NON-LINEAR PROPAGATION FOR MEDICAL IMAGING

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Abstract

The propagation of high amplitude ultrasonic fields, such as those generated by some medical ultrasound systems, is not adequately described by the linear wave equation. Instead it is necessary to consider non-linear propagation if the drive levels are high enough to make non-linear effects significant. As a result of non-linear propagation the transmitted waveforms distort as they propagate, resulting in the generation of harmonics of the initial frequency components transmitted by the transducer. In the nearfield of medical transducers diffraction and focusing effects associated with the source complicate this process.

The basic physics of this non-linear propagation is reviewed and the complex characteristics of the finite amplitude fields generated by different sources are described. This is illustrated with both experimental results and numerical predictions obtained using a finite difference solution (the Bergen Code) to the Khokhlov-Zabolotskaya-Kuznetsov (or KZK) equation. The fields of both ideal sources and real medical systems are demonstrated. The use of harmonics to improve image quality is then considered, with the characteristics of the fields produced by harmonic imaging and pulse inversion systems being compared. The implications of non-linear propagation for medical ultrasound output regulation are also considered.

Introduction

The propagation of ultrasonic waves is often assumed to be a linear process. In this case the waves travel at a constant velocity (c_0) and so maintain their shape as they propagate. However, it is relatively easy to generate high enough pressures in ultrasonic fields for the effects of non-linear propagation to become significant, with the compressional phases of the wave travelling faster than the rarefactions. The resulting wave distortion (see Figure 1) can lead to shock-like waveforms with sudden changes in pressure. This distortion of the wave in the time domain indicates that the waveform now contains additional frequencies; for a single sine wave the propagation results in the generation of harmonics of the initial frequency.

The significance of non-linear propagation in medical ultrasonics was noted by Muir and Carstensen [1] and some of the first measurements of non-linear propagation through tissue were performed by Starritt et al. [2]. Since then a considerable growth in our understanding of non-linear propagation in the field of medical ultrasound has occurred. This has been made possible by a number of factors. Firstly the availability of very wide band hydrophones and high speed digital oscilloscopes has facilitated the accurate acquisition of the distorted waveforms resulting from non-linear propagation. Secondly, the increase in computing power has made it feasible to model the non-linear propagation effects in detail.

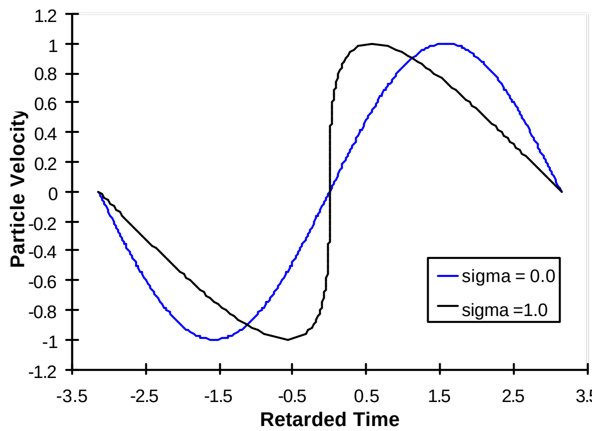

Figure 1. Non-linear propagation of a plane wave. Initial sinusoidal waveform (blue line) and distorted waveform (black thick line) at the point where a shock front has just formed.

A key element that has differentiated this study of non-linear propagation in medical ultrasound is that the non-linear effects occur in the nearfield of transducer beams, where diffraction effects are very important. Although many of these studies have been performed in the context of medical ultrasound systems, the results and modelling techniques may be applied in other areas of ultrasound. Non-linear propagation effects probably occur in other ultrasonic applications, such as non-destructive testing using immersion transducers and the remote sensing of sediment particles in suspension.

This better understanding of non-linear propagation is also finding application in recent developments in ultrasonic imaging, known as tissue harmonic imaging and pulse inversion imaging, which utilise non-linear propagation to improve image quality.

