STUDY OF A "FABRY-PEROT" STRUCTURE CONSISTING OF TWO PHONONIC CRYSTALS

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Abstract

We present the first experimental and theoretical study of resonant tunneling of acoustic waves across a double barrier. The two barriers consist of two identical 2D square phononic crystals immersed in water and brought together to form a Fabry-Pérot cavity. The individual phononic crystals present a stop band around 500KHz in the [10] direction. The transmission measurements performed across this structure in the range of the stop band exhibit Fabry-Pérot-like resonance peaks. At these resonances, the measured group time is much larger than in water and is found to increase with the thickness of the individual crystals, which is related to the increasing time-of-life of the resonant state. On the contrary, outside resonances, the group time is less than in water which is characteristic of a tunneling effect.

Introduction

Phononic band gap crystals are periodic structures, analogous to photonic crystals for electromagnetic waves, that forbid propagation of acoustic waves for a certain range of frequencies. Although a lot of theoretical work has been performed on this subject in the past decade, there have been relatively few experimental studies. These ones have mainly focused on 2D structures. Only recently, transmission of ultrasonic waves through 3D periodic structures has been studied both experimentally and theoretically.

Here, we experimentally show that when two phononic crystals are brought together to form a resonant cavity, the transmission is enhanced at its resonant frequencies. Similar results have been reported for microwaves [1] but here we include an analysis of the time-of-life associated to each resonance. For each of them, we indeed measure the group time. The position of the transmission peaks is well explained using the model of resonant tunneling of an electron across a double barrier in quantum mechanics, but we have to introduce absorption in a simple 1D acoustic model in order to explain the saturation of the peak group time with the individual sample size.

Methods

Each phononic crystal consists of a periodic square arrangement of steel rods immersed in water. The rod diameter is 0.8 mm whereas the lattice constant is 1.5 mm. Thus the surface fraction (22 %) is not enough to observe a complete band gap but it does not matter for our study which only requires evanescent waves in one direction, here [10]. To build a first resonant cavity, we have separated an 8-row sample from the middle with a separation width d of 14 mm. The sample was placed at the Fresnel distance from a 38 mm-diameter planar transducer to ensure a planar illumination to a good approximation. The central frequency of this transducer is 500 KHz and its bandwidth is 60% at -6 dB. We used it to send a short pulse consisting of a few cycles. As a receiver, we used a 2.5-mm-diameter transducer with the same central frequency placed far away from the sample. To digitize the signals, we use an A/D converter manufactured by Picosecond Corp. which samples up to 100 MHz. As our sample is not perfectly periodic, the transmitted signals are averaged out on different positions of the sample to build the coherent wave, i.e. the wave which remains after averaging over disorder. We present in Fig. 1.a and Fig. 1.b typical signals transmitted in water and through a double 3-layer sample.

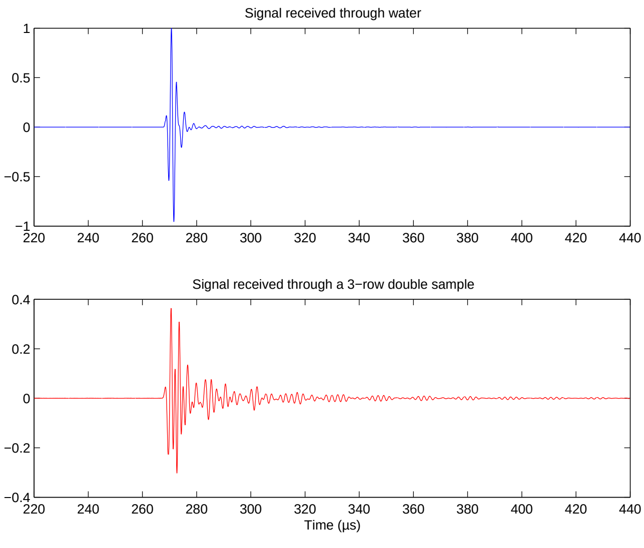

Figure 1: Signals received through water (top) and through a double 3-rows sample (bottom).

We determine the transmission coefficient from the amplitudes of the average transmitted and input signals at each frequency using a Fourier transform technique. Results are presented in Fig. 2.

