NONLINEAR RESONANT ULTRASOUND SPERTROSCOPY WITH TWO WAVE

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Abstract The effect of the influence of a strong acoustic wave on a weak signal wave can be used as one of the methods of the Nonlinear Elastic Wave Spectroscopy. The technique is based on measurements of resonant frequencies and Q-factors for acoustic wave in the presence of a strong, CW pump wave. The presented experiments demonstrate the strong influence of the pump wave on weak probe wave resonances in damaged concrete while there is no influence in the intact concrete. The theory describing a shift of the resonant frequency and an increase in the Q-factor for a high-frequency signal wave in the presence of a low-frequency strong pump wave is the topic of this presentation. The model is based on including small hysteretic loops in the strain-stress dependence for the signal wave. Parameters of hysteretic loops depend on their position on the strain-stress curve for the pump wave.

Introduction

Strong acoustic nonlinearity of microinhomogeneous media, rocks, in particular, manifests itself in a wide spectrum of nonlinear effects such as generation of harmonics and combination frequencies, amplitude-dependent losses, detection, and others. The fields of research is known as the Nonlinear Elastic Wave Spectroscopy (NEWS) includes measurements of amplitude-frequency response of material under study and subsequent diagnostics of its parameters on the basis of these measurements [1-5]. Usually, experimental procedure for NRUS is such: a rod concrete or other material is resonated at fixed input wave when the input wave volume is increased the frequency is swept again. The other version of NEWS can be based on interaction of waves having different amplitudes and different frequencies, for example, strong low frequency pump wave and weak signal. The first experiments presented in [3] revealed that the resonant frequency shift and Q-factor of a weak signal depend on the amplitude of a strong pump wave. This effect was described within the framework of a phenonemological equation of state (the strain-stress dependence G(ε)), taking into account dissipative nonlinearity and did not reflect hysteretic behavior of the material [3]. We present a new experimental results regarding application of the novel NRUS technique for diagnostics of concrete and theoretical approach based on hysteretic stress-strain dependence. The model is based on including small hysteretic loops in the strain-stress dependence for the signal wave. Parameters of hysteretic loops depend on their position on the strain-stress curve for the pump wave.

EXPERIMENTAL RESULTS

The basic principle of the NRUS experiment is measuring the amplitude-frequency response produced by a weak wave in the presence of a strong pump wave. Experiments were carried out with sandstone and cracked concrete. These materials possess strongly nonlinear properties. The principle of the experiment is shown in Fig. 1. The several concrete samples having diameter 50 mm and length 100 mm were tested. One of the samples was loaded by using loading machine until several small cracks appeared. The examined sample was excited independently at two frequencies by two of piezoceramic emitters, the larger of which was loaded by additional mass for increasing stress amplitude. The smaller emitter was used to excite in the resonator acoustic field of a weak wave. Acoustic wave amplitude in the resonator was measured by accelerometer.

The low frequency pump signal was applied near frequency 10820 Hz and it was tuned to the resonance with amplitude increasing. The amplitude frequency responses for signal produced by the prove wave was measured in the wide frequency band for different levels of the low frequency pump wave. Fig.2 shows the amplitude frequency responses in the frequency band 36-38.5 kHz for different amplitudes of the pump signal for the damaged concrete sample. The shift of the resonant frequency of the weak signal at the two probe frequencies *versus* amplitude of the pump excited at the frequency is plotted in Fig. 3. Similar measurements in the intact concrete sample did no show any influence of the pump signal on the probe wave resonance. A nearly linear dependence of the weak signal resonant frequency shift on pump amplitude was revealed in these materials. The variation of the Q-factor of the weak signal, Q_{hf}, was inversely proportional to the amplitude of strong low-frequency pump

Theoretical Model

Consider interaction of a weak high-frequency and strong low-frequency longitudinal wave in a rod resonator. Assume that the strain G and stress ε of the material are related by

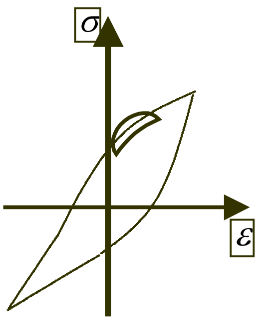
$$G(\varepsilon,\dot{\varepsilon}) = E(\varepsilon_{hf} - \varepsilon_{lf}) + \begin{cases} +\gamma\ (\varepsilon_{hf} - \varepsilon_{lf})(\varepsilon_{hf} - \varepsilon_{lf})^2,\ \dot{\varepsilon} > 0, \\ -\gamma\ (\varepsilon_{hf} - \varepsilon_{lf})(\varepsilon_{hf} - \varepsilon_{lf})^2,\ \dot{\varepsilon} < 0 \end{cases} \quad (1)$$

