ULTRASONIC IMAGING FROM CORRELATIONS OF DIFFUSE FIELDS

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Abstract

We present an imaging technique based on correlations of a randomly scattered ultrasonic wavefield. Usually the Green's function h_{AB} between two points (A,B) is determined by direct pulse/echo measurement. When this is impossible, one can exploit an other idea: if A and B are both passive sensors, h_{AB} can be retrieved from the cross-correlation of the fields received in A and B, the wave field being generated either by deterministic sources or by random noise. Here a randomly scattered field is generated by sending a 3-MHz pulse through a highly scattering slab. Behind the slab is the medium to image: it consists of four liquid layers with different sound speeds. The cross-correlation of the field received on passive sensors located within the medium is used to estimate the speed of sound. Our experimental results show that the sound speed profile can be precisely imaged. Applications to seismology are discussed.

Introduction

In most applications of wave physics (imaging, detection, communication), it is essential to know the Green's function of the medium under investigation. When possible, the Green's function (or impulse response) h_{AB} between two points A and B is determined by a direct pulse/echo measurement. Recent results[1-3] exploited an other idea: when A and B are both passive sensors, h_{AB} can be recovered from the cross-correlation of the fields received in A and B, the wave field being generated either by deterministic sources or by random noise. Mathematical demonstrations were given, based on a discrete modal expansion with random coefficients in the case of a closed reverberant medium[2,3]. We propose a simple physical interpretation of the emergence of the Green's function in the correlations, based on reciprocity and time-reversal symmetry which does not require a modal expansion of the field. We also present experimental results with ultrasound as an example of "small-scale seismology" which show that it is possible to do "passive imaging" from the spatial correlations of the multiply scattered field received on passive sensors through a highly scattering medium.

Physical argument

Why should the direct Green's function h_{AB} suddenly emerge from spatial correlations of fields received in A and B? In order to give a physical interpretation of that phenomenon, let us consider two receiving points A and B and a source C. We will note $h_{IJ}(t)$ the wave field sensed in I when a Dirac $\delta(t)$ is sent by J. If *e(t)* is the excitation function in C, then the wave fields ϕ_A and ϕ_B received in A and B will be $e(t)\otimes h_{AC}(t)$ and $e(t)\otimes h_{BC}(t)$, $\otimes$ representing convolution. The cross-correlation C_{AB} of the fields received in A and B is then

$$C_{AB}(t) = \int \phi_A(t+\theta)\phi_B(\theta)d\theta$$
$$= h_{AC}(-t)\otimes h_{BC}(t)\otimes f(t)$$

with $f(t) = e(t)\otimes e(-t)$. A simple physical argument based on time-reversal (TR) symmetry indicates that the direct Green's function h_{AB} may be entirely retrieved from C_{AB}.

As long as the medium does not move, the propagation is reciprocal i.e. $h_{IJ}(t)=h_{JI}(t)$. So when we cross-correlate the impulse responses received in A and B, the result $C_{AB}(t)$ is also equal to $h_{CA}(-t)\otimes h_{BC}(t)$. Now, imagine that we do a fictitious TR experiment: A sends a pulse, C records the impulse response $h_{CA}(t)$, time-reverses it and sends it back; the resulting wave field observed in B would then be $h_{CA}(-t)\otimes h_{BC}(t)$ which, because of reciprocity, is exactly the cross-correlation $C_{AB}(t)$ of the impulse responses received in A and B when C sends a pulse. So the result C_{AB} of the "real" experiment (fire in C, cross-correlate in A and B) is the same as the result of an imaginary experiment (fire in A, time-reverse in C and observe in B).

We would like the Green's function h_{AB} to appear in this cross-correlation. But in the most general case, C_{AB} has no reason at all to be equal to h_{AB}! Yet we can go beyond: imagine now that we use several points C, and that we place them in such a way that they form a *perfect* TR-device: such would be the case if the sources C were continuously distributed on a surface surrounding A, B and the heterogeneities of the medium (free of absorption). Then a TR operation would be perfect. During the "forward" step, at time t=0 A sends a pulse that propagates everywhere in the medium (including in B where the field received is

$h_{AB}(t)$), may be scattered many times and is eventually recorded on every point C, with no loss of energy. After the TR, the wave should exactly go backwards: it should hit B first and refocus on A at time t=0 [4], which implies that the field received in B (at times t<0) is exactly $h_{AB}(-t)$, the time-reversed version of the Green's function. Once the pulse has refocused on A, it does not stop but diverges again from A and gives rise, at times t>0, to $h_{AB}(t)$ in B. Thus the exact impulse response $h_{AB}(t)$ can be recovered from either the causal (t>0) or the anti-causal part (t<0) of the sum of field-field correlations C_{AB} (the causality issue is discussed in more details in ref. [5]).

