REFLECTION OF A LAMB WAVE IN A BEVELLED PLATE : ULTRASONIC FIELD DESCRIPTION

N. Wilkie-Chancellier, H. Duflo, A. Tinel and J. Duclos
Laboratoire d'Acoustique Ultrasonore et d'Electronique (L.A.U.E.) UMR CNRS 6068,Université du Havre, Le Havre, France
jean.duclos@univ-lehavre.fr

Abstract

We present a two-dimensional study of mode conversions that occur when a harmonic incident wave is reflected at the bevelled edge of a steel plate. A harmonic Lamb wave (A_1 mode) is excited in a steel plate at a frequency-thickness product FE. Firstly, expressing the stress nullity at the end of the plate, the characteristics of the reflected waves (A_0, S_0 and A_1 modes) are computed. Then, the theoretical amplitude shape is composed in the thickness of the plate, particularly on the bevel. So, the Lamb mode ultrasonic field in the bevelled plate is known. These results are compared with those obtained by a finite element simulation. More, energy balances are computed for several bevel angle values by the both methods at FE=2.7 MHz mm.

Introduction

The aim of this paper is the study of the reflection of a Lamb mode at the end of a bevelled plate (angle from 70 to 90 degrees). The reflected wave characteristics are investigated in the case of the harmonic antisymmetric A_1 mode for a frequency-thickness product FE equal to 2.7 MHz mm. The edge of the bevelled end plate has no symmetry and we can attempt reflected modes having the two symetries (antisymmetric and symmetric). The reflection coefficients of the reflected modes (A_0, A_1, S_0 modes) are computed by expressing the stress nullity at the end of the plate. Then, we can compose the theoretical amplitude at any point and determine the mode ultrasonic field at any time in the plate. In order to justify the theoretical calculus, a finite element simulation is also carried out. Thus, a comparison is done between the numerical and theoretical results. The results obtained by both methods are compared in different domains

Theoretical Investigation

The Lamb wave conversion study has been initiated by Torvik [1] who assumes that a harmonic Lamb wave which propagates in a semi-infinite free plate is converted in Lamb waves at the straight cut. We extend to the bevelled plate case the theory carried out by Shen and al [2-3]. We consider an incident wave which is reflected at the end of a stainless steel plate (C_L=5850 m.s^{-1}, C_T=3150 m.s^{-1}) (*Figure 1*). This wave gives rise to few reflected waves having the both symmetries (antisymmetric or symmetric).

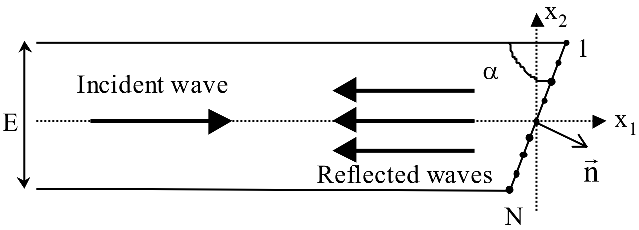


Figure 1: Geometry description

In an infinite plate of thickness E, real, complex and purely imaginary harmonic modes can exist depending on whether their wave vector component $\underline{K}_1$ is real, complex or purely imaginary at the frequency F. A finite number of real solutions of the equation of dispersion has been found for a given frequency-thickness product (FE). These solutions are associated to the Lamb waves. An infinite number of complex modes also exist.

In order to describe each mode at a frequency-thickness product, a Fortran program computes successively the wave number $\underline{K}_1 = K_1^{'} + j\,K_1^{''}$, the amplitudes $\underline{U}_i\left(x_1, x_2, t\right)$, the stresses $\underline{T}_{ij}\left(x_1, x_2, t\right)$ (i and j=1, 2) and the Poynting vector flow Φ through a straight section of the plate (1 meter wide).

