EVALUATION OF THE ACOUSTIC NONLINEAR PARAMETER IN SOLID PLATES.

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Abstract

In this work, a contact nonlinearity parameter β measurement for isotropic solid plates is described. A high frequency (HF) tone-burst signal of 15 MHz is inserted in the material by a contact-transducer. A low frequency (LF) pulse (1.5 MHz) is applied to the other side, in the opposite direction, such that the nonlinear interaction of the two waves takes place during the back propagation toward the HF transducer. This collinear interaction creates a phase modulation of the HF tone-burst which is directly proportional to the β coefficient and the particle velocity v_{LF} of the LF wave at the radiating surface. v_{LF} is determined by self reciprocity calibration of the transducer. The influence of diffraction effects and wave propagation in a solid medium on the phase modulation process is discussed. Measured values of β for fused silica and aluminum are corroborated by literature data.

Introduction

The measurement of the nonlinear acoustic parameter β is of main concern in many fields of application (NDT, biology and medicine…). In particular, it has been shown that this parameter is highly sensitive to slight alteration of the propagation medium such as micro-inhomogeneities [1]. There are several methods to perform the β parameter evaluation: measurement of the second harmonic component with a piezoelectric transducer or a capacitive detector [2], the measurement of the acoustic-radiation stress [3], or parametric interactions [4].

In this work, a contact phase modulation method is presented. This method is an extension of an idea introduced by Barrière and Royer [5] to measure the nonlinearity parameter in liquids. The phase modulation induced on a HF toneburst by its collinear interactions with a low frequency pulse is measured. As the phase modulation index is proportional to the nonlinear parameter of the sample and the particle velocity of the LF wave, the former can be determined if the LF transducer is calibrated.

Firstly, the validity of the phase modulation index expression derived for the interaction of two plane longitudinal waves is examined in the case of wave propagation in an isotropic solid medium.

Secondly, the particle velocity of the contact LF transducer is determined by a self reciprocity calibration method [2] extended in time domain.

Finally, a validation of the method is presented, through nonlinear parameter measurements in fused silica and aluminum showing a good agreement with literature.

Phase modulation method principle

The proposed method is based on the interaction of a low frequency pulse and a high frequency acoustic tone-burst with $f_{HF}>>f_{LF}$. Their collinear interaction ($\theta = 0$) results in a phase modulation of the HF wave (Fig. 1) with a modulation index given by:

$$\Delta\Phi = \frac{\omega_{HF} d}{c_l^2} \beta v_{LF} \qquad (1)$$

where the nonlinear parameter β in solid is defined by:

$$\beta = -\left(\frac{3}{2} + \frac{C_{111}}{2C_{11}}\right) \qquad (2)$$

C_{11} and C_{111} are respectively a second and Brugger's third order elastic modulus [6], ω_{HF} is the angular frequency of the HF wave, d the sample thickness, and c_l the longitudinal wave velocity.

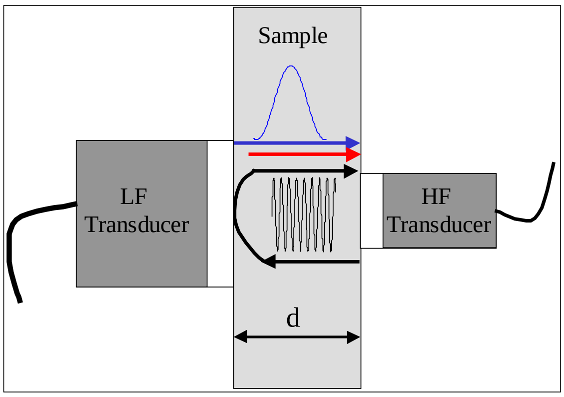

Figure 1 : Principle of the contact phase modulation method.

To evaluate the β coefficient, the ratio $\Delta\Phi / v_{LF}$ has to be determined. Here, v_{LF} is known by calibration of the low frequency transducer, and $\Delta\Phi$ is evaluated by demodulation.

Eq. (1) was initially derived in the case of plane wave propagation in fluids [5]. In order to justify its use in the present study, the interaction of two elastic plane waves in an isotropic solid is investigated in the case of weak nonlinear propagation.

Diffraction effects of the low frequency pulse on phase modulation

It is known [5,7] that both attenuation and diffraction of the high frequency tone-burst have a minor influence on the modulation process as soon as $f_{HF}>>f_{LF}$. Diffraction effects of the LF pulse are investigated in the following section.

