# THE VIBRATION BEHAVIOUR AND DESIGN OF LANGEVIN TRANSDUCER WITH RADIAL COUPLING

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## Abstract

The vibrational behavior of sandwich transducer with coupled vibration is investigated by using commercial FEM software. A design method for coupled vibration transducer is given by apparent acoustic velocity. And a design consideration is presented. The theoretical results are verified experimentally.

## Introduction

The sandwich transducer is the component in common use to produce high intensity ultrasound. It is comprised of the front metal mass, the back metal mass and the central piezoceramics sandwiched between two masses. Therefore sandwich transducer is a composite vibrator. Its traditional design method is based on one dimensional vibration theory under the condition that its lateral dimension is less than a quarter of longitudinal wavelength. In some recent ultrasonic applications such as cleaning and processing, transducers operate at high frequency. At the same time its lateral dimension can not be too small to ensure sufficient energy to be radiated. Thus radial coupling vibration is arisen in the sandwich transducer.

Coupled vibration has been attracting much attention for its importance in practical application. However, it is difficulty to obtain analytical solution due to its complexity. In the past many numerical methods have been proposed such as FEM. For designing transducer the numerical method is not convenient.

To overcome the shortcoming of numerical method Mori E.[1] proposed apparent elasticity method to analyze the coupling vibration between longitudinal and radial modes of short column. Ren[2] and Lin[3] generalized apparent elasticity method to piezoceramics and sandwich transducer and presented design method for symmetric transducer. Wang[4] made a correction for the design method by computing the apparent elasticity of piezoceramics with the second order radial mode for relatively large cross-section area. For understanding the performance of sandwich transducer with coupled vibration, Iula[5] investigated the vibrational behavior of symmetric transducers with different aspect ratios. In this paper from the application point of view, the vibrational behavior of transducers with common structure are investigated. A design method is given by apparent acoustic velocity and a design consideration is presented.

## Vibrational behavior

The basic structure of transducer is shown in Fig.1. And its coupled vibrational modes can be analyzed by commercial FEM software. One

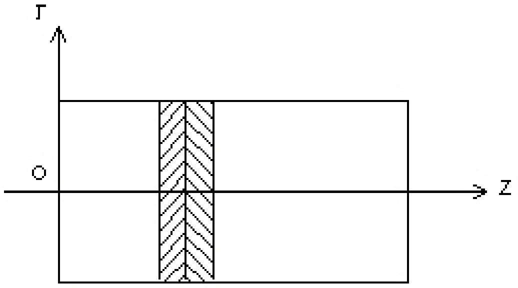
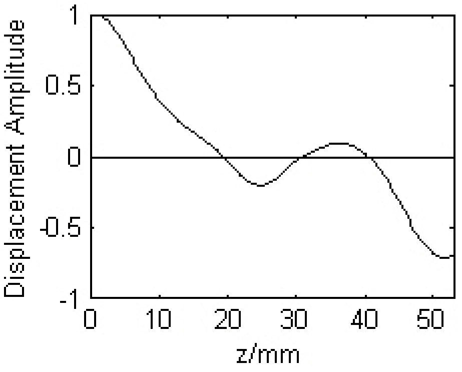


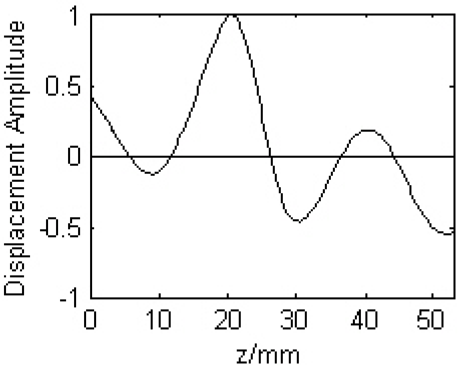
Fig.1 The basic structure of transducer

Calculation result is shown in Fig.2 and Fig.3 of a 131.30kHZ transducer with diameter 37mm comprised of aluminum front mass with length 23.5m, aluminum back mass with length 19.5mm and central PZT-4 piezoceramics with length 10mm. Fig.2 shows the distribution of longitudinal displacement amplitude along the axial direction(the amplitude are normalized) . Fig.3 shows the distribution of longitudinal displacement amplitude along radial in the radiating face.

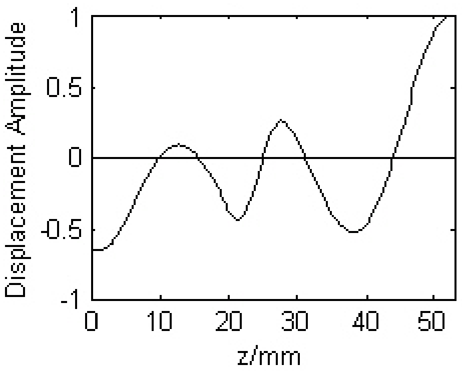
It can be seen from Fig.2 that the mode shapes are different at different places from the axial line due to radial coupling. From Fig.3 it can be seen that there exists a node circle in the radiating face, i.e. the vibrations of all the points in face are not in-phase which will decrease the radiating efficiency and is not desired. How to avoid this anti-phase vibration is a problem for designer.