Numerical Modelling

In order to model the non-linear propagation of a wave in the nearfield of an ultrasonic transducer it is necessary to allow for non-linear propagation, diffraction, and attenuation (including dispersion). A number of different numerical approaches to this problem have been developed depending on the nature of the waveform to be propagated and the geometry of the system.

One approach is to solve the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation, which is a non-linear parabolic equation that consistently accounts for non-linearity and diffraction in sound beams. The parabolic or paraxial approximation assumes that the energy propagates in a fairly narrow beam. This approximation is valid for acoustic sources that are many wavelengths across and for field points that are not too far from the beam axis or too near the source plane. For circular sources this requires $ka >> 1$ (where k is the wavenumber and a is the source radius).

If the z axis is in the direction of the beam propagation, and the transducer lies in the (x,y) plane normal to the z axis, then the KZK equation can be written:

$$\frac{\partial^2 p}{\partial z \partial \tau} = \frac{c_0}{2}\nabla_\perp^2 p + \frac{\delta}{2c_0^2}\frac{\partial^3 p}{\partial \tau^3} + \frac{\beta}{2\rho_0 c_0^3}\frac{\partial^2 p^2}{\partial \tau^2}.$$

Here p is the sound pressure, c_0 is the small signal sound speed, δ is the sound diffusivity, β is the non-linearity coefficient, ρ_0 is the ambient density and $\tau = t - z/c_0$ is retarded time. The Lapacian operator in the (x,y) plane, $\nabla_\perp^2$, can be simplified for axisymmetric sound beams.

The KZK equation is normally solved in the frequency domain for periodic signals by using a finite difference scheme to propagate the wave forward in small steps. Essentially the pressure wave is written as a Fourier series consisting of the fundamental and its harmonics. The series must be truncated at the Nth harmonic for numerical reasons. This enables a set of coupled equations to be derived that enable each harmonic at each grid point in the (x, y) plane at $(z+\Delta z)$ to be in written in terms of the harmonic amplitudes on the previous (x, y) plane at z. This system of equations can then be solved by a variety of finite difference schemes. The first scheme for the circularly symmetric case was implemented by Aanonsen et al. [3], and this code and its successors are referred to as the Bergen code. The computational procedure can become very time intensive since for a distorted wave involving N harmonics the calculation of each harmonic at each grid point will involve some N^2 multiplications. In addition truncating the number of harmonics retained in the calculations below that required will eventually lead to errors in even the lowest order harmonics.

The propagation of short pulses, rather than periodic waves, can be implemented using this frequency domain approach. In this case the fundamental frequency is taken to have a period equal to the time interval between pulses; this interval can be set to be a few times the pulse length rather than the true period. Even so, this can drastically increase the run time. An alternative is to use an approach that performs the non-linear propagation step in the time domain. The effect of the parabolic approximation in the KZK equation can be removed by using a 2D spatial fourier transform to obtain the plane wave spectrum of each harmonic; the plane wave spectrum can then be propagated forward without approximation [4].

Non-linear Propagation in Transducer Wavefields

The aim of this section is to illustrate the non-linear propagation of ultrasound in the nearfield of transducers, with examples for circular, focused and rectangular transducers. Experimental results are compared with numerical predictions obtained using finite difference solutions to the KZK equation.

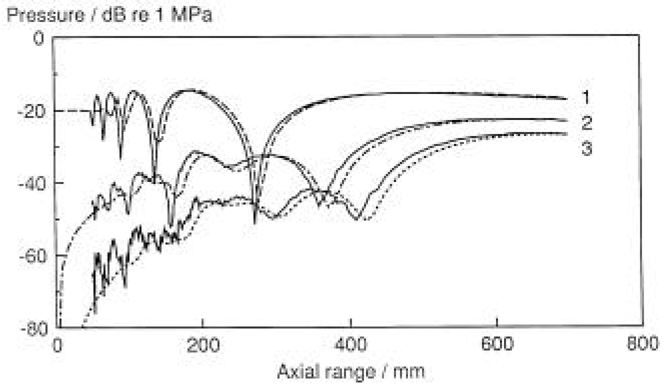

Figure 2. Axial variation of fundamental, second and third harmonics for a plane circular transducer 19 mm in radius. Fundamental frequency 2.25 MHz and source pressure 100 kPa. Experiment (solid lines) and theory (dashed lines).