The boundary condition has to be written to study the reflection phenomenon. This boundary condition is the nullity of the components $\underline{F}_1$ and $\underline{F}_2$ of the force $\vec{\underline{F}}$ resulting from Lamb waves on the inclined section [1]. These components depend on the stresses $\underline{T}_{11}$, $\underline{T}_{12}$, $\underline{T}_{21}$ and $\underline{T}_{22}$ and the components n_1, n_2 of the normal to the surface (Figure 1):

$$\begin{cases} \underline{F}_1 = \underline{T}_{11}n_1 + \underline{T}_{12}n_2 = \underline{T}_{11}\sin(\alpha) - \underline{T}_{12}\cos(\alpha) = 0 \\ \underline{F}_2 = \underline{T}_{21}n_1 + \underline{T}_{22}n_2 = \underline{T}_{21}\sin(\alpha) - \underline{T}_{22}\cos(\alpha) = 0 \end{cases}$$

In order to express the boundary conditions at the bevelled edge of the plate, a sufficient large number of reflected modes must be used. If we only use the real modes, the energy conservation can not be verified. So, to satisfy this principle, we add to the first Lamb modes some complex modes. Their real part $K_1^{'}$ is positive (reflected modes) or negative (incident mode) and their imaginary part $K_1^{''}$ is positive and weak. For FE=2.7 MHz mm, Lamb modes and the first additional complex modes are represented in the complex plane on Figure 2. At this FE product, purely imaginary modes do not exist.

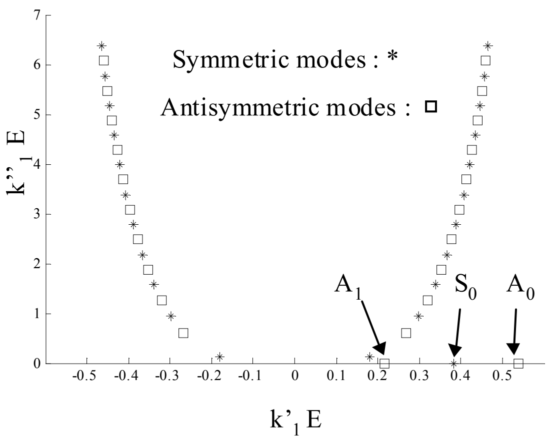


Figure 2: Eigen modes in the complex plane (FE=2.7 MHz mm)

In order to determine the wave amplitudes, we use a collocation method. Weighting coefficient are assigned to the wave displacements, respectively 1 to the incident wave and $\underline{r}_m$ to the mth-additional wave (incident and reflected). The stresses of the incident and reflected waves are numerically computed at N points uniformly distributed at the inclined extremity (Figure 1).So, we suppose M modes are added to the incident mode and the stresses of these modes are calculated at the N points. The stresses nullity at the bevelled extremity is N times written as follows :

$$\begin{cases} \underline{F}_1^{inc}(i) + \sum_{m=1}^{M} \underline{r}_m . \underline{F}_1^m(i) = 0 \\ \underline{F}_2^{inc}(i) + \sum_{m=1}^{M} \underline{r}_m . \underline{F}_2^m(i) = 0 \end{cases} \quad (i = 1 \text{ to N})$$

In order to obtain a good convergence of the $\underline{r}_m$ solutions, we solve the equations with N=800 points and M=45 modes.

As soon as the problem is solved [4-5], we can compute the ultrasonic field representation. We note ${}^{inc}\underline{U}_i^{A_1}$ and ${}^{inc}\underline{U}_i^{m}$ the incident displacement profiles of the A_1 mode and the mth additional modes. If ${}^{ref}\underline{U}_i^{A_1}$, ${}^{ref}\underline{U}_i^{A_0}$, ${}^{ref}\underline{U}_i^{S_0}$ and ${}^{ref}\underline{U}_i^{m}$ are respectively the reflected displacement profiles of the A_1, A_0, S_0 mode and the mth additional modes, we can compose the theoretical incident and reflected amplitude shapes (${}^{inc}\underline{U}_i^{tot}$ and ${}^{ref}\underline{U}_i^{tot}$) in the plate by the relations:

$$\begin{cases} {}^{inc}\underline{U}_i^{tot} = {}^{inc}\underline{U}_i^{A_1} + \sum_{m=1}^{M/2} \underline{r}_m \, {}^{inc}\underline{U}_i^{m} \\ {}^{ref}\underline{U}_i^{tot} = \underline{r}_{A_1} \, {}^{ref}\underline{U}_i^{A_1} + \underline{r}_{A_0} \, {}^{ref}\underline{U}_i^{A_0} + \underline{r}_{S_0} \, {}^{ref}\underline{U}_i^{S_0} + \sum_{m=1}^{M/2} \underline{r}_m \, {}^{ref}\underline{U}_i^{m} \end{cases}$$

So, we have a description of the ultrasonic field plate [6-7].

