THE HIGH-FREQUENCY MODELLING OF THE RADIATING NEAR FIELD OF AN ULTRASONIC TRANSDUCER

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Abstract

This paper is devoted to recent developments in mathematical modelling of radiation of high-frequency elastic fields, with applications to the ultrasonic non-destructive evaluation of industrial materials. It concentrates on new semi-analytical models of the radiating near field of ultrasonic transducers that have produced fast and accurate computer code, which have been already partially validated.

Introduction

The phenomenon of radiation of the high-frequency elastic fields are of interest in applied mathematics and a variety of applications, such as the ultrasonic NDE (Non-Destructive Evaluation) of industrial materials. The underlying mathematical model is an initial boundary-value problem based on a set of hyperbolic partial differential equations in particle displacement, known as the elasto-dynamic or Lam`e equations. The first codes for simulating ultrasonic phenomena have been based on various finite-difference or finite-element schemes designed to solve such equations directly or else on numerical algorithms for direct evaluation of their solutions in the integral form. At high frequencies, the latter procedures involve evaluating integrals of rapidly oscillating functions. Both types of the direct codes - even 2D - have long run-times and because of the computer memory required, many 3D versions are still rather impracticable. For these reasons, in recent years more and more effort has been put into various approximate schemes, such as those based on the ray theory and the Gaussian beam approach. In this paper we concentrate on the former.

It has been pointed out in [1] that the success of the approximate codes, which are based on the ray theory is due to the fact that even the broad-band pulses produced by industrial ultrasonic transducers contain mainly high frequencies, and this means that the modern diffraction theory based on high-frequency asymptotics may be employed to refine them. Numerical experimentation shows that in many cases, for the asymptotics to be applicable, frequencies have to be only relatively high. Transducers are extended sources, whose characteristic sizes are at least a few characteristic wave lengths, which is about one millimetre in steel. We follow [1] in employing the so-called two-tier approach to describe their first Fresnel zone, with the evanescent zone excluded (in other words, their radiating near zone, from a couple to about 80 wave lengths away). The far-field approximations can be obtained by using the same approach, but they have been known for some time, and for this reason, lie outside the scope of this paper.

The two-tier asymptotic approach

The two-tier asymptotic approach to evaluating harmonics of the transducer impulse response comprises two steps: i) the face of the source is assumed to be covered by Huygens' point sources and their far field approximation is found; ii) the uniform stationary phase method is applied to evaluate asymptotic contribution to the resulting surface – Rayleigh – integral of a few critical points or lines. The two-tier methodology is quite common in the NDE literature, but unlike with the fully asymptotic approach advocated here, the second tier usually involves numerical integration over the whole surface. Once the harmonics of the transducer impulse response are found it is convolved in the discretised frequency domain with the transducer pressure pulse and the numerical harmonic synthesis is performed to simulate the radiated pulse trains. Note that the actual received signal as measured by the ultrasonic transducer is proportional to the normal component of the velocity field, that is the time derivative of particle displacement rather than the displacement itself. This is the quantity measured by the ultrasonic transducer.

The procedure has been shown to work by using realistic pulses, similar to those emitted by industrial ultrasonic transducers, and comparing the exact and asymptotic codes – see e.g. [2]. The reason for success is the fact that in the realistic pulses most energy is contained in the high-frequency end of the spectrum. Let us describe various elements of the methodology mentioned above in more detail. We start with

The Uniform Stationary Phase Method

Outside the evanescent zone, each harmonic of the radiated field may be represented as an integral containing a fast-oscillating exponent (see Fig. 1) and a slowly-varying amplitude. It is well known that apart from singularities of this amplitude the main contributions to such integrals come from the stationary points of the phase and various types of critical edge points (see e.g. [3] - [5]). In physical terms, the contributions of the isolated critical points

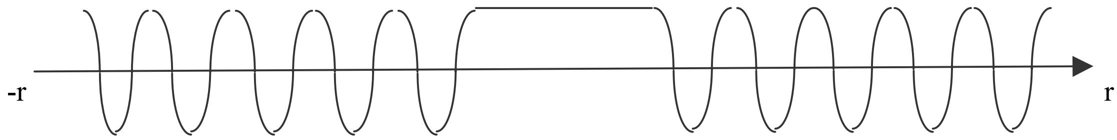

Figure 1: A schematic representation of the real or imaginary part of the fast-oscillating exponent as a function of position.

describe geometrical zones, where we have compressional and shear plane waves as in the GE (Geometrical Elasto-Dynamics), edge waves as in the Keller's GTD (Geometrical Theory of Diffraction - see e.g. [6] , [7]) or else head waves [5]. The coalescing critical points describe various transition zones in-between the geometrical regions. Such transition zones are sometimes called boundary layers [8] , the most well known being penumbrae, which surround the geometrical shadow boundaries and caustic regions, which separate regions reached by different number of diffracted rays. Focal lines are often described as degenerate caustics. There are also boundary layers surrounding the critical head rays [5]. The amplitudes of harmonics propagating in the geometrical zones are described in terms of elementary functions while the standard transition zone asymptotics, in terms of the Fresnel integral in penumbras, Airy Functions near caustics, Bessel functions in focal regions and parabolic cylinder functions in the transition zones surrounding the head critical rays.