Figure 2 shows the development of non-linear distortion along the axis of a plane circular transducer [5, 6]. Measurements of the fundamental and first two harmonics are compared with the numerical predictions of the finite difference model. Close to the source only the fundamental is present with no harmonic components. As the range increases the harmonics build up with maxima and minima reflecting those in the fundamental. The results show very good agreement and indicate how the distortion builds up in the vicinity of the last axial maximum at a range of a^2/λ. In the region prior to this the rapid changes in the fundamental phase prevent the build up of significant distortion. The finite difference solution does not show the rapid nearfield oscillations at short ranges as a consequence of the step size used. It is also possible to model the propagation of pulses using Fourier synthesis and the frequency domain code. Figure 3 illustrates the level of agreement that can be obtained in the time domain for an initially sinusoidal short pulse [7].

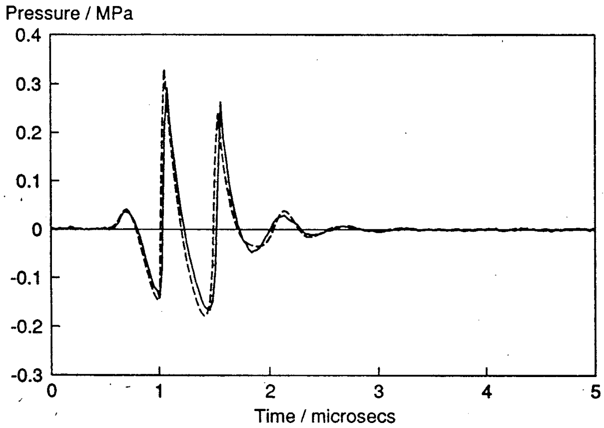

Figure 3. Pressure waveform on axis at 600 mm from a plane circular piston source 19 mm in radius. Experiment (solid lines) and theory (dashed lines).

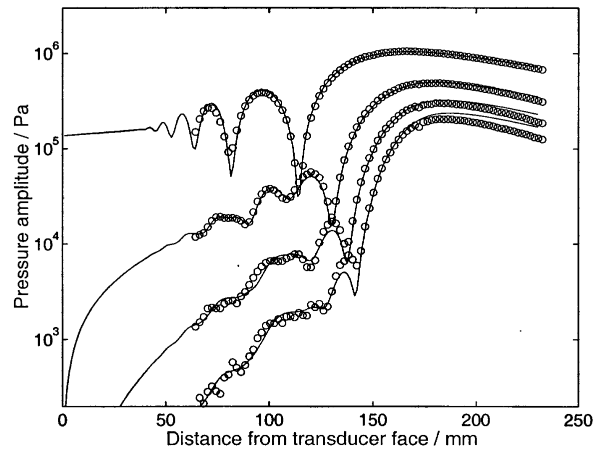

Figure 4. Axial variation of fundamental, second and third harmonics for a focused circular transducer with a gain of 8.5 in water. Fundamental frequency 2.25 MHz and source pressure 135 kPa.

Departures from a plane piston vibration can be accounted for by weighting the initial source amplitude distribution. Likewise focusing can be included by introducing phase shifts to the fundamental component across the face of the transducer in the model [8,9]. Figure 4 gives example results for a system with an amplitude gain of 8.5. The axial variations of the fundamental and first three harmonics in water are shown. Note that the non-linear distortion rapidly builds up in the region between the last axial minimum and the focus; in this case the waveform becomes shocked over a distance of about 50 mm. Studies have also investigated the propagation through attenuating fluids with characteristics more typical of those of body tissues and fluids. (The model can be adapted to include an arbitrary dependence of attenuation on frequency.)