where ε_{hf}, ε_{lf} are stresses at high and low frequencies, $E(\varepsilon_{hf} - \varepsilon_{lf})$ is the effective Young mo-dulus, and $\gamma(\varepsilon_{hf} - \varepsilon_{lf})$ is the parameter of nonlinearety. The proposed model (1) takes into account small hysteretic loops on the G(ε) curve for a weak high-frequency wave, with parameters of the loops depending on their position on the G(ε) curve for a pump wave (Fig. 4). For the case under consideration, interaction of pump and signal can be specified through E(ε) and γ(ε) so that for the weak signal ε_{hf} in (1) we will have

$$G(\varepsilon_{hf},\dot{\varepsilon}_{hf}) = \varepsilon_{hf} E_0 (1 + \alpha|\varepsilon_{lf}(x)|) \pm \quad (2)$$

$$\pm\left(\gamma_1 + \gamma_2|\varepsilon_{lf}|^S \left\{\varepsilon_{hf}^2 - |\varepsilon_{hf}|^2\right)\begin{matrix} + \ for\ \dot{\varepsilon} > 0 \\ - \ for\ \dot{\varepsilon} < 0 \end{matrix}\right.$$

where α, γ_1, γ_2, S are parameters that can be determined in experiment; $|\ \varepsilon_{lf}(x)\ |$, $|\ \varepsilon_{hf}(x)\ |$ are distributions of pump and signal amplitudes in the resonator. This equation of state explains qualitatively the dependences observed in experiment: the linear dependence defects of modulus on the amplitude of



low frequency (*lf*) pump $\frac{\partial G}{\partial \varepsilon_{hf}}\left(|\varepsilon_{lf}|\right)$ and the dependence (average over the period of hf wave) of the hysteretic loop area (loss of weak wave) on the amplitude of lf pump. For qualitative explanation of results of the experiment we obtain an expression for a resonant curve of hf signal. We proceed from the equation for longitudinal waves:

$$\rho\frac{\partial^2 u_{hf}}{\partial t^2} = \frac{\partial G(\varepsilon,\dot{\varepsilon})}{\partial x}, \quad (3)$$

where u_{hf} is longitudinal particle displacement, $\varepsilon = \partial u/\partial x$ is the deformation; $G(\varepsilon,\dot{\varepsilon})$ is determined by the relation (2). Equation (3) must be solved under boundary conditions corresponding to the measurement scheme. Thus, for a resonator with one free $x = l$ and one rigid $x = 0$ wall, these conditions have the form

$$u(0,t) = u_0 \cos\omega_h ut \quad \frac{\partial u}{\partial x}(l,t) = 0\ . \quad (3a)$$

The resonator eigenmodes

$u_m \sim \sin K_m x$, where $K_n = \frac{\Pi}{L}\left(m - \frac{1}{2}\right)$.

For the resonator close to the one with two free walls, the boundary conditions are

$$\begin{aligned} \varepsilon(0,t) &= \varepsilon_0 \cos\omega t \\ \varepsilon(l,t) &= 0 \end{aligned}\ . \quad (3b)$$

The resonator eigenmodes $\varepsilon \sim \sin K_m x$, где $K_m = \Pi m/L$.

We now pass over from equations (3), (3a), (3b) to equations with zero boundary conditions through the following substitution:

$$v(x,t) = u(x,t) - u_0 \cos\omega_{hf} t(1 + \sin K_m x)$$

– for boundary conditions of the first type

$$\varepsilon_v(x,t) = \varepsilon(x,t) - \varepsilon_0 \cos\omega_{hf} t\left(1 - \frac{x}{L}\right)$$

– for boundary conditions of the second type.

We seek a solution to Eqs. (3), (4) for new variables in the form close to the eigenmode of a resonator with unknown amplitude and phase[5].Using the perturbation method we can readily obtain a set of equations for amplitudes and phases of oscillations of a weak signal in the steady-state regime. For example, for boundary conditions of the first type we have:

$$\delta_n v_0 + \delta_m v_0 = -\frac{u_0\omega_m \cos\psi}{\pi(2m-1)} \quad (4)$$

$$\mu_n\omega_m^2 v_0 + \mu\omega_m^2 v_0 = -\frac{u_0\omega_m \sin\psi}{\pi(2m-1)}$$

It should be mentioned that the structure of stationary equations doesn't depend on the boundary conditions (only nonlinear coefficients are changing). According to theoretical estimates the nonlinear hysteresis item is non-zero only for odd numbers of modes. Therefore we will write here only expression for the first type boundary conditions:

$\delta_m = \omega_{hf} - \omega_m$, $\delta_n = -\beta_{mp}\alpha|\varepsilon_{lf}|$ –

linear δ_m and nonlinear δ_n resonant frequency shift, m – the number of eigenmodes for the signal, p – the number of eigenmodes for the pump;

$$\beta_{mp} = \pi(2m-1)\omega^m\left(\frac{1}{[illegible]} + \frac{1}{1/2 [illegible]} + \frac{1}{1/2 [illegible]}\right)$$

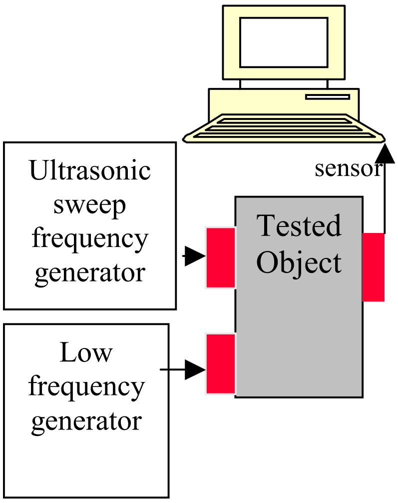


FIGURE 1. A scheme of measurements.

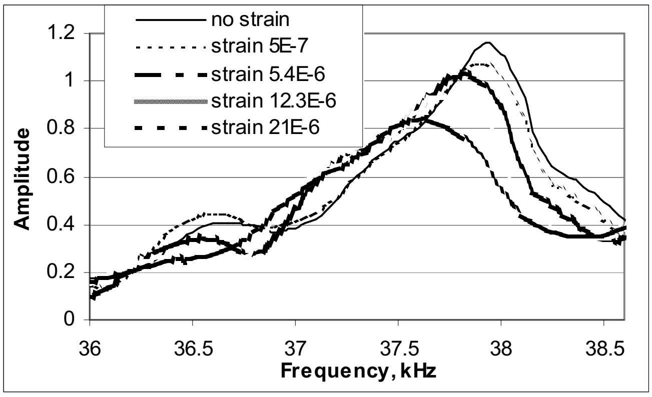


FIGURE 2. Amplitude –frequency response of damaged concrete for different strain in the pump wave.

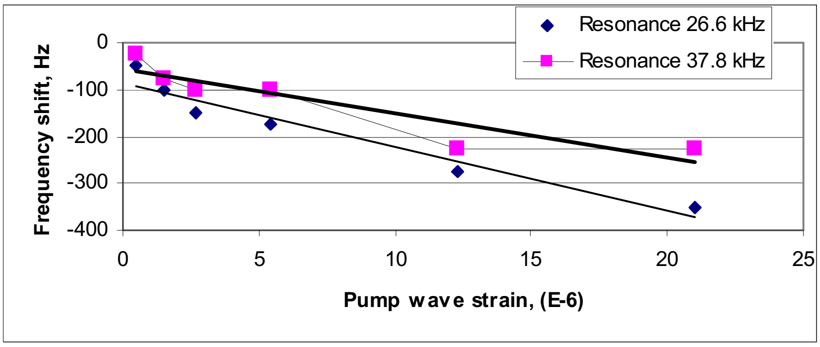


FIGURE 3. Frequency shift dependence on the strain in the pump wave.

$\mu,\ \mu_{mp} = (2m-1)\Pi\gamma_1\frac{3}{4}\left(1+\frac{j\gamma_2}{\gamma_1}\frac{(2m-1)}{(2m+1)}|\varepsilon_{lf}|^2(2p-1)\right)V_0 -$

linear μ and nonlinear μ_{mp} losses.

Then from Eq. (4) we obtain an expression for resonant curve of weake signal at the fundamental frequency

$$v = \frac{A_0 c\ /L}{\sqrt{(\delta_m-\delta_n)^2+(\mu+\mu_n)^2\omega_m^4/4}} \qquad (5)$$

Clearly, parameter α will enter the term proportional to nonlinear resonant frequency shift and determines linear dependence of the shift on pump amplitude $|\varepsilon_{lf}|$.

Parameter S that determines dependence of the nonlinear loss on the amplitude of strong pump should be assumed to be equal to 2. Then, $\Delta Q/Q_0 \sim \mu_n^{-1} \approx 10^{-1}$ will be inversely proportional to pump amplitude $|\varepsilon_{lf}|$. By comparing results of theoretical estimates and data of the experiment one can easily estimate parameters α_1, γ_1 and γ_2. For example, for γ_1 we will have 10^3 with the deformation in the range of 10^{-5}-10^{-7} $\alpha_1 \approx 10^3, \gamma_2 \approx 10^7$

So our supposition of the small loop's position on the larger loop let us explain the amplitude depending losses

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