Thus, the uniform stationary phase method provides a unifying framework for obtaining description of GE and GTD regions as well as the transition zones in between. It can be applied to describe the fields emitted by point and extended sources and scatterers.

Radiating near field of ultrasonic circular transducer

Many industrial components are made of materials that can be considered isotopic on the millimeter scale. One such material is ferritic steel used e.g. in the UK nuclear industry for construction of the nuclear power pressure vessels and in the UK chemical industry for construction of high pressure reaction vessels. For this reason, quite a few direct numerical models of ultrasonic transducers directly coupled to an isotropic half-space have been studied in the literature (see e.g. [9] – [13].) Approximate schemes based on the ray theory and various simplified treatments of amplitude have been offered in e.g. [14] - [16]. An approximate, Kirchhoff, model of a large compressional piston-type (circular) source directly coupled to the isotopic half-space has been offered in [1] (see also [2]). It has been demonstrated to be tens of thousands times faster than any direct scheme but just as accurate. The GE contributions to the Rayleigh integral over the transducer surface come from the so-called specular points on the transducer surrface, while the GTD waves arrive from the neighborhoods of the flash points on the edge (see Fig. 2). Thus, the radiating near field of the ultrasonic transducer may be described in the following terms: In the

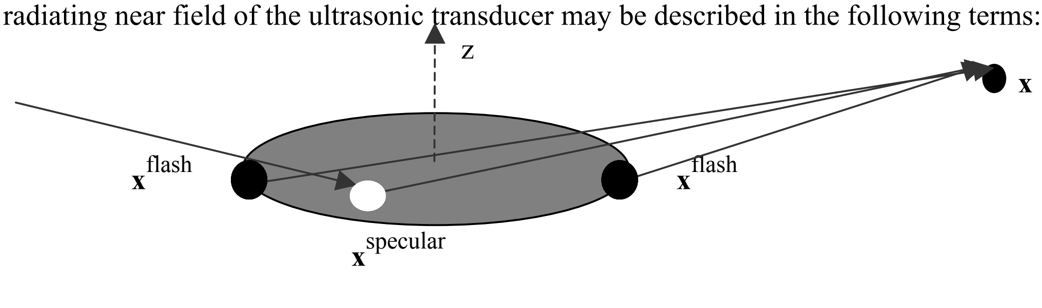

Figure 2: A typical observation point $\mathbf{x}$ and specular and flash points $\mathbf{x}^{specular}$ and $\mathbf{x}^{flash}$ on a circular transducer.

geometrical zones, we have a superposition of contributions of isolated critical points, that is, the main P and S beams and P and S edge waves radiated by the edge of the transducer as well as head waves (see Fig. 3).

(a) (b)

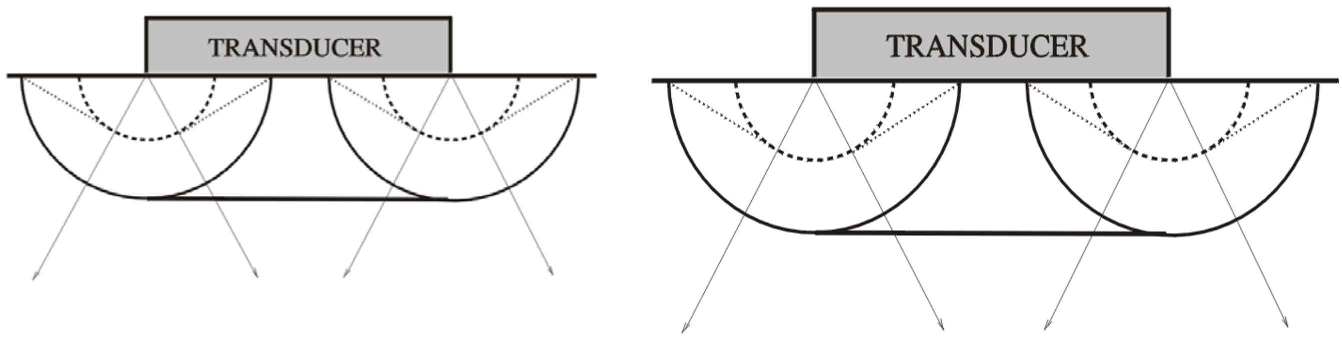

Figure 3: The wave fronts in geometrical zones (a) and transition zones (b) underneath a circular transducer acting normally to the surface of a half-space. Solid line - P fronts, dashed line - S front. Light-shaded area - penumbra (Fresnel's function), dark-shaded area - the axial zone (Bessel's functions). The arrows indicate the wave polarization. The conical head wave front is not indicated.