(a) At axial line r=0



(b) At middle line r=9.25mm



(c) At the edge r=18.5mm

Fig.2 Displacement amplitude along axial direction

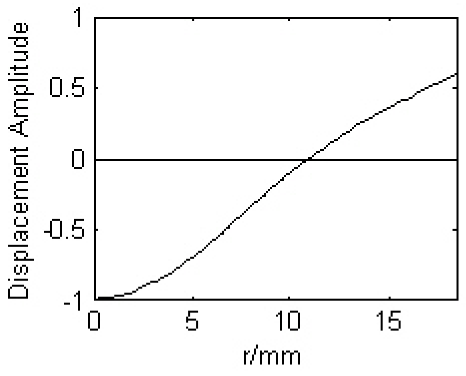
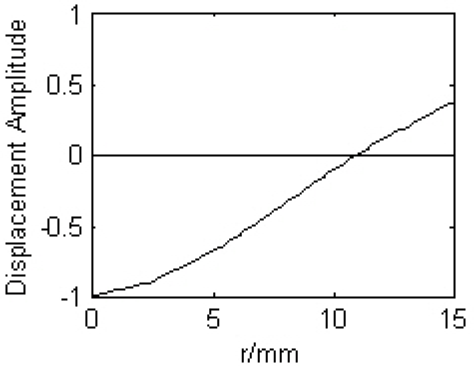


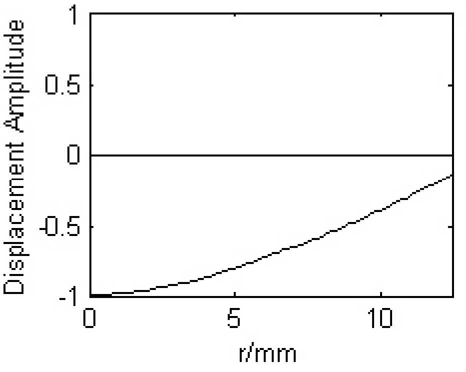
Fig.3 Displacement amplitude along radial direction

It can be seen from Fig.2 that the mode shapes are different at different places from the axial line due radial coupling. From Fig.3 it can be seen that there exists a node circle in the radiating face, i.e. the vibrations of all the points in face are not in-phase which will decrease the radiating efficiency and is not desired. How to avoid this anti-phase vibration is a problem for designer.

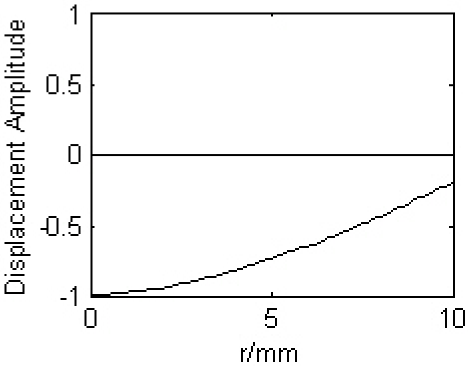
To understand the relation between the longitudinal amplitude distribution along radial direction and the transducer diameter the mode shapes are computed of 131.30KHZ transducer with different diameters. Fig.4 is the results of transducers with



(a) Diameter 30mm



(b)Diameter 25mm



(c)Diameter 20mm

Fig.4 Displacement amplitude along radial direction diameters 30mm， 25mm and 20mm respectively.

It can be seen that anti-phase vibration disappear when the transducer diameter reduced.

## Design method

Great error will arise when design coupled vibration transducer by one dimensional theory. Apparent elasticity method is a very good approximate method considering FEM is not convenient. The elasticity of a vibrator can be expressed by apparent elasticity due to coupling vibration according to Mori E.[1]. The apparent elasticity is related to the dimensions and the operating frequency of a vibrator.

The longitudinal wave velocity in a rod is an important material parameter when designing a transducer. The value of velocity is $\sqrt{E/\rho}$ in a slender rod， here E is the Young's modulus and $\rho$ the density. Since the elasticity can expressed by apparent elasticity then the value of velocity can also expressed by apparent acoustic velocity due to coupling vibration. Next the apparent acoustic velocity will be studied first， then the design method will be illustrated.

### *The apparent acoustic velocity*

The longitudinal apparent elasticity can be expressed by equation (1) of an isotropic metal cylinder[2]:

$$E_Z^S = \left\{ \frac{1}{E}\left[ 1 + \frac{2\sigma^2(1+\sigma)}{\left(R^S\right)^2 \big/ \left[a_2{}^2\omega_0{}^2\rho_2/E\right] - \left(1-\sigma^2\right)} \right] \right\}^{-1} \quad ,(1)$$

where E is Young's modulus， $\sigma$ Poisson's ratio， $\rho_2$ density， $a_2$ radius， $R^S$ the root of equation $xJ_0(x) = (1-\sigma)J_1(x)$ and $\omega_0$ the angular frequency.

