NONLINEAR ACOUSTICS OF PHASE CONJUGATE WAVES IN HETEROGENEOUS MEDIA (NGA Approach)

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Abstract

Nonlinear geometrical acoustics approach is developed for description of retro-focusing of phase conjugate ultrasound beams in nonlinear inhomogeneous media. Both spatial variations of refractive index and of nonlinear parameter of medium are taken into account. Selective phase conjugation of the second acoustic harmonics of nonlinear focused beam is considered as an example. Evolution of transversal distributions of fundamental and second harmonics of phase conjugate wave during their back propagation to the focus of the incident beam is described. Wave propagation in presence of transversally inhomogeneous layer or of nonlinear inclusions is studied in detail. High quality retro-focusing of phase conjugate wave in spite of partial break of time reversal invariance is obtained for these cases. Imaging of nonlinear inclusion in C-scan mode using selective phase conjugation of acoustic harmonics is discussed.

Introduction

Retro-focusing of ultrasound in the focal area of an incident beam by wave phase conjugation (WPC) or time reversal techniques is a well known phenomenon when propagation mode is linear. Retro-focusing takes place even when the medium introduces strong phase aberrations to the incident wave. Compensation of phase aberrations is one of manifestations of time reversal invariance (TRI) of acoustic field. On the other hand, the TRI is clearly violated in nonlinear mode of propagation when the conjugate wave is amplified relatively to the incident one. Partial break of TRI can take place also because of the limited frequency bandwidth of real conjugators that prevents the total reproduction of the frequency spectrum of the incident nonlinear wave. In such cases the problem of ultrasound retro-focusing becomes non trivial.

Recently it was demonstrated theoretically [1] and experimentally [1-3] that the property of automatic retro-focusing with compensation of phase distortions can be saved also for nonlinear mode of propagation of the phase conjugate waves in heterogeneous non-dispersive media. The effect results from the spatio-temporal phase matching of harmonics generated in cascade by the conjugate waves of finite amplitudes [1]. In the present paper theoretical approach [1] based on nonlinear geometrical acoustics (NGA) approximation is developed for selective narrowband phase conjugation of the second harmonics of nonlinear incident wave. Nonlinear back propagation of the conjugate wave accompanied by harmonic generation in inhomogeneous media is described.

NGA equations of nonlinear retro-focusing of second harmonics

The typical geometry of the retro-focusing problem is shown on fig.1. Focused transducer (T) emits an incident ultrasound wave with frequency ω towards the phase conjugator (C). The incident wave passes through an inhomogeneous area between the focal plane and conjugator.

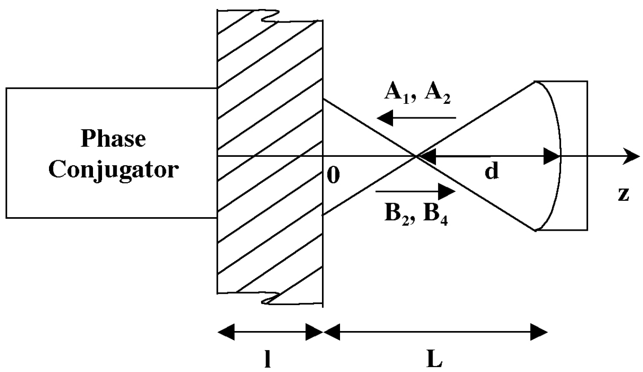

Fig.1 Retro-focusing of harmonics of phase conjugate wave passed through inhomogeneous medium of thickness l (geometry of the problem).

We will assume that amplitude of the incident wave is high enough for harmonic generation due to nonlinearity of medium and the conjugator provides narrowband selection, phase reversal, amplification and reemission of the incident second harmonics only. These conditions correspond to experiments carried out recently in ref.[3-5]. We suppose also that the reemitted phase conjugate wave (PCW) of frequency 2ω is strong enough for observable generation of its second harmonics (4ω) during back propagation towards the transducer. For description of nonlinear propagation of ultrasound in inhomogeneous and non-dispersive media we will use the equation of KZK-type [4]:

$$\frac{\partial}{\partial \tau}\left[\frac{\partial p}{\partial z} - p\frac{\partial}{\partial z}\ln\sqrt{\rho c} + \frac{1}{2c}\phi(\vec{r})\frac{\partial p}{\partial \tau}\right] - \frac{c}{2}\Delta_\perp p = \beta(\vec{r})\frac{\partial^2 p^2}{\partial \tau^2} \quad (1)$$