More, we must check that the energy of the incident Lamb wave is wholly transported by the reflected waves. Because the complex modes do not transport energy, only reflected real modes have to be taken in consideration in this sum. Indeed, far from the end of the plate, only Lamb waves exist.

In order to realise an energy balance, the Poynting vector flow of the mth reflected and the incident modes are respectively noted P_m and P_{inc} ($P_{inc} = \phi_{inc}$, $P_m = r_m . r_m^* \, \phi_m$, where r_m^* is the complex conjugate value of r_m). So, the reflection coefficients can be written: $R_m = \frac{P_m}{P_{inc}} = r_m . r_m^* \frac{\phi_m}{\phi_{inc}}$. By this way, the relative energy of each reflected mode is computed and finally an energy balance can be done.

In the last part of this paper, results are presented for a Lamb wave incident in a stainless steel bevelled plate.

Finite element computation

A finite element simulation has been used to compare with the previous results. Computations are made with the ANSYS Finite Element code. The studied stainless steel plate is 40 mm long and 2 mm thick and its characteristics are: Young modulus E=2.0043.10^{11} N.m^{-2}, Poisson coefficient σ=0.29 and density ρ=7800 kg.m^{-3}.

To describe the Lamb wave propagation, a modelisation by a two-dimensional mesh (Ox_1x_2) is sufficient. The limited plate mesh consists of rectangular element (400×20). Of course, the mesh is finer in the bevel. With this kind of mesh, whatever Lamb wave is propagating, the node density is more than twenty elements per wavelength.

To generate a Lamb mode in the plate with the Finite Element Method, a transient analysis method is performed. The normal and tangential components of the theoretical Lamb wave displacements are imposed at the first plate nodes (Fig 3). A fifteen period burst is applied in a rectangular window. So, the amplitude of each mode present in the plate must be constant along the burst (end and beginning excepted). Then we collect the temporal evolution of the displacements at the surface of the plate in order to obtain a time-space image.

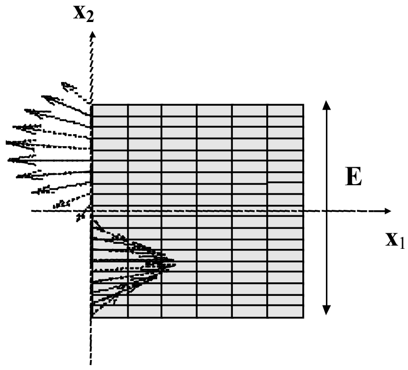


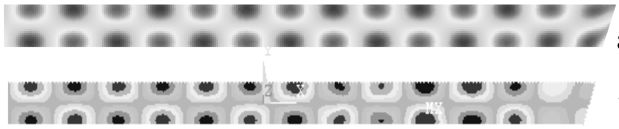
Figure 3: First displacement profile.

By the meaning of two successive Fast Fourier transforms (temporal and spatial), the waves present in the plate can be performed in the dual k-FE space [5;8]. On this representation, we can perform a cut at a precise FE product. So, the relative amplitudes can be known versus the wave number and linked to the transported energy [9]. Indeed, the normal displacements at the surface of the plate are linked to the corresponding energy by the mean of the Poynting vector flow through a straight section of the plate. So, for different bevelled plates, energy balances are performed and compared to the theoretical studies.

Several comparisons

Now, let us consider the A_1 Lamb mode in the stainless steel plate incident at FE=2.7 MHz mm and reflected at a 70 degrees bevel. We can compare the ultrasonic field computed by the both methods: theory and finite element investigation (Figure 4 and 5).

On the Figure 4, we present the visualisation obtained when the incident wave arrives at the bevelled edge. The comparison between the images a (theoretical computation) and b (finite element simulation) is not possible at any point. Indeed, the finite element computation does not give a perfect permanent sinusoidal mode. The comparison between the incident waves is only possible where the sinusoidal mode is established (so without the beginning and the end of the burst). In this domain, we observe similar periodicity for the displacements. However, grey scales are not exactly the same.



a

b

U_1 displacements

Figure 4 : Incident displacement shapes obtained by both methods : theory (a) and Finite Element (b) (A_1 incident mode, FE=2.7 MHz mm, α=70 degrees)

The comparison (Fig.5) between the reflected waves is more difficult. At a given time, a common spatial domain is necessary in which the several reflected Lamb modes are established. This domain is surely more limited than for the incident modes.

a

b

U_1 displacements

Figure 5 : Reflected displacement shapes obtained by both methods : theory (a) and Finite Element (b) (A_1 incident mode, FE=2.7 MHz mm, α=70 degrees)

The images (a and b) differ because the FEM computations are less precise: mesh size and calculation time are limited.