The longitudinal apparent elasticity can be expressed by equation (2) for piezoceramics cylinder[2] of PZT-4 and PZT-8:

$$E_Z = \left\{ S_{33}^E \left[ 1 + \frac{2\sigma_{13}\sigma_{31}(1+\sigma_{12})}{R^2 \big/ \left[a_1{}^2\omega_0{}^2\rho_1 S_{11}^E\right] - \left(1-\sigma_{12}{}^2\right)} \right] \right\}^{-1} \quad ,(2)$$

where $S_{ij}^E$ is the elastic constants， $\sigma_{12} = -S_{12}^E/S_{11}^E$ ， $\sigma_{13} = -S_{13}^E/S_{11}^E$ ， $\sigma_{31} = -S_{13}^E/S_{33}^E$ ， $\rho_1$ is the density， $a_1$ the radius，and $R$ is the root of equation $xJ_0(x) = (1-\sigma_{12})J_1(x)$.

The apparent acoustic velocity can be calculated with formula $\sqrt{E_Z/\rho}$ by using equations (1) and (2) of metal and piezoceramics cylinders of a transducer.

As validation and insight into apparent acoustic velocity values of velocity are calculated of several metal cylinders. The values of apparent acoustic velocity are also calculated by using FEM. Table 1 are the results of apparent acoustic velocity calculated by apparent elasticity method (AEM) and FEM.

Table 1 Apparent acoustic velocity

| Length (mm) | Diameter (mm) | AEM (m/s) | FEM (m/s) | Error (%) |
|---|---|---|---|---|
| 127.5 | 3 | 5098 | 5099 | 0.02 |
| 127.5 | 30 | 5084 | 5079 | 0.10 |
| 127.5 | 63.75 | 5019 | 5005 | 2.80 |
| 127.5 | 127.5 | 4681 | 4645 | 0.77 |
| 20 | 40 | 3260 | 3222 | 1.20 |
| 4 | 40 | 696 | 696 | 0 |

It can be seen from table 1 that the both results agree well.

### *The method*

A design method is illustrated as following for sandwich transducer with coupled vibration:

(1) Frequency: It is assumed that the resonant frequency is 93.5kHz of the transducer to be designed.

(2) Material and some size parameters: The piezoceramics is chosen to be PZT-4(Density 7.5× $10^3$kg/m$^3$， elastic constants(× $10^{-12}$m$^2$/N) $S_{11}^E$=12.3， $S_{12}^E$=-4.05， $S_{13}^E$=-5.31， $S_{33}^E$=15.5，and Poisson's ratio $\sigma_{12}$=0.33， $\sigma_{13}$=0.43， $\sigma_{31}$=0.34)， diameter 35mm， length 10mm; Metal masses is chosen to be Aluminum(Density 2.7 × $10^3$kg/m$^3$ ， Young's

modulus 7.02×$10^{10}$N/$m^2$， Poisson ration 0.34)， the diameters are 37mm for both front and back metal mass.

(3) Calculating apparent acoustic velocity: By using the Poisson ratio 0.34 of aluminum the first root of equation $xJ_0(x)=(1-\sigma)J_1(x)$ is $R^S$=2.0735. The apparent acoustic velocity can then be calculated to be $c_Z^S$=2.087×$10^3$m/s by substituting $R^S$， the value of radius and material parameters into equation (1).

By using the Poisson ratio $\sigma_{12}$ =0.33 of piezoceramics the first root of equation $xJ_0(x)=(1-\sigma_{12})J_1(x)$ is 2.0674. Thus the apparent acoustic velocity is 7.81×$10^3$m/s accordingly. But this value of velocity is greater than 2.93×$10^3$m/s when there is no coupling vibration. This is meaningless. In this case the second root of above equation is chosen[4]， which is $R$=5.40. And the calculated acoustic velocity is $c_Z$=2.694×$10^3$m/s.

(4)Determining the length of back metal mass: The apparent wavelength in aluminum is calculated to be 22.32mm from the apparent acoustic velocity and frequency. The back metal mass will be too short if the length is chosen to be less than a quarter of wavelength. The length of back metal mass is chosen to be 25.6mm for engineering consideration.

(5)Calculating the length of front metal mass: By using similar design method as one dimensional theory the length of front metal mass can be calculated to be 25.6mm now that the apparent acoustic velocity and the lengths of back metal mass and piezoceramics are known.

Now the design of transducer is completed. The measured frequency is 92.8kHz of a trial-made transducer. The experimental result agree well with the theoretical one.

In addition， it is pointed out above that anti-phase vibrations may occur in the transducer radiating face if the transducer diameter is large. Study showed that the transducer radiating face will vibrate in-phase if only the apparent acoustic velocity $c_Z^S$ and the longitudinal acoustic velocity $c^S$ in slender rod of metal satisfy $0<c_Z^S<c^S$.

## Concluding remarks

In this paper the mode shapes of coupled vibrations are investigated of sandwich transducer. A design method based on apparent acoustic velocity is proposed. And a design consideration is presented for transducers with in-phase vibrations in radiating face. The coupled vibrations of sandwich transducer is very complicated. Further study is needed for designing transducers with excellent performance under coupling.

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