where p is the acoustic pressure, $\tau = t \pm z/c$ is the local time, $\beta(\vec{r})$ is the nonlinear parameter , $\Phi(\vec{r})$ is perturbation of the refractive index, ρ and c are the density of medium and sound velocity respectively. The amplitudes of the first two harmonics A_n (n=1,2) of the incident (direct) nonlinear wave and B_n (n=2,4) of PCW (backward) waves satisfy the NGA equations [1]:

$$\hat{L}\{A_1\} = 0, \quad \hat{L}\{A_2\} = 2i\beta(\vec{r})\omega A_1^2$$
$$\hat{L}\{B_2\} = 0, \quad \hat{L}\{B_4\} = 4i\beta(\vec{r})\omega B_2^2 \tag{2}$$

where

$$\hat{L} = \frac{\partial}{\partial z} + \nu(\vec{r}) + (\vec{\xi}.\vec{\nabla})$$
$$\nu(\vec{r}) = -\frac{\partial}{\partial z}\ln\sqrt{\rho c} + \frac{c}{2}(\nabla_\perp^2 \psi)$$

and $\vec{\xi} = c\nabla_\perp \psi$ (3)

here the eiconal equation same for all the harmonics in non-dispersive medium is:

$$\frac{\partial \psi}{\partial z} + \frac{1}{2c}\phi(\vec{r}) + \frac{c}{2}(\nabla_\perp \psi)^2 = 0 \tag{4}$$

The boundary conditions for equations (2,4) are:

$$\psi\big|_{z=L} = r_\perp^2 / 2cd\ , \quad A_1\big|_{z=L} = A_{10}\exp\left\{-r_\perp^2/\varepsilon^2\right\}$$
$$A_2\big|_{z=L} = 0 \tag{5}$$
$$B_2\big|_{z=-l} = \Gamma A_2\big|_{z=-l}, \quad B_4\big|_{z=-l} = 0$$

where Γ means gain of phase conjugator. Gaussian approximation for incident focused beam on transducer of aperture radius ε is assumed, d is the focal distance. Solutions of the equations (2) can be found in form:

$$A_2(\vec{r}) = 2i\beta_0\omega Q(\vec{r})A_1^2(\vec{r})$$
$$B_2(\vec{r}) = 2i\beta_0\omega[Q(\vec{r}) - R(\vec{r})]\Gamma A_1^2(\vec{r}) \tag{6}$$
$$B_4(\vec{r}) = 4i\beta_0\omega R(\vec{r})B_2^2(\vec{r})$$

where β_0 corresponds to nonlinear parameter of the medium without inhomogeneous perturbations. The functions Q(**r**) and R(**r**) describe longitudinal evolution and transversal distortions of harmonics of the direct and backward waves and satisfy the equation:

$$\left(\frac{\partial}{\partial z} + (\vec{\xi}.\vec{\nabla}_\perp) - \nu(\vec{r})\right)\begin{Bmatrix} Q \\ R \end{Bmatrix} = \beta_0^{-1}\beta(\vec{r}) \tag{7}$$

The boundary conditions for solutions of equations (7) are: $Q|_{z=L} = 0$, $R|_{z=-l} = 0$. The solutions have also to be continuous on interfaces of heterogeneous medium. As it follows from equations (6) time reversal invariance is evidently broken for second harmonics even for homogeneous media (see also [5]) because of absence of the fundamental harmonics (ω) in the spectrum of PCW. Below we will consider two particular cases: propagation of phase conjugate waves trough transversally inhomogeneous layer and their propagation in presence of a nonlinear inclusion of arbitrary shape.

Retrofocusing through transversally inhomogeneous layer

The main properties of the two first harmonics (2ω and 4ω) of the backward wave can be explained on an example of cylindrically focused incident beams and transversely inhomogeneous layered structure of the medium (Φ(**r**)= α*y and β(**r**) = β(y) if $-l < z < 0$, Φ(**r**)=0 and β(**r**) =β_0 if $0 < z < L$). The distortion functions for this case can be found as:

$$Q(z,y) = 2[(z - L + d) - d^{1/2}\cdot|z + d - L|^{1/2}]$$
$$R(z,y) = 2[(z - L + d) - |L - d|^{1/2}\cdot|z + d - L|^{1/2}] + \tag{8}$$
$$+ \beta_0^{-1}(z - L + d)^{1/2}\int_{-l}^{0} dz'(z' - L + d)^{-1/2}\beta\left(y\frac{z' - L + d}{z - L + d} + \frac{\alpha}{4}f(z', z)\right)$$