In the range of the gap, one observes resonant peaks which manifest themselves in the time domain as

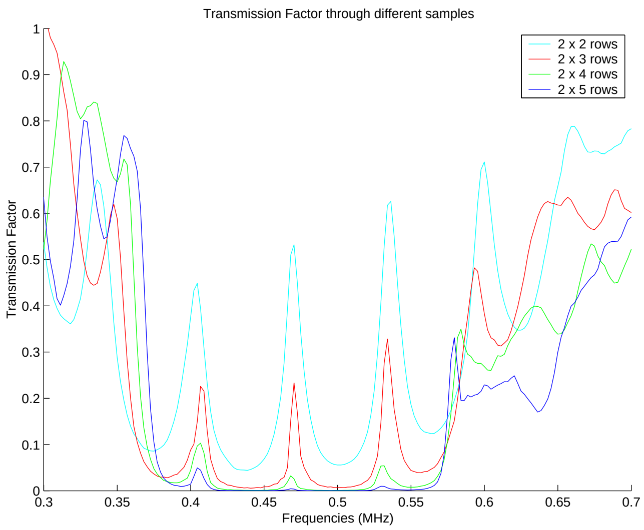

Figure 2: Transmission factor through four different samples of increasing size. Resonant peaks tend to disappear when individual sample size is increased.

wavepackets separated by the time necessary to go forth and back inside the cavity (Fig. 1b). The resonant peaks arise at frequencies sligthly different from the ones planned for a Fabry-Pérot interferometer of thickness d, i.e. $\nu_n = n\frac{C}{2d}$ where C is the velocity in the cavity and d the cavity width. Indeed, the theoretical frequency separation between resonances would be 70 kHz whereas one found 60 kHz experimentally. These results can be well explained in the framework of tunneling through a double barrier in quantum mechanics. Indeed, as we show now, the resonances should appear exactly for distances between the two barriers equal to multiple integer of half a wavelengh in the limit of infinite barrier height. The analogy between a quantum barrier and a phononic band gap has been justified by Page et al. in an earlier paper [2] where they have shown that the group velocity inside the gap increases linearly with the sample thickness (for sufficiently thick samples) which is characteristic of tunneling effect. Consider a plane wave e^{ikx} incident on a potential barrier of width a. In the first medium, the resulting wave is the sum of incident and reflected wave. Inside the barrier (in which the wave number is K), there are only evanescent waves and finally, the wave in the third medium is the transmitted one. The amplitudes in the first and third medium are related by the tranfer matrix M so that $\begin{pmatrix} 1 \\ r \end{pmatrix} = M \begin{pmatrix} t \\ 0 \end{pmatrix}$. If we apply the quantum mechanics boundary conditions, that field and its derivative are continuous at the boundaries. It follows that $M_{11} = M_{22}^* = \left(ch(Ka) + i\frac{\epsilon}{2}sh(Ka)\right)e^{ika}$ and $M_{12} = M_{21}^* = i\frac{\eta}{2}sh(Ka)$ with $\epsilon = \frac{K}{k} - \frac{k}{K}$ and $\eta = \frac{K}{k} + \frac{k}{K}$. The results for a double barrier can be extrapolated from the previous one in defining the translation matrix $T = \begin{pmatrix} e^{ikd} & 0 \\ 0 & e^{-ikd} \end{pmatrix}$. Hence: $\begin{pmatrix} 1 \\ r \end{pmatrix} = MT^{-1}MT\begin{pmatrix} t \\ 0 \end{pmatrix}$. After some calculations, the transmission coefficient t is then found to be $t = \frac{e^{-2ika}}{1+sh^2(Ka)\left(1-\frac{\epsilon^2}{4}\right)+i\epsilon ch(Ka)sh(Ka)+sh^2(Ka)\left(1+\frac{\epsilon^2}{4}\right)e^{2ikd}}$ If $Ka >> 1$, the range terms e^{2Ka} cancel if: $e^{2ikd} = -\frac{(1-\epsilon^2/4)+i\epsilon}{(1+\epsilon^2/4)}$ i.e., assuming $\epsilon^2/4 > 1$ as in our experiments, under the condition : $kd = \frac{1}{2}atan\left(\frac{4\epsilon}{4-\epsilon^2}\right) + n\pi$

If $\epsilon = 0$ or if $K \to \infty$ (large barriers), we encounter the classical Fabry-Pérot resonance condition $d = n\lambda/2$. In our experiment, we found $K \approx 0.4\,mm^{-1}$for the three resonant frequencies in the gap, which gives a theoretical separation of 61 kHz between them, value compatible with the experimental results. Following the procedure described in [3], we also calculated the group time as a function of frequency by digitally filtering the transmitted signal with a narrow band gaussian filter, so that we could accurately define the arrival time of the resulting wave packet. The results are presented in Fig. 3 and 4.