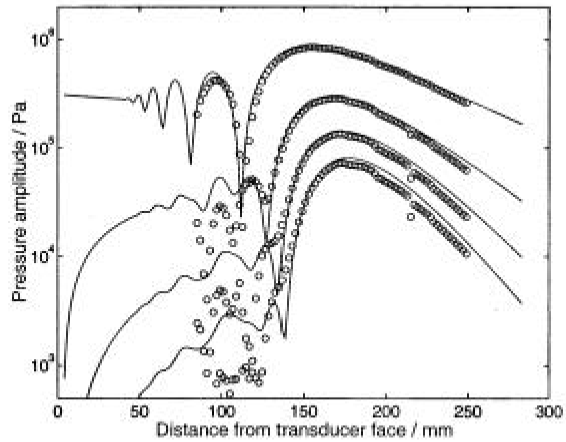

Figure 5. Axial variation of fundamental and first three harmonics for a focused field in a soft tissue mimic. Fundamental frequency 2.25 MHz and source pressure 310 kPa. Experiment (points) and theory (lines).

Figure 5 shows an example result for the propagation of a focused field through a soft tissue mimic which was measured to have an attenuation of 0.3 dB cm^{-1} at 1 MHz and a frequency power law dependence of 1.07. Again it should be noted that the distortion rapidly builds up in the focal region, although the relative amplitude of the harmonics is reduced. This and other experimental/numerical studies indicate that although the very high frequencies may not be generated in tissue, significant generation of lower harmonics can occur.

The finite difference models are not limited to cases with axial symmetry, although full 3-D models do take significantly longer to run. This is illustrated in Figure 6 which shows the field of a square transducer, of side 20 mm, driven at 3.5 MHz. The field is shown in a plane that is defined by the normal to the transducer and the face diagonal. The predicted levels of the fundamental, second harmonic and tenth harmonic are shown. The high level of harmonic build up off-axis before the last axial maiximum should be noted.

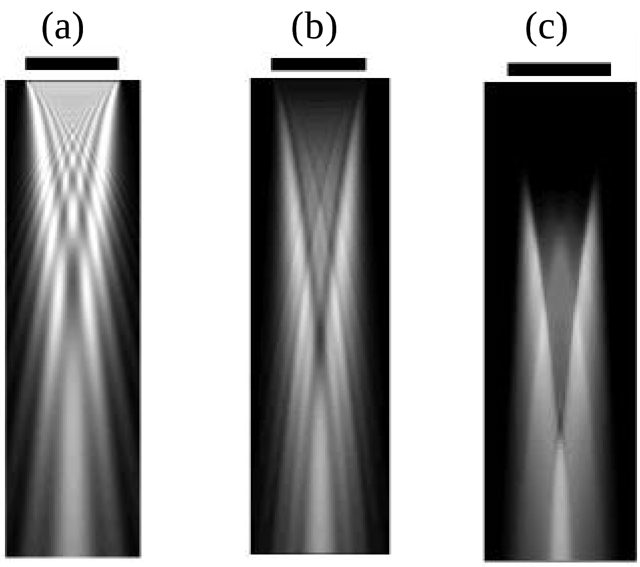

Figure 6. Field component amplitudes in plane through the diagonal of a square transducer (side length = 20 mm). The beam propagates down the page for 200mm. (a) Fundamental (2.5 MHz), (b) second harmonic and (c) tenth harmonic. Model predictions.

Implications of Non-linear Propagation

Non-linear propagation has a number of implications for the field of medical ultrasonics, some of which are of significance to other fields. Firstly the generation of harmonics, which may extend to many times the fundamental frequency, makes the calibration of medical ultrasound systems more difficult. When making calibration measurements in water it is necessary to have a hydrophone (receiver) and receiving system capable of responding to the wide range of harmonic frequencies that are present. The accurate reproduction of waveforms is especially difficult because of the phase response of the hydrophone.

Secondly, the generation of higher harmonics, which are preferentially attenuated because of the increase in attenuation with frequency, can result in an enhanced loss of energy from the beam over that expected on the basis of linear propagation. This enhanced attenuation can lead to saturation, enhanced streaming and enhanced heating. In the last case the temperature rise generated by ultrasonic absorption may exceed that predicted on the basis of linear acoustics.