But this requires that the sources C are placed so that they would form a perfect TR device, which has been confirmed by numerical simulations[6]. But in real life, whatever the type of waves involved, this condition is hard to meet, mostly because the number of sources is limited. In seismology for instance the displacement field at the earth surface is recorded by seismic stations (A, B) but the sources (C) of the earthquakes are far from being arranged as a perfect TR device, they are mostly aligned along faults. Yet Campillo and Paul recently showed [1] that the elastic Green's function can be at least partially recovered using correlations of the late seismic "codas" produced by distant earthquakes. The waveforms of these coda show strong multiple scattering due to the heterogeneities within the earth's crust. We have designed a laboratory experiment (Figure 1) to test the feasibility of imaging from correlations of coda waves.

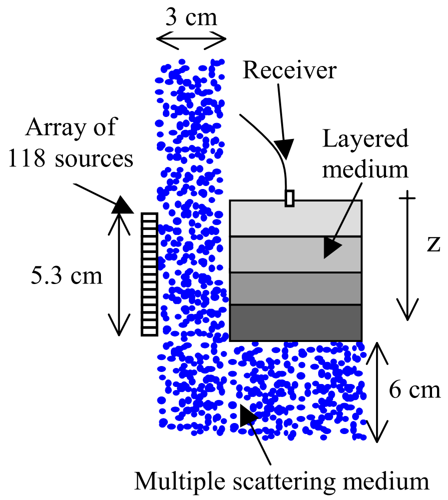


Figure 1 : Experimental set-up.

Experiments

The experiment takes place in a water tank. We use a 118-element array to create 118 "earthquakes" : each time, one of the elements sends a short pulse (3 MHz center frequency) that traverses a highly scattering medium. This scattering medium consists of a random arrangement of steel rods (average density 29.5 rods/cm^2). The sample's mean free-path is 3 mm, whereas its smallest dimension is 30 mm, therefore the waves undergo many scatterings before they can get out of the scattering slab and reach the receiver.

Behind the slab, we place the medium that we want to image: it consists of four liquid layers (alcohol,oil, water, sugar syrup) with different sound speed. A piezoelectric transducer is translated downwards along the z-axis, and records the scattered signals ("seismograms") that are generated each time one of the elements on the array fires a pulse. One of these signals is represented on Fig. 2a. It lasts more than 300 µs, i.e. 300 times the initial pulse and shows a high degree of multiple scattering, similarly to the "coda" of real seismograms.

The 118 "earthquakes" are generated and recorded for each position z of the receiver. The scattered waveforms are $h_N(z,t)$, with N the index of the earthquake, z the position of the receiver and t denotes time. Next, we choose a pair of receiver positions (z_1 z_2) and we cross-correlate the seismograms due to one of the earthquakes :

$$C_N(z_1,z_2,t)= h_N(z_1,-t)\otimes h_N(z_2,t)$$

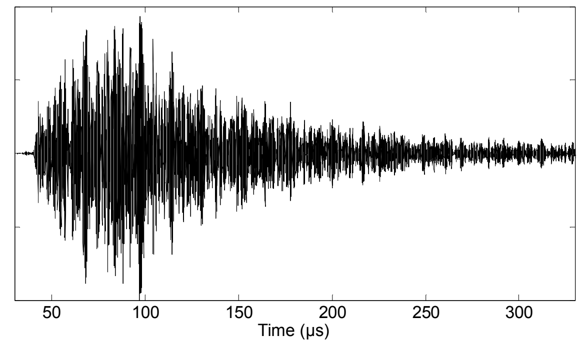
A typical result is shown on Fig. 2(b), for N=60, z_1=36 mm, z_2=16 mm. Nothing seems to emerge from the correlation.

Then we repeat this for the 118 "earthquakes" and we average the result :

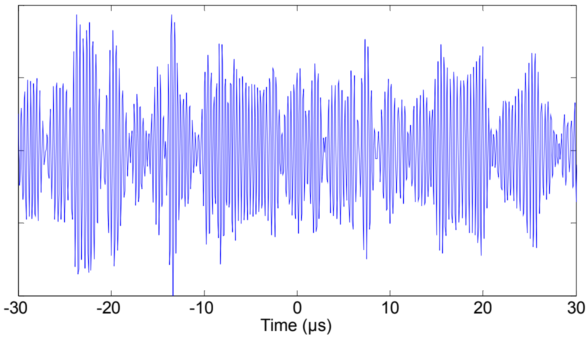
$$C(z_1,z_2,t)=\sum_{N=1}^{118} C_N\left(z_1,z_2,t\right)$$