where :

$$f(z', z) = \frac{(z - z')[zz' + (z + z')(d - L)]}{(z - L + d)}$$

Analysis of the solutions (7) and (8) show that inhomogeneous refraction itself (when α#0 and β(**r**) = β_0) does not lead to the transversal distortions of the backward beam. This conclusion is analogous to the result obtained previously [1] for phase conjugation of the fundamental harmonics of the incident wave. On the other hand as it follows from (8) inhomogeneity of the nonlinear parameter creates transversal distortions in the output of the inhomogeneous medium. This property of PCW is illustrated by fig.2,3. The results are obtained for the following parameters: α = 1.5 cm^{-1} , β(**y**) = β_0[1+χ*y] , χ = 3 cm^{-1}, l =1 cm, d = 3cm, L = 5 cm, ε = 0.5 cm. Analysis of evolution of the backward wave harmonics show that as approaching the focus distortions of both harmonics decrease and become negligible near the focal area. More over the

second harmonics (4ω) is better concentrated near the focus than the first one (2ω) that is quite typical effect for nonlinear propagation of acoustic beams (see [6]).
Solution of equations (2) for longitudinally layered media ($\Phi(\mathbf{r})= \Phi(z)$ and $\beta(\mathbf{r}) =\beta(z)$) shows that inhomogeneity of this kind does not introduce transversal distortions in harmonics of PCW neither via inhomogeneous refraction no via inhomogeneous nonlinear parameter.

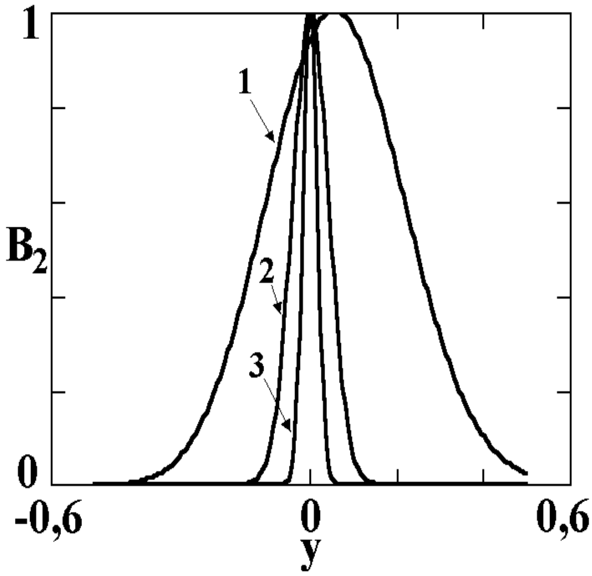

Fig.2 Evolution of the transversal structure of selectively phase conjugate harmonics (2ω) (1- z=0; 2- z= 1.5 cm; 3- z= 1.8 cm).

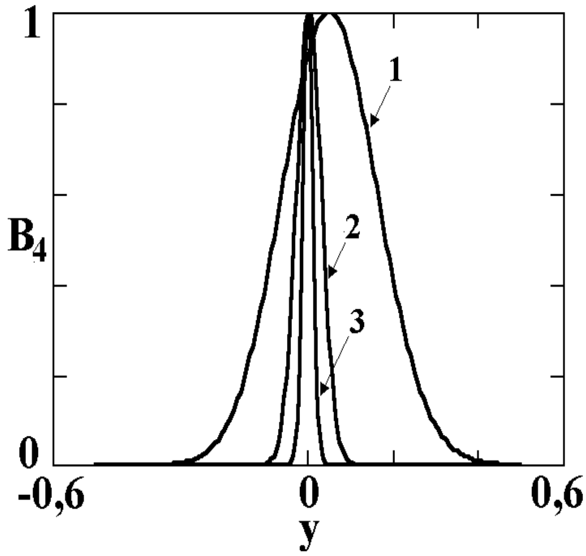

Fig.3 Evolution of the transversal structure of the second harmonics (4ω) generated during nonlinear propagation of PCW (1- z=0; 2- z= 1.5 cm; 3- z= 1.8 cm).