Thirdly, the non-linear propagation complicates the consideration of safety indices and safety limits for medical ultrasound systems. Reference measurements made in water may be subject to non-linear effects, such as enhanced attenuation, that do not occur to the same extent as in tissue. It should be noted that although the attenuation of tissue can significantly reduce the generation of higher harmonics the non-linear distortion in tissue can still be significant.

Tissue Harmonic Imaging

The non-linear propagation of ultrasound can also be used to advantage in imaging situations. Consider the characteristics of the second harmonic generated by non-linear propagation. It has a narrower beam cross-section than that of the fundamental transmitted by the transducer and lower sidelobe levels (see Figure 7). In principle it is possible to filter out the harmonic generated in the propagation medium from the fundamental transmitted. (In order to ensure that the harmonic is generated in the medium it is necessary to keep the transmitted harmonic to a minimum.) These factors give the use of the second harmonic some potential advantages for imaging.

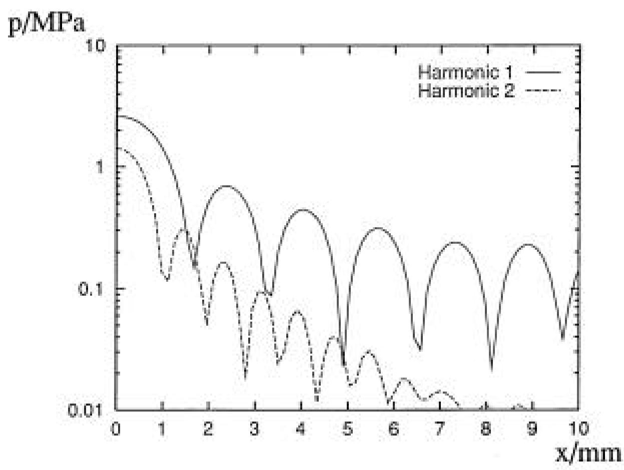

Figure 7. Theoretical normalised beam cross-sections in tissue for a 3.0 MHz array (15 mm by 10 mm), with a single focal length of 50 mm. The calculations were performed for a source pressure of 1.0 MPa and show the cross-section in the focal plane.

The possible use of harmonics for imaging has been suggested by a number of authors including Muir [10] and Bjørnø and Lewin [11], and the generation of harmonics can be used to improve resolution in acoustic microscopy. The possibility of using the technique for medical imaging was demonstrated by Ward et al. [12,13] and is described in references [14] and [15].

Commercial manufacturers initially produced systems with very wide bandwidth transducers, capable of imaging the second harmonic, in order to detect the harmonic generated by contrast agents oscillating non-linearly. It was then observed that these scanners were capable of imaging without a contrast agent present and such harmonic imaging systems are now being produced by a number of manufacturers. They typically use very wideband transducers transmitting, for example, 2 MHz and forming an image using the received energy at 4 MHz. These systems can give significantly improved images, particularly in "hard to image" patients, where they appear to be able to reduce the amount of clutter present in images.

Ward et al. demonstrated the potential of harmonic imaging using a laboratory system consisting of a focused transmitter and receiver, where the receiver consisted of a large area PVdF membrane hydrophone in order to give the required frequency response on reception [12,13]. In this case the system had a fundamental of 2.25 MHz and a focal length of 262mm. The system was used to image an array of nylon lines immersed in water using both the fundamental and water generated second harmonic. The images showed the potential improvement that can be obtained (Figure 8).