Plane wave interaction in isotropic solids

A transducer laid in contact with a solid enables various wave types generation: longitudinal direct and edge waves, shear edge waves, and head waves (Fig. 2). The latter ensure continuity between longitudinal and shear edge waves.

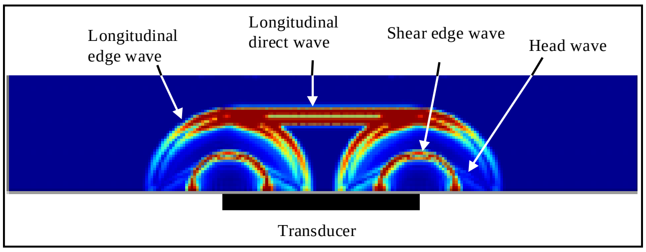

Figure 2 : Calculated radiation of a transducer laid in contact with an isotropic solid.

Now, the diffraction effects are discussed through the consideration of the interactions between the various wave types propagating in an isotropic solid.

Considering elastic plane waves (longitudinal L, and shear S) in an isotropic solid, the only possible interactions must satisfy the following conservation laws [8]:

$$\begin{aligned} \omega_{\pm} &= \omega_{HF} \pm \omega_{LF} \\ \vec{k}_{\pm} &= \vec{k}_{HF} \pm \vec{k}_{LF} \end{aligned} \qquad (3)$$

where $\omega_{\pm}$ and $\vec{k}_{\pm}$ are the sum and difference angular frequencies and wave vectors, respectively.

If $f_{HF} >> f_{LF}$ and $c_s/c_l \approx 0.5$ (shear to longitudinal phase velocities ratio), the only allowed interactions are :

$$L+L \rightarrow L \qquad (4a)$$
$$L+S \rightarrow L \qquad (4b)$$

where L and S refer to Longitudinal and Shear waves respectively.

Furthermore, the shear edge pulse propagates slowly and so, does not mix with the longitudinal HF tone-burst in our experimental setup (aluminum $c_s = 3150$m/s, transducer radii HF: 3.1 mm, LF: 9.5 mm).

Thus, the only interactions to consider in our modulation measurements are those between the HF longitudinal tone-burst and the LF longitudinal pulses (Eq. 4a), that are direct and edge waves.

Influence of longitudinal edge waves

Based on a previous study upon the interactions between two intersecting ($\theta \neq 0$) longitudinal plane waves [8], a nonlinear parameter $\beta(\theta)$, depending on the second and third orders elastic moduli of the material, has been introduced in the equation giving the phase modulation index of the HF tone-burst:

$$\Delta\Phi = \frac{\omega_{HF} d}{c_0} - \frac{\omega_{HF}}{c_0^2} \int_0^d \beta(\theta(z))\, v_{LF}(z,t)\, dz \qquad (5)$$

The nonlinear parameter $\beta(\theta)$ for aluminum is plotted in Fig. 3 as a function of the angle.

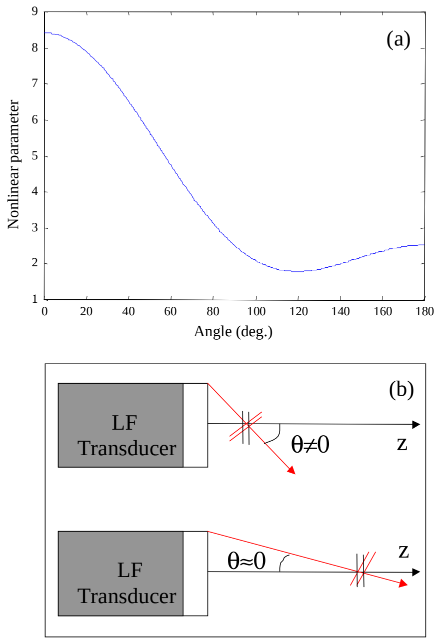

Figure 3 : Nonlinear parameter in aluminum as a function of the interaction angle θ between two longitudinal plane waves.

When the angle between the two waves increases, the nonlinear parameter value is decreasing. Consequently, the edge waves contribution can be neglected near the LF transducer, and increases with the axial distance as represented in Fig. 3.

Hence, when transducers are close to each other (thin samples), the direct wave is the only one that matter in the phase modulation process. This allows defining the validity field of Eq. 1.