PCW propagation in presence of nonlinear inclusion

In a number of experiments [1-5] inhomogeneities introduced between focal plane and conjugator have the form of layer with arbitrary or random surface. In this chapter we consider propagation of PCW harmonics through an inclusion with negligible perturbation of the refractive index ($\Phi(\mathbf{r}) = 0$) but with essential difference of nonlinear parameter $\beta(\mathbf{r}) = \beta$ from surrounding medium $\beta(\mathbf{r}) = \beta_0$. The surface of inclusion directed to the focus is described by arbitrary function $z_0 = f(y_0)$. The function R(y,z) in this case is equal to:

$$R(z,y)=2(z-L+d)+ \\ |z+d-L|^{1/2}\cdot 2\left[(L-d-z_0)^{1/2}\cdot(1-\beta/\beta_0)+\beta/\beta_0\cdot(L-d+l)^{1/2}\right] \quad (9)$$

where $z_0 = f[y_0(\zeta)]$ and $\zeta= y/(z-L+d)$. The function Q(z,y) is still defined by the equation (8). The amplitudes of PCW harmonics one can calculate by the equation (6). The features of PCW propagation in such case can be illustrated by an example of inclusion of a wedge shape $z_0= \mu\ (y_0 - h)$, were h describes position of the inclusion relatively to the transducer axis y=0. Calculation of amplitudes $B_2(z,y)$ and $B_4(z,y)$ gives the patterns of retrofocusing qualitatively similar to the fig.2,3. Analysis of signals $U_{2,4}(h) \approx \int B_{2,4}(L,y)dy$ received by the transducer show their essential sensitivity to the relative position h of the transducer and insertion. More over the signal $U_4(h)$ corresponding to the second PCW harmonics is much more sensitive to h than $U_2(h)$. This feature is illustrated by the fig.4.

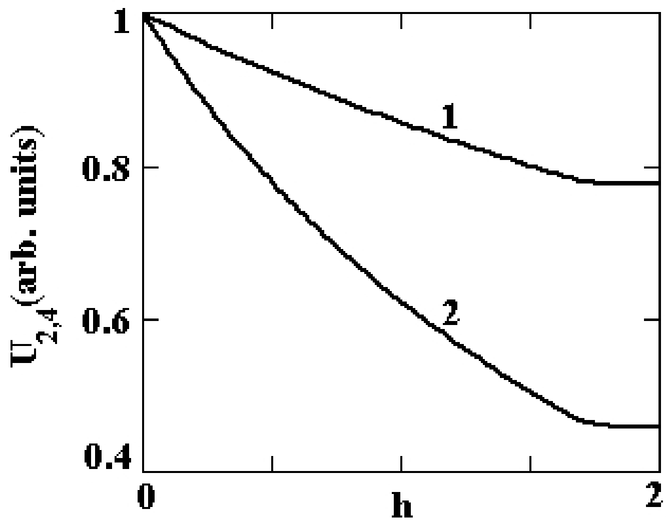

Fig.4 Dependence of the signals induced by PCW fundamental U_2 (1) and second U_4 (2) harmonics on relative position of the transducer and wedge nonlinear insertion.

Calculations are carried out for the following parameters: L=5cm, d=4.5cm, $\beta/\beta_0=3$, $\mu= 3^{-1/2}$ and $\varepsilon = 0.5$ cm. C-scanning of the transducer provides detection of the insertion shape. Analogous result was obtained for unfocused transducer of the same aperture ε. But the dynamic range in this last case was smaller 3 and 15 times for U_2 and U_4 signals respectively because of weaker nonlinearity of waves.

Conclusion

The results obtained in the frame of NGA approximation demonstrate effective retro-focusing of phase conjugate ultrasonic waves in inhomogeneous media under selective narrowband phase conjugation of the second harmonics of nonlinear incident wave. Inhomogeneity of nonlinear parameter of medium introduces visible distortions in transversal distribution of harmonics of back propagating phase conjugate wave near the output of the inhomogeneous medium. Nevertheless the intensity distribution of the conjugate wave is well localized in the focal area of the incident beam and compensation of distortions for harmonics takes place near the focus. This conclusion is in agreement with the recent experiments [3-5]. NGA theory of propagation of PCW harmonics in presence of nonlinear inclusion show also retro-focusing ability of selective phase conjugation. Analysis of signals of PCW harmonics received by emitting transducer in C-scan mode provides detection of the inclusion shape. Sensitivity of detection is essentially higher for the second PCW harmonics then for the fundamental one. The effect is of interest for imaging of nonlinear inclusions when their linear acoustic parameters are close to ones of the surrounding medium.

Acknowledgments

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