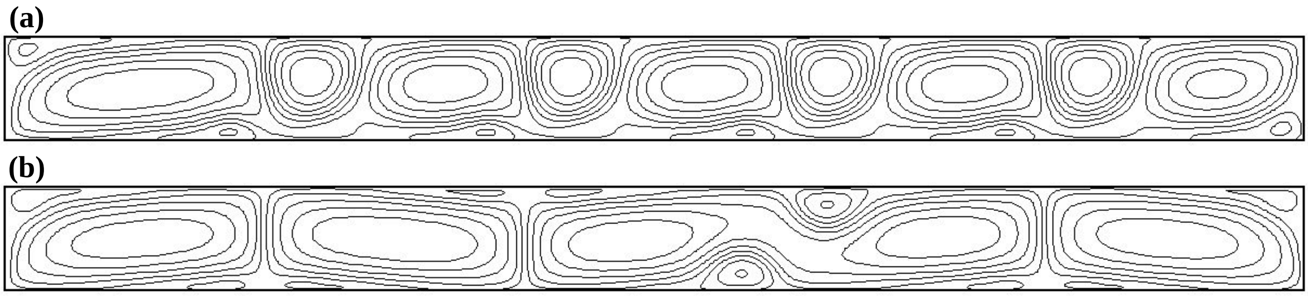

Figure 5: Streamlines for Re $= 700$. Different type asymmetric flows.

The computations are continued up to the time when the numerical solution converges at a steady state. Since we solve the initial value problem, unstable steady solutions cannot be found. In other words, the numerical solutions obtained by this procedure may be regarded as the stable steady solutions.

Figures 2(a) and 3(a) shows the classical symmetric streaming pattern. The classical symmetric pattern seems to survive as a stable steady solution at least up until Re $= 300$ in the case of $n = 5$ (see Fig. 6). The flow pattern shown in Fig. 2(b) has the 180-degrees rotational symmetry, but the symmetry around the central axis $(y = w/2)$ is lost. The rotational symmetry in Fig. 2(b) disappers in the flow shown in Fig. 3(b) at

Re $= 300$. The asymmetric flow in Fig. 3(b) belongs to the same type as that in Fig. 4(b) at Re $= 600$, although this pattern is also modified as shown in Fig. 5(b) at Re $= 700$.

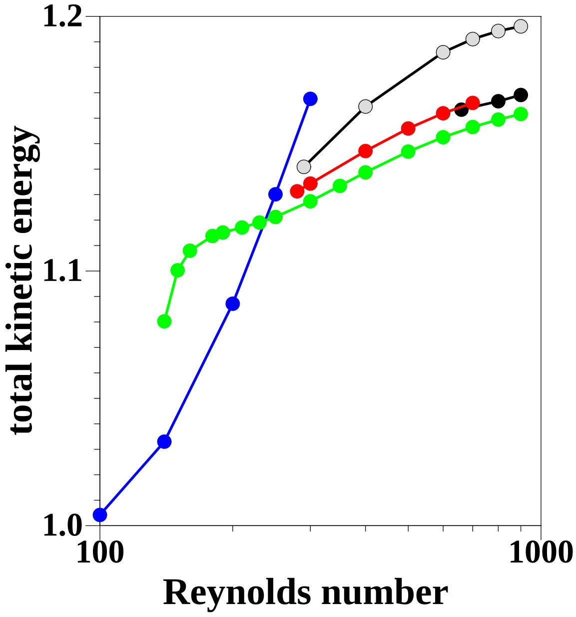

Figure 6: Multiple existence of steady streaming induced by the fifth mode standing wave ($n = 5$).

The symmetric flows as shown in Figs 2(b) first appears at Re $= 140$ and remains to be stable at least up to Re $= 1000$ (see Fig. 6). The asymmetric flows as shown in Figs. 3(b), 4(a), and 4(b) emerge at about Re $= 300$, while the asymmetric flows as shown in Fig. 5(b) are not obtained for Re < 700 in the present numerical procedure.

We shall remark that the flow patterns shown in Figs. 4(a) and 5(a) somewhat resemble an experimentally obtained flow pattern [8]. We also remark that all the flow patterns in $n = 5$ can be regarded as combinations of those in $n = 2$ obtained in Ref. 7.

Conclusions

We shall summarize the main results:

1. Multiple existence of steady streaming is numerically confirmed in the fifth mode standing wave field in a 2-dimensional relatively long rectangular box.

2. Some flow patterns resemble those observed in experiments.

3. All flow patterns obtained in the case of $n = 5$ can be regarded as combinations of those in the case of $n = 2$ obtained in the previous numerical analysis.

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