Low frequency transducer calibration

In order to determine the LF velocity emitted in the sample by the LF transducer, a self-reciprocity calibration is used [2, 9]. The low frequency velocity at the transmitter surface is given by [9]:

$$v_{LF}(t) = IFFT\left[I_{out}(f)H_v(f)\right] \qquad (6)$$

where:

$$H_v(f) = \sqrt{-\frac{1}{2AZ_0}\frac{E_{in}}{I_{out}}\frac{1}{D(f)}} \qquad (7)$$

is a transfer function, A is the transducer active surface, Z_0 is the acoustic impedance, D is the exact diffraction correction, and:

$$E_{in} = V_{in} - \frac{V_{out}}{I_{out}} I_{in} \qquad (8)$$

The needed electrical measurements (I_{in}, V_{in}, I_{out}, and V_{out}) are made as shown in Fig. 4. More detailed information can be found in reference [9].

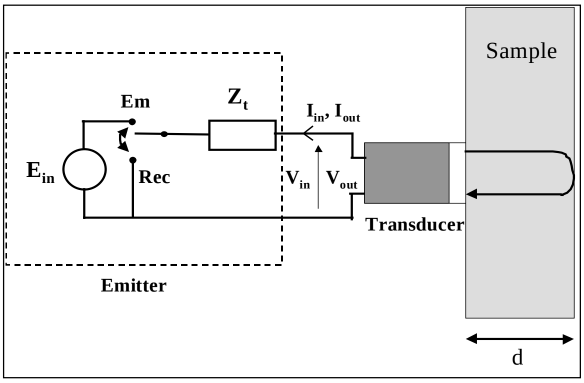

Figure 4 : Self-reciprocity calibration setup

Experimental results

The experimental set-up is described in Fig. 5. First, the LF transducer (1.5 MHz, 0.75” diameter) is laid in contact with the sample. It is driven by a burst generated by a HP 3314A waveform generator and amplified by an ENI A150 power amplifier (55 dB). Its calibration is made by electrical measurement as described in the preceding paragraph. The measured surface velocity of the LF transducer as a function of the power amplifier input voltage is shown in Fig. 6.

Then, a planar contact transducer (0.25” diameter) placed on the opposite side of the sample is driven by a 15 MHz toneburst created by a HP 3314A waveform generator. An HP 33120A function generator trig the LF burst emission in order to create collinear interactions between the LF burst and the HF toneburst during its back propagation toward the HF

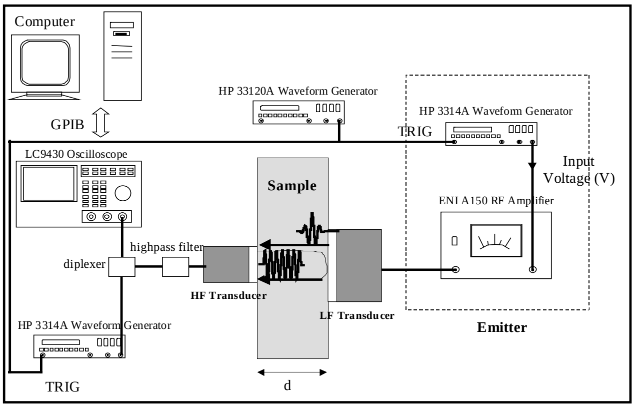

Figure 5 : Experimental apparatus of the contact phase modulation method.

transducer. The induced phase modulation of the HF received signal is extracted (Fig. 7) with a phase demodulation algorithm programmed in *Matlab*®.

Measured modulation index in a 3 cm Fused Silica sample for various input voltage are displayed in Fig. 7a. For each input voltage, it is shown that only the phase modulation amplitude changes, not its form. Moreover, comparison of the modulation index measured in Fused Silica and Aluminum (Fig. 7b) shows that the present method is sensitive to the nonlinear parameter sign.