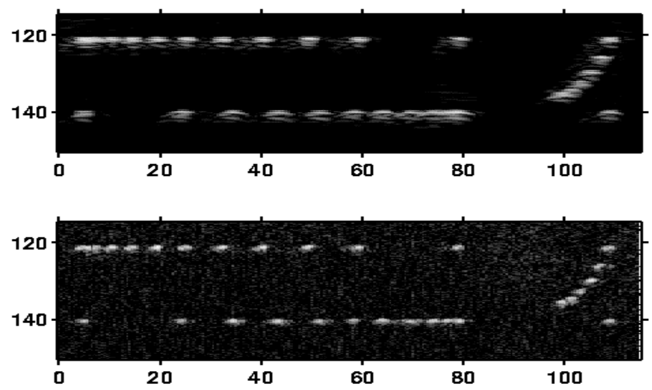

Figure 8. Reflected signal images of an array of nylon lines in water for the fundamental (top) and second harmonic (bottom). Dimensions in mm.

It is interesting to consider the reasons why harmonic imaging gives rise to the improved image quality observed *in vivo*. The reasons may vary, but the following factors are all potentially significant:

Narrower main lobe beam width

The reduced second harmonic beam width can lead to improved lateral resolution. Of course a narrower beam could be obtained by direct transmission of the second harmonic from the transducer. The essential point here is that in an attenuating medium, in which the attenuation increases with frequency, it can be as efficient to generate the second harmonic remotely using non-linear propagation as to transmit it directly.

Reduced side lobe level

The performance of imaging systems is often limited by scattering from structures that are outside the main beam of the transducer. These unwanted signals, known as reverberation, arrive at the same time as the main echo and can not be resolved in time. Consequently they make it harder to distinguish the real structures being imaged. As the amplitude of the reverberation will depend on the transducer's side lobe level, the relative level of the side lobes is of particular significance. Figure 7 clearly shows the reduced sidelobe level that is obtained for the second harmonic using a realistic array geometry.

Although array technology and apodisation can be used to reduce these side lobe levels in the scan plane of diagnostic scanners it is much harder to reduce them in the transverse plane. It is probably the reduction of sidelobe scatter from these out of plane scatterers that is one of the major advantages of harmonic imaging. It is interesting to note that the introduction of 1.5D and 2D arrays, with the potential of apodisation in both planes, may reduce some of the advantages of harmonic imaging.

Reduced aberration due to body wall inhomogenities

When imaging through surface layers inhomogenities in the body wall may reduce the coherence of the transmitted wave and distort the beam generated by the transducer array. In particular this may result in an increase in the sidelobe level. The use of a lower primary frequency with longer wavelength reduces the influence of these inhomogenities, while the subsequent harmonic generation enables the resolution to be maintained. This has been investigated in reference [14].

Reduced reverberation in surface layers

Multiple reflections from the layers within the body wall near to the transducer can give rise to secondary pulses that follow the main transmission. In normal imaging these can not be distinguished and give rise to additional clutter in the image. When using harmonic imaging these lower amplitude trailing pulses do not result in significant harmonic generation and can be simply filtered out, reducing the reverberation present.

Pulse Inversion Harmonic Imaging

One of the potential difficulties with harmonic imaging in practice is the filtering out of the second harmonic components from the fundamental components. This is especially true for the short pulses used in imaging as the transmitted pulse may have a wide frequency spread with significant frequency content at twice the centre frequency.

One approach to this problem is use "pulse inversion imaging". This involves transmitting two pulses along each scan line, with the second pulse an inverted copy of the first. The resultant echoes are then added together. If the propagation were linear the echoes for the second pulse would be the inverse of the first, so the addition would give zero output. In this way the effect of the 'fundamental' transmitted from the transducer can be cancelled out. The non-linear propagation of two pulses will, however, produce second harmonic components that are approximately in phase. Hence the addition of the two echoes will result in the summation of the non-linearly generated second harmonic components.

In this way the non-linearly generated second harmonic can be enhanced relative to the transmitted (fundamental) signals. Of course, this process is complicated by non-linear propagation, especially for short pulses. This can be illustrated by numerical calculations for a model system.