The head waves and waves in the in the vicinity of the critical head rays are relatively small and can be neglected, but for completeness, the uniform stationary phase method has been applied in [17] to evaluate the corresponding high-frequency asymptotics. So far, apart from the uniform circular transducers, the approach has been applied to rectangular transducers [18] as well as transducers of various apodizations [19] and [20]. By the same token, an alternative fast and accurate asymptotic method has been proposed for simulating the transient field radiated by a circular normal transducer directly coupled to a homogeneous and isotropic elastic half-space based on the wave-front expansions of the impulse response (rather than the high-frequency asymptotics of its harmonics), and then its numerical time convolution with the transducer pressure pulse [21] (see Fig. 4, where the model parameters are chosen to approximate realistic conditions, namely the wave speeds are $c_P = 5840$ m/s and $c_S = 3170$ m/s; the solid density $\rho = 7770$ kg/m^3; and the pressure amplitude $P_0 = 1$ MPa.) For simplicity of presentation, the pressure input is assumed to be a narrow band pulse, one cycle of *sin* $(2\pi f t)$ with $f = 5$ MHz.) The uniform rectangular transducers produce a much simpler response in the near field: The biggest contribution is the plane wave. The pulse trains produced by the circular transducers are more complicated due to focussing of both P and S edge waves on the transducer axis.

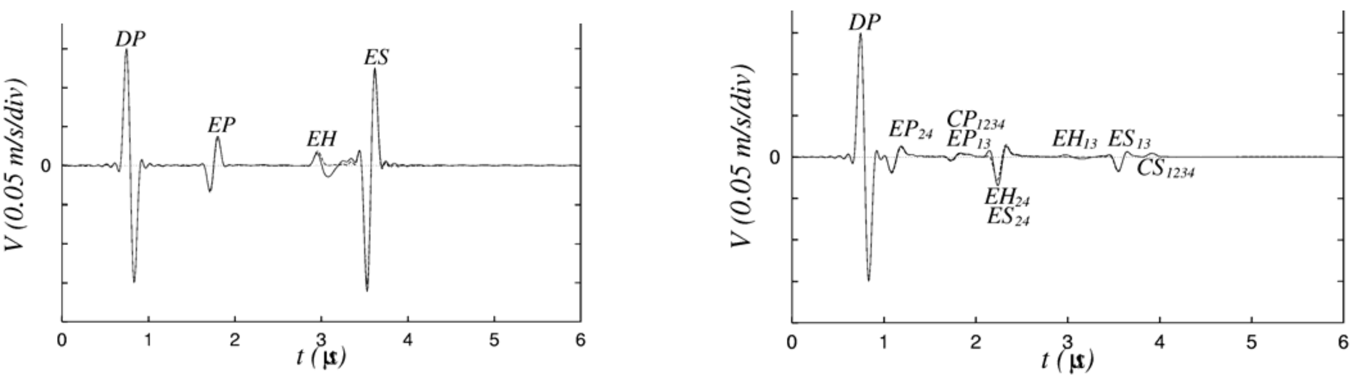

Figure 4: Typical waveforms of the particle velocity at the $\mathbf{e}_3$ - axis of a (a) circular and (b) rectangular transducer. Solid and dashed lines are the exact and asymptotic solutions respectively. DP - the direct P wave, EP, ES or EH - the edge P , S or head wave from the point E_i and CP_i, CS_i or CH_i - the corner P, S or head wave from the point C_i.

Conclusions

We have developed codes based on high-frequency asymptotics to model the near radiating fields of normal ultrasonic transducers directly coupled to isotropic or transversely isotropic component. Similar codes are being developed for angled beam fluid-coupled transducers (all we need are the corresponding boundary conditions, then we can find the corresponding Lamb's Green's function and apply the two-tier asymptotics). The asymptotic codes have been tested against direct codes: They are tens of thousands times faster and within their region of applicability, are practically just as accurate. The speed is partly due to the fact that only contributions of a few specular and flash points are taken into account. The region of applicability is wider than that of many other approximate schemes or even direct codes. The approach elucidates the physics of the problem and gives the explicit dependence on model parameters. It would be possible to develop the asymptotic code further to simulate propagation through curved and multiple interfaces as well as materials with continuously varying properties. The next step is the full radiation – scatter – reception model in the radiating near field.

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