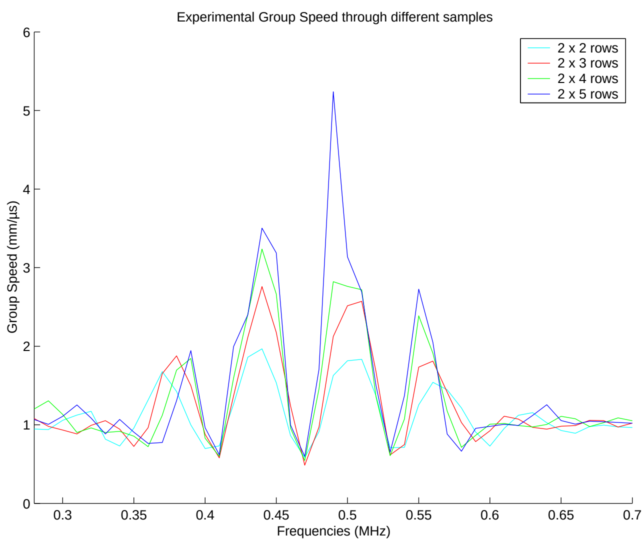

Figure 3: Group velocity measured through different samples of increasing size. “Slow Sound” appears at resonances while “Fast Sound” occurs in between.

At the resonances, the group time is found to be larger than in water which indicates that some time-of-life can be associated to each resonance. On the contrary, outside the resonances, we find group time less than in water which corresponds to a superluminal group velocity, as encoutered in quantum tunneling. These results can be explained at least qualitatively in the framework of our quantum model. Around the resonance, we can expand the phase factor $e^{2ikd} = e^{2ik^*d}(1 + 2i\Delta kd)$. Then, the transmission becomes :

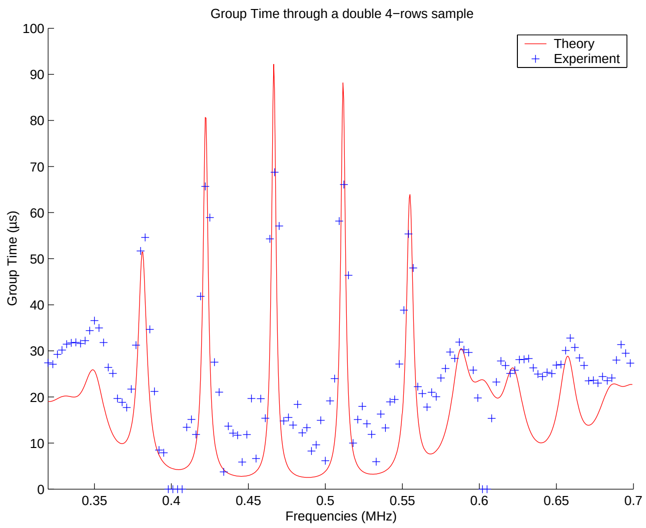

Figure 4: Group time measured through a double 4-rows sample (blue crosses) compared to the predictions of the 1D model (red curve).

$t = \frac{e^{-2ika}}{1-s^2\left[(1-\frac{\epsilon^2}{4}+i\epsilon)\right]2i\Delta kd}$ from which we deduce the resonance width $\Delta k = \frac{2e^{-2Ka}}{d\left(1+\frac{\epsilon^2}{4}\right)}$. From this result, we obtain the delay time associated to the resonance width $\frac{1}{c}\frac{d\varphi}{dk} \approx \frac{d}{c}e^{2Ka}$, where φ denotes the phase, which is exponentially big compared to the propagation time in water $\frac{d}{c}$. Although we do observe an increase of the group time for the smallest thicknesses, this exponential behavior is not retrieved in our experiments and a saturation occurs for larger thickness (see Fig. 5).

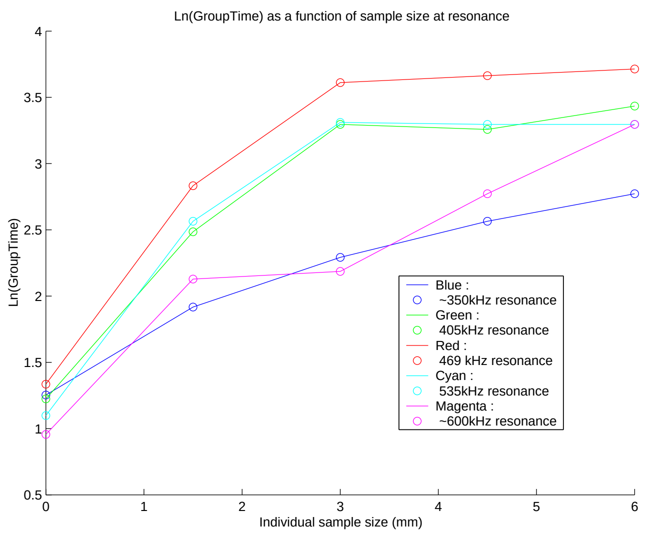

Figure 5: Evolution of group time at different resonance frequencies with individual sample size.