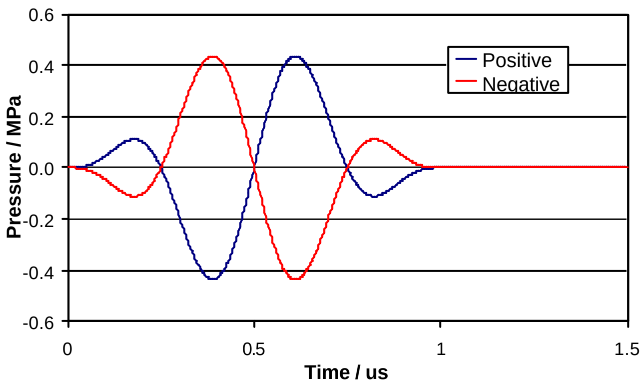

Figure 9. Initial waveforms used for pulse inversion simulation.

Figure 9 shows the two initial short pulses based on a 2.0 MHz sine wave that will be considered. These pulses will be referred to as 'positive' and 'negative'. The transducer is assumed to be square with dimensions of 10 mm x 10 mm with focal lengths of 50 mm in both planes. The resulting distorted waveforms in the focal plane are shown in Figure 10. Quite clearly these are not the exact inverse of each other, showing the complexity of non-linear propagation for short pulses.

In Figure 10 the sum of the two waveforms is also plotted. This can be considered to be the effective pulse that is used in this imaging mode. It can be seen that the fundamental frequency of the sum waveform is basically twice that of the original signal. This is evident from Figure 11, which shows the spectra of the positive, negative and sum waveforms. These show how the second harmonic is enhanced for the sum signal while the primary frequency is reduced. Of course, the non-linear nature of the propagation results in the positive and negative pulse spectra not being the same at low frequencies; hence the cancellation is not complete in this region.

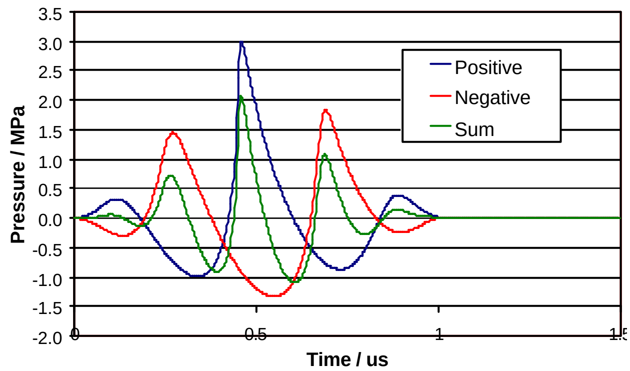

Figure 10. Positive, negative and sum waveforms on axis in the focal plane (50 mm).

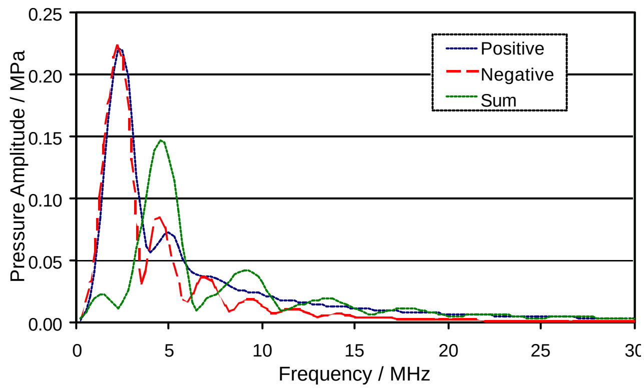

Figure 11. Positive, negative and sum spectra on axis in the focal plane for propagation through water.

The resulting beam cross-sections for the positive pulse and the effective sum pulse are shown in Figure 12. This shows how the narrower beam and lower relative side lobe levels are retained for the second harmonic for the sum signal, while the fundamental signal is reduced by 20 dB in the inversion mode.

This is also illustrated in Figure 13, which shows the amplitude of the effective wavefield components in the (x, z) plane through the acoustic axis. (The transducer is on the left and the acoustic axis along the top of each plot.) In the 2 MHz (fundamental) image for the positive pulse (a) the side lobes are clearly visible while the whole field in significantly reduced for the sum pulse (c). The 4MHz images (b) and (d) show the build up with distance *z* of the harmonic, its narrower beam and significantly lower side lobe levels.

