# HYSTERESIS BEHAVIOUR OF MECHANIC AND ACOUSTIC PROPERTIES AT THE FATIGUE TORSION DEFORMATION

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# Abstract

Mechanic and acoustic hysteresis of metal rods under alternating torsion deformation are investigated. It is shown that in plastic region the static deformation hysteresis and the sound velocity one are well described by elastic hysteron model worked out in [1,2] if to take into account the signs of hysterons and to extend the Preisach-Mayergoytz diagram into negative region of loading.

#### Introduction

Acoustic methods are often used in research of materials undergoing by fatigue load up to strength limit. Deformation process in plastic range is accompanied by change of material defect structure. Dislocations density increase leads to the change of sound velocity. But due to a wide range of relaxation times of defects the behavior of static and dynamic elastic properties is essentially different.

At the cyclic loading the hysteresis of static and dynamic elasticity are observed; under repeated (fatigue) loading the defects are accumulated up to destruction of material.

Torsion deformation gives the possibility to obtain the large rotation angles of opposite signs at the relatively small load and does not accompanied by change of sample dimensions. Therefore the variation of sound velocity is determined only by the accumulation of internal defects.

#### **Experimental arrangement**

Block-diagram of the experimental arrangement is represented on Figure 1. Bronze or aluminium samples had a form of spool, so that the deformation took place only in thin part free of clips. The length of samples was 7 cm, the length and the diameter of the narrow part were 4 cm and 5 mm respectively. A longitudinal wave (f = 10 MHz) propagated along the sample axis. Standard pulse method was used; the relative sound velocity was measured up to  $10^{-4}$ . The samples were subjected to alternating torsion deformation with  $\varphi_{max} \sim 0.1 \varphi_{destruct}$ . Number of load cycles before destruction was 10 - 100.

Mechanical and acoustical hystereses were measured simultaneously. The first one is the dependence  $M(\varphi)$  where M is torque,  $\varphi$  is rotation angle. The second one is  $\Delta v(\varphi)/v$  or  $\Delta v(M)/v$  where v is sound velosity.



Figure1. Block-diagram of experimental arrangement

#### Model

For account of hysteresis loop it is possible to utilize the model of elastic hysterons, offered in [1,2] for longitudinal strain in rocks. As to torsion deformation the next variant of this model is proposed.

Regardless to a physical nature it is considered that such "torsion" hysterons have a rectangular hysteresis loop. The hysteron is opened at the torque  $M \ge M_{o}$ , and their rotation angle becomes to be equal  $\Delta \varphi$ ; at the inverse path it is closed at the pressure  $M_c$ , and its angle becomes equal to zero. It is assumed that all hysterons have the same angle and the common rotation in plastic area is proportional to the sum of the open hysterons, which depends on a previous history of loading and hysteron statistics. The Priesach-Mayergoytz (P-M) diagram in coordinates  $M_o$  and  $M_c$  is used for calculation of open hysterons number. On this diagram the density of hysterons with different  $M_o$  and  $M_c$  is represented.

For torsion deformation the model offered in [1,2]is added to some further assumptions.

P-M diagram is expanded into negative region of  $M_c$  because some hysterons are closed only under torque of opposite sign. Thus the residual strain and fatigue effect may be taken into account.

We introduced also two hysterons systems of opposite signs corresponding to different direction of

rotation. The complete rotation in plastic area is proportional to the algebraic sum of open hysterons (without the account of elastic deformation).

The shape of a hysteresis is determined by a statistics of hysterons. From the form of loading curves it is clear that hysterons density increases with load, and P-M diagram has the form represented on Figure 2.



Figure 2. P-M diagram for torque deformation

Density distribution depends on two parameters: on index  $\alpha$  of exponent for a hysterons density on a diagonal P-M diagram ( $\rho_i \sim \exp(\alpha i)$ ) (i = 1,2,...) and dispersion constant *D*, provided that the hysterons density in columns P-M diagram varies as  $\rho_{im} = \rho_i \exp(-m^2/2D)$  (m = 0,1,... is counted from a diagonal). The same statistics takes place for negative hysterons.

On Fig. 3,4 the loops of mechanical hysteresis  $M(\phi)$  for different  $\alpha$  and D are shown. The value *M* is given in relative units, the value  $\varphi$  is proportional to  $N_+ - N_-$ , which are calculated as the sum of density in all open sells. When torque increases from 0 to  $M_{\text{max}} = n$  all hysterons in *n* columns are opened. When unloading is produced the hysterons with  $|M_{\rm c}| < 0$  remain open (residual strain); when the loading changes sign the negative hysterons are opened and positive ones are closed. After each cycle the equal number of positive and negative "fatigue" hysterons remains; for them  $|M_c| > |M_{max}|$ . Thus the loop of the mechanical hysteresis is closed and in plastic area does not depend on the cycle number if the load is symmetrical and has the constant amplitude. Really, the form of experimentally observed mechanic hystereses does not change up to destruction.



Figure 3. Mechanical hysteresis ( $\alpha = \text{const}$ )



Figure 4. Mechanical hysteresis (D = const)

As to a sound velocity, it depends only on the number of defects at does not depend on these sign due to symmetry of problem. Therefore it is possible to assume, that relative change of sound velocity is proportional to the arithmetical sum of open hysterons and  $\Delta v/v \sim -(N_+ + N_-)$ . On Figures 5-8 acoustical hysteresis  $(\Delta v/v(M))$  and  $(\Delta v/v(\varphi))$  respectively for the parameters are represented. Acoustical same hysteresis may have loops but it is unclosed due to fatigue hysterons. If this effect is small the acoustical hysteresis has a form of 'butterfly'. At each cycle in considered model the total number of open hysterons increases linearly with number of a cycle. Acoustical loops are unclosed and the sound velocity in each phase of cycle decreases linearly with the cycle number due to fatigue defects. That effect is confirmed experimentally (Figure 9).



Figure 5. Acoustical hysteresis  $\Delta v(\varphi)/v$ ( $\alpha = \text{const}$ )



Figure 7. Acoustical hysteresis  $\Delta v(\varphi)/v$ (D = const)

In comparison of theoretical sound velocity loops and experimental ones it is necessary to note that the experiment gives the cross section averaged value. Besides, near the destruction the large cracks change the wave-guide form and influence on the form of acoustical hysteresis loop.

On Figures 10-13 the experimental results are represented. The destruction of samples happen for  $\varphi_{max} \sim 800^{\circ}$  (unidirectional deformation) and  $\varphi_{max} \sim 200^{\circ}$ , n = 150 (alternating deformation).

On Figure 10, 11 mechanical and acoustical hystereses in bronze sample are shown;  $\phi_{max} = 200^{\circ}$ . The solid lines are the results of calculation for  $\alpha = 1.2$ , D = 160.



Figure 6. Acoustical hysteresis  $\Delta v(M)/v$ ( $\alpha = \text{const}$ )



Figure 8. Acoustical hysteresis  $\Delta v(M)/v$ (D = const)



Figure.9. Linear dependence of sound velocity on cycles number at the points of maximal load

On Figure 12, 13 acoustical hysteresis loops in aluminium are shown when the maximal load increases with the number of cycle. The experimental curve coincides qualitatively with theoretical one for  $M_{\text{max}}$ = 3, 5, 7 ( $\alpha$  = 0.8, D = 20).



Figure 10. Mechanical hysteresis in bronze sample



Figure 11. Acoustical hysteresis in aluminium sample

## References

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Figure 12 . Acoustic hysteresis with increasing load  $(\alpha=0.8,\,D=20)$ 



Figure 13 . Acoustic hysteresis with increasing load In aluminium