The second comparison we have done is about the energy balances. Indeed, computations have been performed when the A_1 Lamb mode is incident at a frequency-thickness product FE equal to 2.7 MHz mm in plates bevelled by different angles (α from 70 to 85 degrees). The energy curves of each reflected mode (A_0, S_0 and A_1 modes for this FE product) are reported in Figure 6. Results obtained by the both methods are presented.

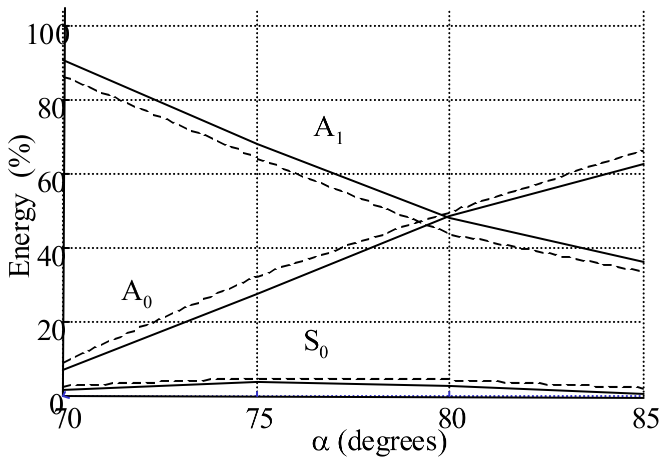


Figure 6: Comparison Theory (-) – Finite Element (--) A_1 incident mode, FE=2.7 MHz mm.

We can see in this quantitative study that the finite element computations confirm the theoretical results obtained to the collocation method. Indeed, the trend of the curves is the same in both cases.

Conclusion

This paper presents a study of the Lamb wave conversion at the bevelled edge of a stainless steel plate. The study includes a theoretical calculus and a finite element computation. They give the same energy balance within a few percents. The theoretical calculus shows that reflection of Lamb waves is well described by a sum of numerous eigen modes of the plate. Among these, the complex modes have a great importance at the bevel.

References

[1] P.J. Torvik, *"Reflection of Wave Trains in Semi-Infinite Plates"*, Journal of the Acoustical Society of America, Vol.41, 1967, pp 346-353.

[2] S.Y. Zhang, J.Z. Shen, C.F. Ying, *"The reflection of the Lamb Wave by a free plate edge: Visualization and theory"*, Materials Evaluation, Vol. 46, 1988, pp 638-641.

[3] J.Z. Shen, S.Y. Zhang, C.F. Ying, *"The reflection of the Lamb wave in a semi-infinite plate"*, Chinese J. Acoustics, Vol. 9, 1990, pp 27-35.

[4] N. Wilkie-Chancellier, H. Duflo, A. Tinel, J. Duclos, "Theoretical study of Lamb wave conversion at the edge a different angles bevelled plates", Forum Acusticum, Seville, 17-21 September 2002, Proceedings on CD-ROM.

[5] B. Morvan, N. Wilkie-Chancellier, H. Duflo, A. Tinel and J. Duclos, "Lamb wave reflection at the free edge of a plate", Journal of the Acoustical Society of America, 2003, 113(3), pp.1417-1425.

[6] T. Hayashi, S. Endoh, "Calculation and visualization of Lamb wave motion", Ultrasonics (2000), 38, pp. 770-773.

[7] H.U. Li, K. Neghishi, "Visualization of Lamb mode patterns in glass plate", Ultrasonics (1994), Vol.32 (4), pp. 243-248.

[8] D. Alleyne. and P. Cawley, *"A two-dimensional Fourier transform method for the measurement of propagating multimode signals"*, Journal of the Acoustical Society of America, 1991, 89(3), pp.1159-1168.

[9] N. Wilkie-Chancellier, "Réflexion et conversion d'une onde de Lamb à l'extrémité biseautée d'une plaque", Thesis, Université du Havre, France, 2003.