This effect can be ascribed mainly to absorption and to a lack of periodicity of our samples. Indeed, whatever effect cuts the longest paths corresponding to the multiple forth and back in the cavity contributes to artificially decrease the time-of-life of the resonances.

In order to resolve the influence of absorption in our samples, we have built a simple 1D model based on the use of acoustic impedances in a multilayer sample, following the procedure described in [4]. Absorption was then easily introduced by making the wave number go complex, thus the phase speed and the impedances too. The best match between the experimental results and the theoretical ones was found with no absorption in $75\mu m$ thickness steel layers and a global figure of $6m^{-1}$ for “absorption” in water (see Fig.6 for a comparison between the experimental and theoretical transmission curves). This value for absorption may seem a little high compared to the tabulated value of absorption in water at 0.5MHz, but one has to keep in mind that this figure accounts for many other effects including local scattering from impurities and diffraction.

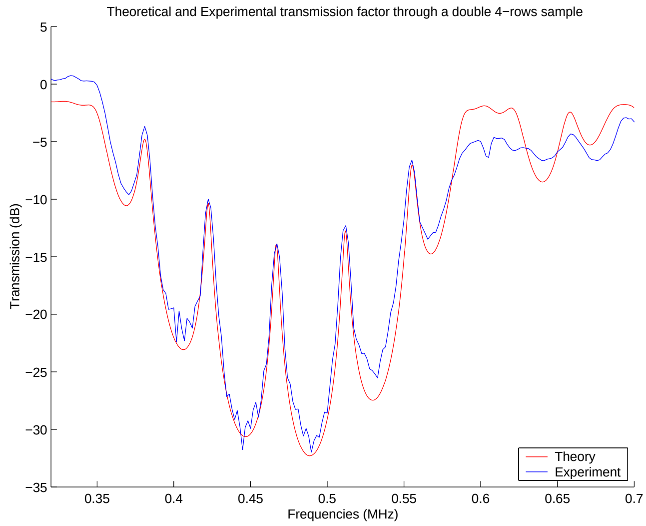

Figure 6: Measured transmission factor through a double 4-rows sample (blue curve) compared to the theoretical prediction of the 1D model (red curve).

This model allows us to draw the theoretical evolution of the group time at one particular resonance with the individual sample size at any absorption rate (Fig.7). The saturation of the group time at resonance is well explained by this simple model.

We have presented both theoretical and experimental results for the resonant tunneling of acoustic waves across a double barrier made of two phononic band gaps. The transmission measurements performed across this structure show resonant peaks whose distance between them is well described invoking a simple quantum analogy. Compared to the classical Fabry-Pérot relation, a correction depending on the penetration depth in the individual crystal is introduced. At the resonances, the measured group time exceeds the one in water by a factor depending on the associated time-of-life whereas apart, the group time is less than in water because of tunneling. A phenomenon that cuts the

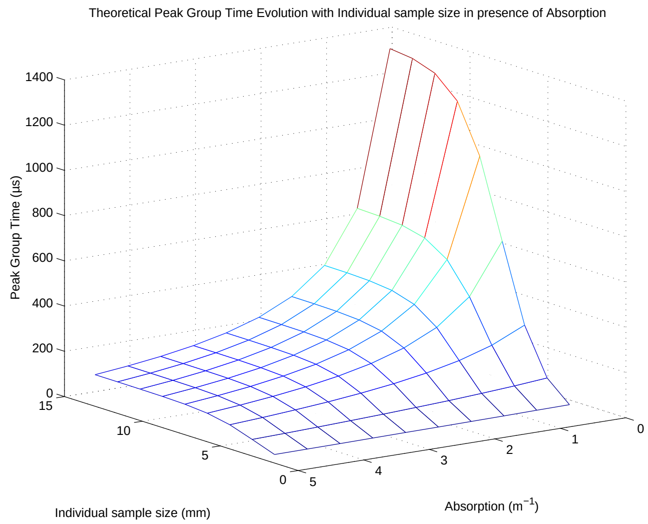

Figure 7: Theoretical dependance of the group time at 469kHz (central peak) with individual sample size and absorption.

longest paths for the acoustic wave has to be introduced in order to explain the saturation of the group time at resonance with the sample size.

References

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