A typical result is plotted on Fig. 2(c). This time, we see that a strong peak emerges from the correlation, at time t=-13.6 µs. This is the signature of the direct Green's function between z_1 and z_2 ! Indeed, here the expected Green's function between z_1 and z_2 is a well-defined pulse arriving at time $|z_1-z_2|/c$, followed later by lower reflections on the rods. The experimental results show that the main feature of the Green's function emerges from the correlation if the numbers of sources ("earthquakes") is large enough, so that the "time-reversal criterion" can be partially fulfilled. In this example, the distance between the receivers is 20 mm, so from the arrival time t=-13.6 µs we get an estimation of the sound velocity between the passive receivers z_1 and z_2 : 1.47 mm/µs.

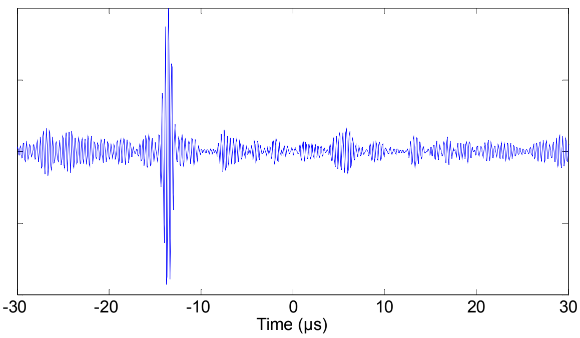
Even though the sources of the "earthquakes" were not arranged as a perfect time-reversal device, we were able to retrieve at least the main feature of the direct Green's function. This is due to multiple scattering in the random sample. It is now well known that time-reversal "works better" (meaning that time-reversal focusing is more efficient) in the presence of strong scattering or reverberation [7]. Therefore, a finite aperture time-reversal mirror is closer to perfection in the presence of scattering or reverberations.



(a)



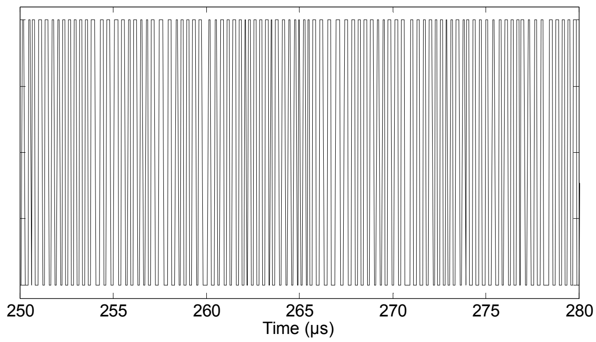
(b)



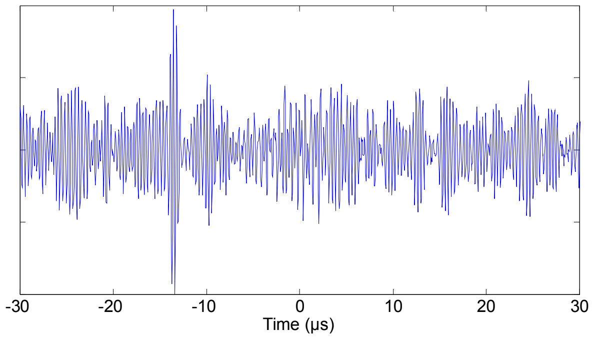
(c)

Figure 2 : Typical experimental results. (a) Waveform $h_{60}(z_1$=36 mm, t) received by the passive sensor at depth z_1=36 mm when source #60 fires a pulse. (b) Cross-correlation $C_{N=60}(z_1$=36 mm, z_2=16 mm, t). (c) Cross-correlation $C(z_1$=36 mm, z_2=16 mm,t) averaged over the 118 sources.

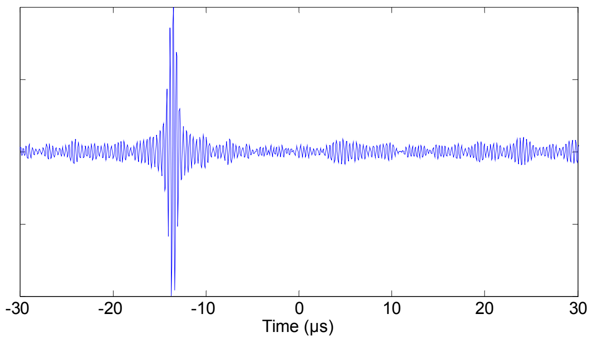
Now, we can repeat the procedure for every pair of neighboring observation points (z_1,z_2), and estimate the velocity profile of the layered medium. We have done so all along the vertical axis, with a step of 2 mm. We have obtained a velocity profile which constitutes an image of the layered medium. (Fig. 4). Note that, at room temperature, it is difficult to distinguish between the water layer and the oil layer. Interestingly, if we repeat the experiment after heating the sample by 8°C, we see that the measured velocity of the water layer increases whereas that of the oil layer decreases, which is consistent with what is known from these two liquids, and the two layers are better separated on the profile (Fig. 4).