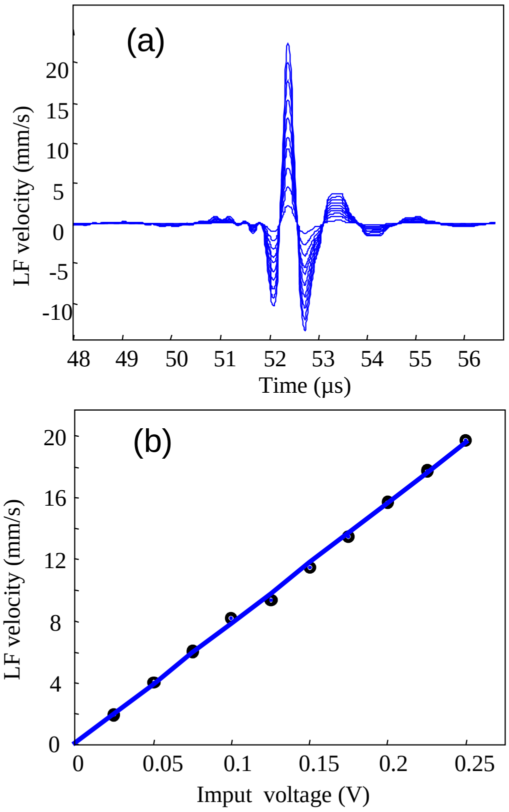

Figure 6 : LF transducer surface velocity versus: (a) time, (b) the amplifier input voltage.

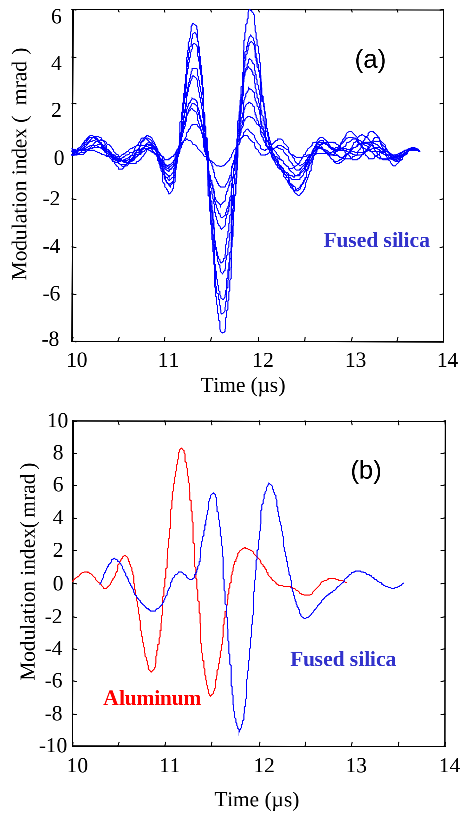

Figure 7 : Modulation index versus time.
(a) in fused silica, for various input voltages ;
(b) comparison between aluminum and fused silica.

For each input voltage, $\Delta\Phi$ and v_{LF} are evaluated and the modulation index variations as a function of the LF particle velocity is displayed, as shown in Fig. 8 for the two samples. The ratio $\Delta\Phi/v_{LF}$ is then extracted from the linear fit of measurements made with various input voltages. The obtained nonlinear parameters for the Fused Silica and Aluminum are shown in Tab. 1, and compare favorably with other measurements described in the literature.

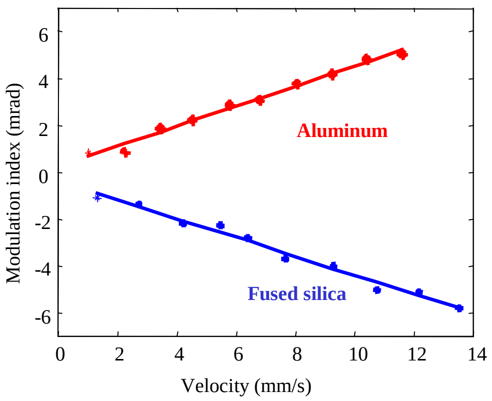

Figure 8 : Modulation index of the HF signal for fused silica and aluminum.

Table 1 : Comparison of measured nonlinearity parameter with literature values. (slope and associated standard deviation)

β	Present Work	Ref. [2]	Ref. [3]	Ref. [4]
Fused Silica	-6.0±0.3	-6.2±0.1	-6.35±0.7	-4.49
Aluminum	5.9±0.2	4.5±0.2		

Conclusion

A contact phase modulation method has been used to measure "in situ" the nonlinearity parameter of solid plates without immersing the sample. This method is able to determine the sign of the nonlinear parameter and has been validated on samples of Fused Silica (negative β) and Aluminium (positive β). The main perspective of this method is the contact characterisation of fatigue in glass and composite samples.

References

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