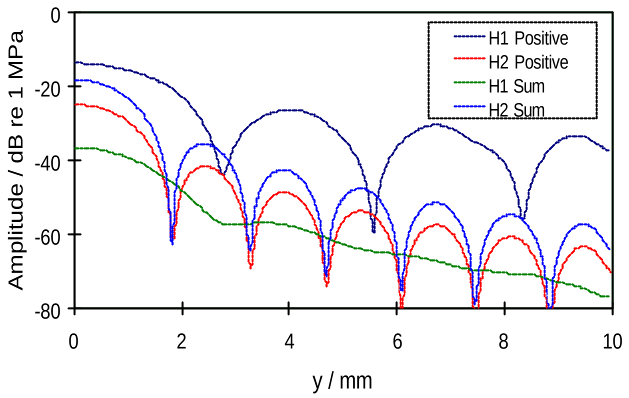

Figure 12. Beam cross-sections in the focal plane for positive pulse at 2.0 MHz (H1 Positive) and 4.0 MHz (H2 Positive), and sum of pulses at 2.0 MHz (H1 Sum) and 4.0 MHz (H2 Sum).

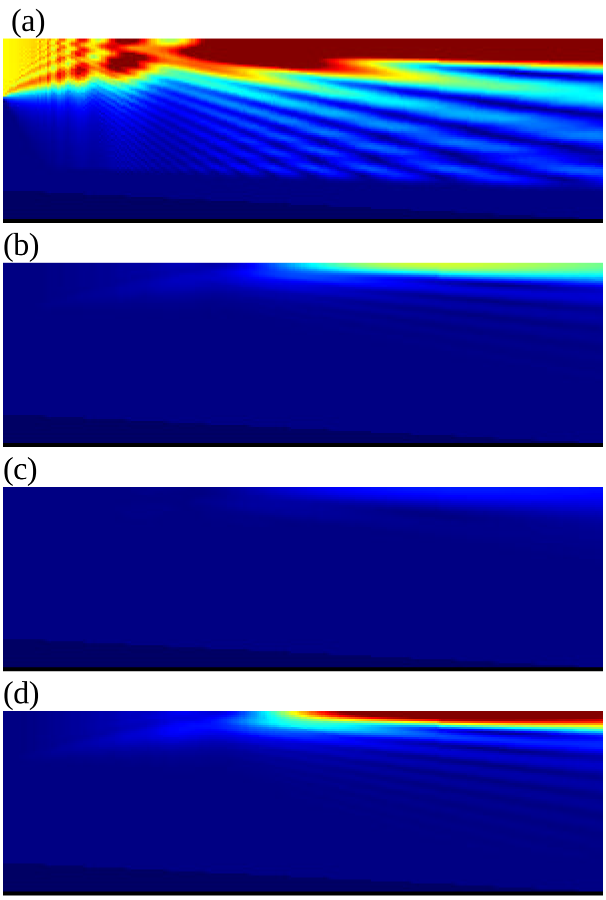

Figure 13. Beam component amplitude plots in the (*y*, *z*) plane; (a) positive pulse 2.0 MHz, (b) positive pulse a 4.0 MHz, (c) sum pulse 2.0 MHz and (d) sum pulse 4.0 MHz.

Conclusions

Our understanding of non-linear propagation in ultrasonic beams has grown considerably in recent years. This has been aided by the development of wideband hydrophones and numerical models; the latter are now able to predict accurately the behaviour of such fields, provided that the initial transducer behaviour and material properties are known sufficiently accurately. There is, however, still a need for more efficient algorithms and for simpler models to be able to predict the effects of non-linearity in a given field. Tissue harmonic imaging has developed to exploit this non-linear propagation and continues to develop with new systems being designed to obtain optimal results. Developments include the use of improved filtering techniques and the use of pulse inversion techniques to improve the rejection of the fundamental component from the signals used to form the image. As such, it is probable that harmonic imaging still has considerable potential for improvement. Non-linear propagation also complicates the measurement of ultrasonic fields and has implications for measurement standards.

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