(a)



(b)



(c)

Figure 3 : Same as Figure 2, except that only the sign (+1/-1) of the scattered wave forms have been recorded and cross-correlated.

It should be noted that this technique also works with *one-bit correlations*: instead of recording the entire waveforms $h_N(z,t)$, we only record and cross-correlate their sign, i.e. a series of +1/-1 as shown on Figure 3. One-bit correlations seem to give even better results than "normal" correlations (see Figure 2b and 3b for a comparison), because they tend to give more importance to the longest scattering path which are most efficient in a time-reversal experiment. One bit correlations and time-reversal are discussed in more details in [8].

The same experimental procedure was also applied to a two-layer medium (oil/sugar syrup). Initially the liquids are at rest, and the velocity profile clearly shows the two layers (Fig. 5). Then the medium is scrambled to form an emulsion : the velocity profile

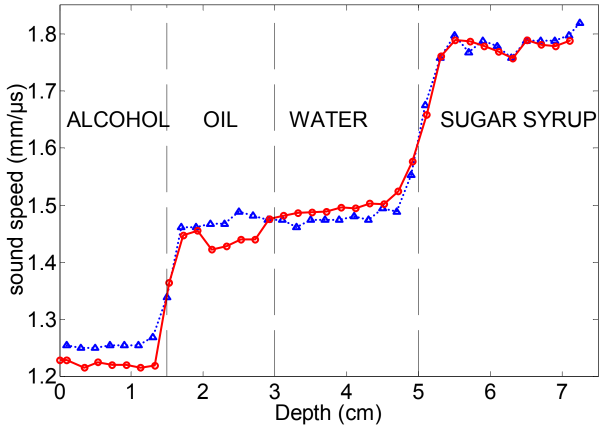


Figure 4 : effect of a temperature change on the image of the medium. Sound speed profile before (blue triangles) and after (red circles) heating up the sample by 8° C.

we obtain shows an apparently homogeneous medium with a sound speed of 1.57 mm/µs. The experiment is repeated on the same sample while the two liquids progressively separate one from the other. After 12 hours the separation is complete. Thus, the images we obtained from correlation of the scattered field were able to monitor the evolution of a medium undergoing a slow structural change.

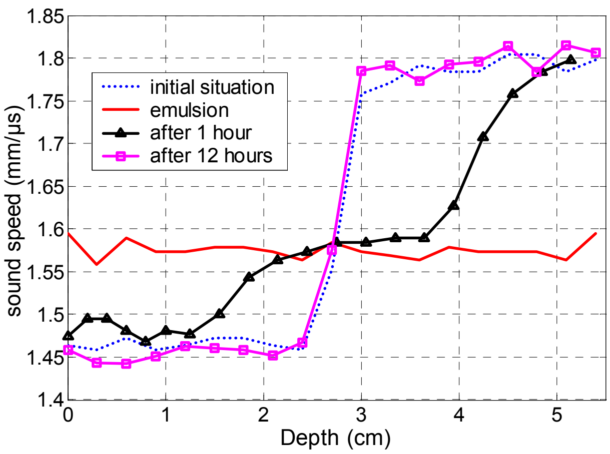


Figure 5 : Sound speed profiles obtained as the medium under investigation changes. Initially, there are two well-separated layers (oil/syrup). Then they are mixed together to form an apparently homogeneous emulsion. Progressively, the two phases of the emulsion separate again, and the process can be monitored by the sound speed profiles. Twelve hours later, the separation is complete.

Conclusion

We have given a physical interpretation of the emergence of the direct Green's function from the correlations of wavefields received on passive sensors. The experimental results we have presented here confirmed that it is possible to use this property and take advantage of multiple scattering to do "passive imaging". This is particularly interesting in applications, like seismology, where the source (earthquakes) cannot be controlled, whereas the wavefield is measured at several receiving points (seismic stations). This technique could be applied to retrieve the direct Green's function between all the passive sensors, which would be the first step to a "passive tomography". The interest of one-bit correlation was also demonstrated

Further studies are needed to establish a more rigorous theoretical basis, to examine the possibility of retrieving not only the sound speed but also the attenuation, and to predict exactly what amount of averaging is necessary in order to retrieve enough information about the direct Green's function.

Acknowledgments

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